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TOPOLOGICAL AND NON-TOPOLOGICAL SOLITONS OF A GENERALIZED DERIVATIVE NONLINEAR SCHRÖDINGER'S EQUATION WITH PERTURBATION TERMS

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Abstract. This paper studies a generalized form of the derivative nonlinear Schrödinger equation, describing Alfven waves in plasmas with perturbation terms. The perturbation terms that are considered are dissipative term and higher order nonlinearity term. The solitary wave ansatz is used to carry out the integration and the exact topological and non-topological soliton solutions are obtained. Parametric restrictions for the existence of the exact solutions are given.

Key words: topological and non-topological solitons, nonlinear Schrödinger equation.

1. INTRODUCTION

The rapid progress over the past two decades in the theory of solitons has led to the research of new dynamical models, governing the propagation of nonlinear waves in many physics areas [1–29]. Several nonlinear evolution equations (NLEEs) have been formulated depending on the physical situation. Besides, many such equations are generalized to study their general behaviour so that the special cases are truly meaningful both from the physical and mathematical point of view. As a matter of fact, these models exhibit a rich variety of shape-preserving waves with interesting properties, opening the door to a series of applications in various fields.

The aspect of integrability of NLEEs is one of the most important questions of nonlinear science and theoretical physics because of its direct connection with the understanding of the physical phenomena and dynamical processes in nonlinear dynamical systems. In many practical physics problems, the resulting nonlinear wave equations of interest are nonintegrable [15]. In some particular cases they may be close to an integrable one [15]. It is remarkable that non-integrability is not necessarily related to the nonlinear terms [16]. Higher order dispersions, for example, also can make the system to be non-integrable (while it remains Hamiltonian) [16]. The finding of soliton solutions for such models is of particular interest, because the soliton approach is universal in different fields of modern physics. These exact solutions when they exist can help one to well understand the mechanism of the complicated physical phenomena and dynamical processes modelled by these nonlinear evolution equations [23].

As is well known, a soliton is a pulse that propagates in a dispersive medium in such a way that a nonlinear effect compensates dispersion, and the pulse remains unchanged during propagation [14]. Solitons are ubiquitous in nature, appearing in diverse systems such as shallow water waves, DNA excitations, matter waves in Bose-Einstein condensates, and ultrashort pulses (or laser beams) in nonlinear optics [6].

Optical solitons in Kerr nonlinear media have been the subject of intense current research motivated by their important applications to high-capacity fiber telecommunications and to all optical switches due to their capability of propagating long distances without attenuation and changing their shapes [8]. In the picosecond domain, the propagation of optical soliton pulses in the single-mode optical fibers is governed by the nonlinear Schrödinger (NLS) equation. This equation is completely integrable by the inverse scattering transform. This means that it is possible to find both solitary wave and multi-soliton solutions [16]. The key factors, which determine the solutions of the NLS equation, are the dispersion D and nonlinear N coefficients. Physically these two coefficients represent the curvature of the frequency versus wave number dispersion and the change in the carrier frequency with signal amplitude, respectively [5]. The cubic NLS equation takes the form [5]:

$$\mathrm{i}E_z - \frac{\beta_2}{2}E_{tt} + \gamma|E|^2 E = 0, \tag{1}$$

where $E(z,t)$ is the slowly varying amplitude of the pulse envelope, β_2 is the group velocity dispersion (GVD) coefficient, and γ is the nonlinear parameter responsible for self-phase modulation (SPM). z represents the distance along the direction of propagation, and t represents the retarded time (measured in the group velocity frame). By means of the coupled amplitude-phase formulation, fundamental bright and dark soliton solutions have been obtained for both anomalous dispersion regimes ($\beta_2 < 0$) and normal dispersion regimes ($\beta_2 > 0$) in Ref. [7].

