Romanian Reports in Physics, Vol. 64, No. 3, P. 663–671, 2012

THEORETICAL PHYSICS

OPTICAL GAUSSONS IN BIREFRINGENT FIBERS AND DWDM SYSTEMS WITH INTER-MODAL DISPERSION

LAILA GIRGIS[1], DANIELA MILOVIC[2], SWAPAN KONAR[3],
AHMET YILDIRIM[4,5], HOSSEIN JAFARI[6], ANJAN BISWAS[7]

[1]Delaware State University, Department of Mathematical Sciences,
Dover, DE 19901-2277, USA
[2]University of Nis, Faculty of Electronic Engineering, Department of Telecommunications,
Aleksandra Medvedeva 14, 18000 Nis, SERBIA
[3] Birla Institute of Technology, Department of Applied Physics,
Mesra, Ranchi-87131, INDIA
[4]Ege University, Department of Mathematics,
35100 Bornova, Izmir, Turkey, E-mail: ahmetyildirim80@gmail.com
[5]University of South Florida, Department of Mathematics and Statistics,
Tampa, FL 33620-5700, USA, E-mail: ahmetyildirim80@gmail.com
[6]University of Mazandaran, Department of Mathematics, Babolsar, IRAN
[7]Delaware State University, Department of Mathematical Sciences,
Dover, DE 19901-2277, USA
E-mail: biswas.anjan@gmail.com

Received December 24, 2011

Abstract. This paper studies the optical solitons in birefringent fibers and DWDM systems in presence of log-law nonlinearity with inter-modal dispersion. The Gaussian ansatz is used to carry out the integration of the governing equation. The exact solutions are obtained and the constraint conditions, for the existence of these Gaussons, fall out during the course of derivation of the solution. A brief discussion on Thirring solitons is also included.

Key words: optical solitons, birefringent fibers, DWDM systems, thirring solitons.

1. INTRODUCTION

Optical solitons is one of the major areas of research in the area of nonlinear fiber optics [1–20]. A lot of studies has been going on in the context of birefringent fibers as well as in the area of dense wavelength division multiplexed (DWDM) systems. It is interesting to note that most of these studies are being made with Kerr and other laws of nonlinearity. Surprisingly, there are very few papers that are published for birefringent fibers and DWDM systems with log law nonlinearity.

There are advantages of studying the optical solitons with log law nonlinearity. In this case, solitons are free from radiations and therefore there is no energy shedding during pulse transmission through optical fibers [1–6]. This paper will address this issue and an exact Gaussian soliton solution, or *Gausson* in birefringent fibers and DWDM systems. There will be several constraint conditions that will fall out during the course of derivation of the exact solutions. These constraint conditions must hold in order for the Gaussons to exist.

2. BIREFRINGENT FIBERS

An ideal circular-core fiber supports simultaneously supports one polarization mode which consists of two orthogonal polarization modes. However, optical fibers that are employed in modern optical communication systems are not perfectly circularly symmetric. This asymmetry introduces small refractive index differences for two polarization states known as birefringence. Due to this effect, one polarization mode will travel faster than the other and this phenomena is known as differential group delay.

Polarization mode dispersion (PMD) becomes very significant in high bit rate ultra long-haul optical communication systems that operate at bit rates in excess of 5 Gbps. In such systems PMD needs to be compensated by introducing certain degree of birefringence which will nullify the effect of PMD over transmission length. PMD compensation is an useful technique for error-free transmission through long-haul and metropolitan area networks at bit rates higher than 10 Gbps.

In soliton transmission systems, random birefringence causes solitons to generate dispersive waves, that degrade the transmission performance. Dispersive wave cause solitons to continuously lose energy, and thus induces pulse broadening. Additionally, these dispersive waves interact with other soliton pulses and cause distortion of a sequence of pulses.

One can overcome the problem with birefringence by using a polarization maintaining fiber which is a fiber with strong built-in birefringence, also known as high birefringence fiber, or single polarization mode fiber. Polarization of light launched into the fiber is aligned with one of the birefringent axes and this polarization state remains preserved even if the fiber is bent. The physical principle behind this can be understood in terms of coherent mode coupling. The propagation constants of the two polarization modes are different due to strong birefringence, so that the relative phase of such co-propagating modes rapidly drift away. Therefore, any disturbance along the fiber can effectively couple both modes only if it has significant spatial Fourier component with a wavenumber which matches the difference of the propagation constants of the two polarization modes. If this difference is large enough, the usual disturbances in the fiber very slowly vary to do effective mode coupling.

2.1. MATHEMATİCAL ANALYSİS

The governing equation for the propagation of Gaussons through birefringent optical fibers is given by the following dimensionless coupled vector NLSE [7]:

$$\mathrm{i}q_t + \mathrm{i}\alpha_1 q_x + a_1 q_{xx} + \left(b_1 \log|q|^2 + c_1 \log|r|^2\right) q = 0, \tag{1}$$

$$\mathrm{i}r_t + \mathrm{i}\alpha_2 r_x + a_2 r_{xx} + \left(b_2 \log|r|^2 + c_2 \log|q|^2\right) r = 0. \tag{2}$$

Here in (1) and (2), $q(x,t)$ and $r(x,t)$ represents the wave profile of the two split pulses, while a_l, for $l = 1, 2$ are the group velocity dispersion coefficients, b_l represent the coefficients of self-phase modulation (SPM) and c_l represent the coefficients of cross-phase modulation (XPM) and finally α_l are the coefficients of the inter-modal dispersion terms. The first term in (1) and (2) is the evolution term for the optical pulse.

In order to solve (1) and (2), the following hypothesis are made [1–6]:

$$q(x,t) = P_1(x,t)\,\mathrm{e}^{\mathrm{i}\phi_1} = A_1 \mathrm{e}^{-\tau^2} \mathrm{e}^{\mathrm{i}\phi_1}, \tag{3}$$

$$r(x,t) = P_2(x,t)\,\mathrm{e}^{\mathrm{i}\phi_2} = A_2 \mathrm{e}^{-\tau^2} \mathrm{e}^{\mathrm{i}\phi_2}, \tag{4}$$

where A_l are the amplitudes of the two polarized solitons and ϕ_l represents the phase components of the two pulses. Also,

$$\tau = B(x - vt), \tag{5}$$

where B is the width of the two pulses and v is the velocity with which the two polarized pulses travel. Moreover,

$$\phi_l = -\kappa_l x + \omega_l t + \theta_l, \tag{6}$$

for $l = 1, 2$. Here in (6), κ_l represents the frequency of the two solitons, ω_l are the wave numbers and θ_l are the phase constants.

Substituting (3) and (4) reduces (1) and (2) respectively to

$$\begin{aligned} &\mathrm{i}\frac{\partial P_l}{\partial t} - \omega_l P_l + a_l\left(\frac{\partial^2 P_l}{\partial x^2} - 2\mathrm{i}\kappa_l \frac{\partial P_l}{\partial x} - \kappa_l^2 P_l\right) + \\ &+\left(2b_l \ln P_l + 2c_l \ln P_{\bar{l}}\right) P_l + \mathrm{i}\alpha_l\left(\frac{\partial P_l}{\partial x} - i\kappa_l P_l\right) = 0, \end{aligned} \tag{7}$$

for $l = 1, 2$ and $\bar{l} = 3 - l$. Now decomposing (7) into real and imaginary parts respectively gives the following pair of equations

$$-\left(\omega_l+\alpha_l\kappa_l+a_l\kappa_l^2\right)P_l+\left(b_l+\lambda_l\kappa_l\right)P_l^3+c_lP_lP_{\bar{l}}^2+a_l\frac{\partial^2 P_l}{\partial x^2}=0 \tag{8}$$

and

$$\frac{\partial P_l}{\partial t}+\left(\alpha_l-2a_l\kappa_l\right)\frac{\partial P_l}{\partial x}=0. \tag{9}$$

Now from (9), using (3) and (4), gives the velocity of the Gaussons as

$$v=\alpha_l-2a_l\kappa_l \tag{10}$$

for $l=1,2$, while the real part given by (8) reduces to

$$\begin{aligned}&-\left(\omega_l+a_l\kappa_l^2+2a_lB^2-\alpha_l\kappa_l-2b_l\ln A_l-2c_l\ln A_{\bar{l}}\right)+\\&+\left(4a_lB^2-2b_l-2c_l\right)\tau^2=0.\end{aligned} \tag{11}$$