Recently, a large variety of powerful methods were developed to obtain exact solutions of many NLEEs of all kinds *e.g.* the nonlinear Schrödinger equation, the Korteweg-de Vries equation, the Boussinesq equation and many others. Among these methods we can cite the coupled amplitude-phase formalism [7, 18], the hyperbolic tangent method, Hirota bilinear method [24], the sub-ODE method [22], the solitary wave ansatz method [2–5] and other analytical methods as well. Moreover, several numerical methods, such as the Petrov-Galerkin method [12], the collocation method [13], were employed for numerical treatments of the nonlinear problems. However, some of these analytical and numerical solutions methods are not easy to use and sometimes require from tedious works and calculations. It is then essential to use appropriate techniques without much complicated calculations for the obtention of explicit solutions of NLEEs of physical relevance. What is important here is whether the method is efficient to construct closed form solutions of a given nonlinear evolution equation. The solitary wave ansatz method [2–5], and other methods of integrability have shown great success and progress in this area of research. The solitary wave ansatz method rather heuristic and possesses significant features that make it practical for the determination of soliton-type solutions for a wide class of NLEEs in a direct method. This technique has recently been applied successfully to wide range of NLEEs [2–5].

In this paper, the solitary wave ansatz will be exploited to find the exact soliton solutions of a generalized form of the derivative nonlinear Schrödinger equation with generalized evolution term and pertubation contributions in the form of a dissipative term and higher order nonlinearity effect.

The considered generalized derivative NLS equation with perturbation terms is as follows:

$$\mathrm{i}\left(q^m\right)_t+\alpha\left(q^m\right)_{xx}+\mathrm{i}\beta\left(|q|^{2n}q^m\right)_x=\mathrm{i}\gamma|q|^{2n}\left(q^m\right)_x+\delta|q|^{4n}q^m. \tag{2}$$

Here in (2), the first term is the generalized evolution term, while the second and third terms respectively represent the dispersion and nonlinear terms. The perturbation terms on the right-hand side of Eq. (2) represent the dissipative term and the higher order nonlinearity term. Also, $\alpha,\ \beta,\ \gamma,\ \delta\in R$ are constants, while $m,\ n\in\ Z^+$. When the constants $\gamma=\delta=0,$ this equation is the generalized form of the so-called derivative NLS equation, where, in particular, the case $\alpha=\beta=m=n=1$ leads to the derivative NLS equation [1, 11]:

$$\mathrm{i}q_t+q_{xx}+\mathrm{i}\left(|q|^2q\right)_x=0. \tag{3}$$

Generally, Eq. (2) is not integrable. It is always useful to construct exact analytical solutions and it is worthwhile to investigate the exact solutions (in particular soliton solutions). In this paper, we deal with the existence of exact soliton solutions of Eq. (2) as it appears, namely for general values of m and n and in presence of perturbation terms. To our knowledge, the exact analytic soliton solutions of Eq. (2) have not previously obtained. Importantly, it is not possible to integrate (2) by the inverse scattering transform for any general values of the exponents m and n since the Painlevé test of integrability will fail in this situation.

Thus, equation (2) will be integrated in this paper, and the search will be for bright and dark solitons that are also known as non-topological and topological solitons respectively. These topological solitons are of interest only in the nonlinear optics community. The bright solitons for equation (2) models Alfven waves in plasmas, in presence of dissipation and higher order nonlinearity as described above [1]. Additionally, the equation represents the propagation of solitons through optical fibers for transcontinental and transoceanic distances in presence of perturbation terms that are given by self-steepening term (to avoid shock wave formation) and nonlinear dissipation. These two terms are respectively given by the coefficients of β and γ. The full nonlinearity factor m is considered since there can arise a situation, from real life scenario, where the evolution or the dispersion or the dissipation is not exact or linear due to the noise or fiber imperfections. Setting $m=1$ always collapses the analysis to the actual model. A simplified version of this equation with $m=n=1$ models wave propagation on discrete electrical transmission line based on the modified complex Ginzburg-Landau equation that is derived in the smal amplitude and long wavelength limit by the aid of standard reductive perturbation technique and complex expansion on the governing nonlinear equations [13].