Hence, from the linearly independent functions, after setting their respective coefficients to zero yields

$$\omega_l=2\left(b_l\ln A_l+c_l\ln A_{\bar{l}}\right)-a_l\left(\kappa_l^2+2B^2\right)+\alpha_l\kappa_l \tag{12}$$

and

$$B=\sqrt{\frac{b_l+c_l}{2a_l}}. \tag{13}$$

Now, equating the velocities of the solitons from (10), for $l=1,2$, yields the constraint condition

$$\alpha_1-\alpha_2=2\left(a_1\kappa_1-a_2\kappa_2\right) \tag{14}$$

and then equating the width of the Gaussons leads to the constraint relation

$$\frac{b_1+c_1}{a_1}=\frac{b_2+c_2}{a_2}. \tag{15}$$

Another restriction that immediately follows from (13) is given by

$$a_l\left(b_l+c_l\right)>0 \tag{16}$$

for $l=1,2$. Hence finally, the Gaussons solutions in a birefringent fiber is given by (3) and (4) where the amplitudes A_l are arbitrary, while the width B and the Gausson velocity is given by (13) and (10) which leads to the constraint conditions (14–16) that must hold in order for the Gaussons to exist.

2.1.1. Integrals of motion

For birefringent fibers, there are at least two integrals of motion or the conserved quantities that are energy (E) and linear momentum (M) that are respectively given by [2]

$$E = \int_{-\infty}^{\infty} \left(|q|^2 + |r|^2 \right) \mathrm{d}x = \frac{1}{B}\sqrt{\frac{\pi}{2}} \left(A_1^2 + A_2^2 \right) \tag{17}$$

and

$$\begin{aligned} M &= \mathrm{i}\int_{-\infty}^{\infty} a_1 \left(u^* u_x - u u_x^* \right) + a_2 \left(v^* v_x - v v_x^* \right) \mathrm{d}x = \\ &= -\frac{\sqrt{2\pi}}{B} \left(a_1 \kappa_1 A_1^2 + a_2 \kappa_2 A_2^2 \right). \end{aligned} \tag{18}$$

2.2. THIRRING SOLITONS

This is a special case of solitons in birefringent fibers when the SPM is negligible and hence it is discarded. Thus the governing equations in this case, with the SPM term eliminated and XPM term term surviving, is given by [10]

$$\mathrm{i}q_t + \mathrm{i}\alpha_1 q_x + a_1 q_{xx} + c_1 \log|r|^2 q = 0, \tag{19}$$

$$\mathrm{i}r_t + \mathrm{i}\alpha_2 r_x + a_2 r_{xx} + c_2 \log|q|^2 r = 0. \tag{20}$$

Therefore, equations (19) and (20) are special cases of (1) and (2) where b_l are set to zero, for $l = 1, 2$. Hence, the Thirring Gaussons are still given by (3) and (4) where the velocity of the Thirring Gausson stay the same as in (10), while the wave numbers and the width of the solitons are given by

$$\omega_l = 2c_l \ln A_{\bar{l}} - a_l \left(\kappa_l^2 + 2B^2 \right) + \alpha_l \kappa_l \tag{21}$$

and

$$B = \sqrt{\frac{c_l}{2a_l}} \tag{22}$$

respectively, where $l = 1, 2$ and $\bar{l} = 3 - l$. Hence, the constraint conditions in this case reduce to

$$\frac{c_1}{a_1} = \frac{c_2}{a_2} \tag{23}$$

and

$$a_l c_l > 0 \tag{24}$$

while (14) stays intact, for Thirring Gaussons.

3. DWDM SYSTEMS

DWDM (dense wavelength division multiplexed system) is a fiber optic transmission technique that allows the transmission of a variety of information over the optical layer. The DWDM uses dispersion-flattened fibers where the dispersion weakly depends on operating wavelength.

DWDM technology is efficiently used for increasing the capacity and reliability of fiber optic communication systems. Unlike previous generation optical networks, where the information is carried by a single light beam, DWDM carves up large bandwidth of an optical fiber into many wavelength channels making spectral band use more efficient. Each of the optical carriers wavelength carries information individually but their spacing needs to be properly chosen to avoid inter-channel interference. DWDM can increase the information carrying capacity by about 10-100 times without the need of a new optical fiber.

DWDM systems are also bit rate and format independent and can accept any combination of interference rates on the same fiber at the same time. This technology can be applied to different areas in the telecommunications networks, that includes the backbone networks, the residential access networks and also local area networks.

3.1. MATHEMATICAL ANALYSIS

For a DWDM system, the corresponding coupled dimensionless NLSE that dictates the propagation of solitons through log law optical fibers is given by

$$iq_t^{(l)} + i\alpha_l q_x^{(l)} + a_1 q_{xx}^{(l)} + \left\{ b_l \log\left|q^{(l)}\right|^2 + \sum_{n \neq l}^{N} \lambda_{ln} \log\left|q^{(n)}\right|^2 \right\} q^{(l)} = 0, \tag{25}$$

where $1 \le l \le N$. Equation (25) is the model for bit-parallel WDM soliton transmission. Here λ_{ln} are known as the XPM coefficients. In equation (25), $q^{(l)}$ represents the dimensionless form of the wave profile in the l^{th} component. Also, α_l and b_l are the inter-modal dispersion and SPM coefficients. Finally, λ_{ln} are the XPM coefficients. Again, here in (25), the first term represents the evolution of the pulse in the l^{th} channel.

In order to solve (25), the following hypothesis is picked [1–7, 12]

$$q^{(l)}(x,t) = P_l(x,t)\,\mathrm{e}^{\mathrm{i}\phi_l} = A_l \mathrm{e}^{-\tau^2}\mathrm{e}^{\mathrm{i}\phi_l}, \tag{26}$$

for $1 \le l \le N$. Then, substituting (26) into (25) gives

$$\begin{gathered}\mathrm{i}\frac{\partial P_l}{\partial t} - \omega_l P_l + a_l\left(\frac{\partial^2 P_l}{\partial x^2} - 2\mathrm{i}\kappa_l\frac{\partial P_l}{\partial x} - \kappa_l^2 P_l\right) + \\ +2\left(b_l \ln P_l + \sum_{n\neq l}^{N}\lambda_{ln}\ln P_n\right)P_l + \mathrm{i}\alpha_l\left(\frac{\partial P_l}{\partial x} - \mathrm{i}\kappa_l P_l\right) = 0.\end{gathered} \tag{27}$$

Similarly, as in the case of birefringent fibers, discussed in the previous section, decomposing (27) into real and imaginary parts and carrying out the analysis yields

$$v = \alpha_l - 2a_l\kappa_l \tag{28}$$

$$\omega_l = 2\left(b_l \ln A_l + \sum_{n\neq l}^{N}\lambda_{ln}\ln A_n\right) - a_l\left(\kappa_l^2 + 2B^2\right) + \alpha_l\kappa_l \tag{29}$$

and

$$B = \sqrt{\frac{b_l + \sum_{n\neq l}^{N}\lambda_{ln}}{2a_l}}. \tag{30}$$

for $1 \le l \le N$. Hence, these induce the constraint relations

$$\alpha_1 - 2a_1\kappa_1 = \alpha_2 - 2a_2\kappa_2 = \cdots = \alpha_N - 2a_N\kappa_N, \tag{31}$$

$$\frac{1}{a_1}\left(b_1 + \sum_{n=2,\,n\neq 1}^{N}\lambda_{ln}\right) = \frac{1}{a_2}\left(b_2 + \sum_{n=1,\,n\neq 2}^{N}\lambda_{ln}\right) = \cdots = \frac{1}{a_2}\left(b_2 + \sum_{n=1,\,n\neq N}^{N-1}\lambda_{ln}\right), \tag{32}$$

and

$$a_l\left(b_l + \sum_{n\neq l}^{N}\lambda_{ln}\right) > 0, \quad \text{for } 1 \le l \le N. \tag{33}$$

3.1.1. Integrals of motion

In this case also, there are at least two conservation laws, namely the energy (E) and linear momentum (M) that are given by [2]

$$E = \sum_{l=1}^{N}\int_{-\infty}^{\infty}\left|q^{(l)}\right|^2 \mathrm{d}x = \frac{1}{B}\sqrt{\frac{\pi}{2}}\sum_{l=1}^{N}A_l^2 \tag{34}$$

and

$$M = \mathrm{i}\sum_{l=1}^{N}\int_{-\infty}^{\infty} a_l \left\{ q^{(l)*} q_x^{(l)} - q^{(l)} q_x^{(l)*} \right\} \mathrm{d}x = -\frac{\sqrt{2\pi}}{B}\sum_{l=1}^{N} a_l \kappa_l A_l^2 . \qquad (35)$$

4. CONCLUSIONS

The dynamics of optical Gaussons in birefringent fibers, and DWDM systems, is studied in this paper. An exact Gausson solution is obtained in each of these cases. The constraint conditions fell out during the course of derivation of the exact solutions. These constraints must remain valid for the propagation of Gaussons down the optical fibers. Additionally, a succinct discussion on Thirring solitons is included as a by-product of birefringent fibers.