2. NON-TOPOLOGICAL SOLITON SOLUTION

For solving Eq. (2), we use a solitary wave ansatz of the form

$$q(x,t)=\frac{A}{\left[D+\cosh\tau\right]^{p}}\mathrm{e}^{\mathrm{i}\varphi}, \tag{4}$$

where

$$\tau=B\left(x-vt\right) \tag{5}$$

and

$$\varphi=-kx+\omega t+\theta. \tag{6}$$

Here, in (4–6), A is the amplitude of the soliton, while v is the velocity and B is the inverse width of the soliton. Also, κ is the frequency of the soliton, while ω is the wave number of the soliton and θ is the phase constant. The exponent p is unknown at this point and its value will fall out in the process of deriving the solution of this equation. From the ansatz (4), one obtains

$$\left(q^m\right)_t = \left\{\frac{pmBvA^m \sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}} + \frac{\mathrm{i}m\omega A^m}{\left[D+\cosh\tau\right]^{pm}}\right\}\mathrm{e}^{\mathrm{i}m\varphi}, \tag{7}$$

$$\left(q^m\right)_x = \left\{-\frac{pmBA^m \sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}} - \frac{\mathrm{i}mkA^m}{\left[D+\cosh\tau\right]^{pm}}\right\}\mathrm{e}^{\mathrm{i}m\varphi}, \tag{8}$$

$$\begin{aligned}\left(q^m\right)_{xx} = &\left\{\frac{pm\left(pm+1\right)B^2A^m\left(D^2-1\right)}{\left[D+\cosh\tau\right]^{pm+2}} + \frac{A^m p^2m^2B^2}{\left[D+\cosh\tau\right]^{pm}} -\right.\\ &\left.-\frac{pm\left(2pm+1\right)DB^2A^m}{\left[D+\cosh\tau\right]^{pm+1}} + \frac{2\mathrm{i}kpmBA^m\sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}} - \frac{A^mk^2}{\left[D+\cosh\tau\right]^{pm}}\right\}\mathrm{e}^{\mathrm{i}m\varphi},\end{aligned} \tag{9}$$

$$\left\{\left|q\right|^{2n} q^m\right\}_x = \left\{-\frac{p\left(2n+m\right)A^{2n+m}\sinh\tau}{\left[D+\cosh\tau\right]^{p(2n+m)+1}} - \frac{\mathrm{i}kmA^{2n+m}}{\left[D+\cosh\tau\right]^{p(2n+m)}}\right\}\mathrm{e}^{\mathrm{i}m\varphi}, \tag{10}$$

$$\left|q\right|^{2n}\left(q^m\right)_x = \left\{-\frac{pmBA^{2n+m}\sinh\tau}{\left[D+\cosh\tau\right]^{p(2n+m)+1}} - \frac{\mathrm{i}kmA^{2n+m}}{\left[D+\cosh\tau\right]^{p(2n+m)}}\right\}\mathrm{e}^{\mathrm{i}m\varphi}. \tag{11}$$

Substituting Eqs. (7)-(11) into Eq. (2), we find

$$\begin{aligned}&\mathrm{i}\left\{\frac{pmBvA^m\sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}} + \frac{\mathrm{i}m\omega A^m}{\left[D+\cosh\tau\right]^{pm}}\right\} +\\ &+\alpha\left\{\frac{pm\left(pm+1\right)B^2A^m\left(D^2-1\right)}{\left[D+\cosh\tau\right]^{pm+2}} + \frac{A^mp^2m^2B^2}{\left[D+\cosh\tau\right]^{pm}} - \frac{pm\left(2pm+1\right)DB^2A^m}{\left[D+\cosh\tau\right]^{pm+1}} +\right.\\ &\left.+\frac{2\mathrm{i}kpmBA^m\sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}} - \frac{A^mk^2}{\left[D+\cosh\tau\right]^{pm}}\right\} +\\ &+\mathrm{i}\beta\left\{-\frac{p\left(2n+m\right)A^{2n+m}\sinh\tau}{\left[D+\cosh\tau\right]^{p(2n+m)+1}} - \frac{\mathrm{i}kmA^{2n+m}}{\left[D+\cosh\tau\right]^{p(2n+m)}}\right\} =\\ &= -\frac{\mathrm{i}\gamma pmBA^{2n+m}\sinh\tau}{\left[D+\cosh\tau\right]^{p(2n+m)+1}} + \frac{\gamma kmA^{2n+m}}{\left[D+\cosh\tau\right]^{p(2n+m)}} + \frac{\delta A^{4n+m}}{\left[D+\cosh\tau\right]^{p(4n+m)}}.\end{aligned} \tag{12}$$