These exact Gausson solutions in the context of birefringent fibers and DWDM systems, are new and are being reported for the first time in this paper. These exact solutions will be extremely useful in future in further studies with Gaussons. For example, the quasi-stationary Gaussons will be obtained by the aid of multiple-scale analysis, in presence of several other perturbation terms. Additionally, these exact Gaussons will be of great help in formulating the quasi-particle theory, in the study of soliton-soliton interactions. These results will all be reported in future publications.

Acknowledgments. The work of the second author (DM) presented in this paper was supported by the Serbian Ministry of Education and Science (Project III44006) and this support is sincerely appreciated.

REFERENCES

1. I. Bialynicki-Birula, J. Mycielski, Physica Scripta, **20**, 539–544 (1979).
2. A. Biswas, D. Milovic, Communications in Nonlinear Science and Numerical Simulation, **15**, 3763–3767 (2010).
3. A. Biswas, C. Cleary, J. E. Watson Jr., D. Milovic, Applied Mathematics and Computation, **217**, 2891–2894 (2010).
4. A. Biswas, C. M. Khalique, Waves in Random and Complex Media, **21**, 554–558 (2011).
5. A. Biswas, M. Fessak, S. Johnson, S. Beatrice, D. Milovic, Z. Jovanoski, R. Kohl, F. Majid, Optics and Laser Technology, **44**, 1775–1780 (2012).
6. A. Biswas, D. Milovic, R. Kohl, *Inverse Problems in Science and Engineering* (in press).
7. D. Grecu, A. Visinescu, R. Fedele, S. D. Nicola, Romanian Journal of Physics, **55**, 585–600 (2010);
   D. Grecu *et al*., Romanian Journal of Physics, **55**, 980–994 (2010).
8. P. Guerrero, J. L. Lopez, J. Nieto, Nonlinear Analysis: Real World Applications, **11**, 79–87 (2010).
9. M. H. Jakubowski, K. Steiglitz, R. K. Squier, Complex Systems, **10**, 1–21 (1996).
10. S. Jana, S. Konar, Physics Letters A, **362**, 435–438 (2007).
11. R. Kohl, A. Biswas, D. Milovic, E. Zerrad, Optics and Laser Technology, **40**, 647–662 (2008).

12. H. Leblond *et al*., Mathematics and Computers in Simulation, **69**, 378–388 (2005); I. V. Melnikov *et al*., Physical Review A, **56**, 1569–1576 (1997).
13. H. Leblond, D. Mihalache, Romanian Reports in Physics, **63**, 1254–1266 (2011); D. Mihalache, Romanian Reports in Physics, **63**, 9–24 (2011).
14. H. Leblond *et al*., Physical Review A, **83**, 063802 (2011).
15. H. Leblond *et al*., Optics Communications, **285**, 356–363 (2012).
16. I. V. Mel'nikov *et al*., Optics Communications, **181**, 345–351 (2000).
17. D. Mihalache, Proceedings of the Romanian Academy A, **11**, 142–147 (2010).
18. D. Mihalache, Journal of Optoelectronics and Advanced Materials, **13**, 1055–1066 (2011); E. A. Ostrovskaya *et al*., IEEE J. Select. Topics in Quantum Electron., **8**, 591–596 (2002); D. Mihalache, D. Mazilu, L. Torner, Phys. Rev. Lett., **81**, 4353–4356 (1998); L. Torner *et al*., Opt. Commun., **138**, 105–108 (1997).
19. A. B. Nassar, Physical Review A, **33**, 3502 (1986).
20. J. Oficjalski, I. Bialynicki-Birula, Acta Physica Polonica B, **9**, 759–775 (1978).