Separating the real and imaginary parts, we get

$$\frac{pmBvA^m \sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}}+\frac{2k\alpha pmBA^m \sinh\tau}{\left[D+\cosh\tau\right]^{pm+1}}-\frac{\beta p\left(2n+m\right)A^{2n+m}\sinh\tau}{\left[D+\cosh\tau\right]^{p(m+2n)+1}}= \\ =-\frac{\gamma A^{2n+m}pm\sinh\tau}{\left[D+\cosh\tau\right]^{p(m+2n)+1}} \tag{13}$$

and

$$-\frac{m\omega A^m}{\left[D+\cosh\tau\right]^{pm}}+\frac{\alpha pm\left(pm+1\right)B^2A^m\left(D^2-1\right)}{\left[D+\cosh\tau\right]^{pm+2}}+\frac{\alpha A^m p^2m^2B^2}{\left[D+\cosh\tau\right]^{pm}}- \\ -\frac{\alpha pm\left(2pm+1\right)DB^2A^m}{\left[D+\cosh\tau\right]^{pm+1}}-\frac{\alpha A^m k^2}{\left[D+\cosh\tau\right]^{pm}}+\frac{\beta kA^{2n+m}}{\left[D+\cosh\tau\right]^{p(m+2n)}}= \\ =\frac{\gamma mkA^{2n+m}}{\left[D+\cosh\tau\right]^{p(m+2n)}}+\frac{\delta A^{4n+m}}{\left[D+\cosh\tau\right]^{p(m+4n)}}. \tag{14}$$

By equating the exponents $pm+1$ and $p\left(m+2n\right)$ in Eq. (14), we get

$$pm+1=p\left(m+2n\right). \tag{15}$$

This gives

$$p=\frac{1}{2n}, \tag{16}$$

with

$$n\neq 0. \tag{17}$$

It needs to be noted that the same value of p is yielded when the exponents $pm+2$ and $p\left(m+4n\right)$ are equated with each other.

Now, from (13), the linearly independent functions are $\sinh\tau/\left[D+\cosh\tau\right]^{pm+1}$ and $\sinh\tau/\left[D+\cosh\tau\right]^{p(m+2n)+1}$. Therefore, setting their respective coefficients to zero yields

$$v=-2k\alpha, \tag{18}$$

and

$$\frac{\gamma}{\beta}=\frac{2n+m}{m}, \tag{19}$$

which serves as a constraint relation between the model coefficients and the exponents. Also, from (14), the linearly independent functions are $1/\left[D+\cosh\tau\right]^{pm+j}$ for $j=0,1,2$ (with $pm+1=p\left(m+2n\right)$). Therefore, setting their respective coefficients to zero yields

$$-m\omega A^{m}+\alpha A^{m}p^{2}m^{2}B^{2}-\alpha A^{m}k^{2}=0, \tag{20}$$

$$\alpha pm\left(pm+1\right)B^{2}A^{m}\left(D^{2}-1\right)=\delta A^{4n+m}, \tag{21}$$

$$-\alpha pm\left(2pm+1\right)DB^{2}A^{m}+\beta kA^{2n+m}=\gamma mkA^{2n+m}. \tag{22}$$

Solving the above equations gives

$$\omega=\frac{\alpha\left(m^{2}B^{2}-4n^{2}k^{2}\right)}{4mn^{2}}, \tag{23}$$

$$D=\frac{\delta A^{2n}\left(m+n\right)+\sqrt{k^{2}\left(m+2n\right)^{2}\left(\beta-\gamma m\right)^{2}+\delta^{2}A^{4n}\left(m+n\right)^{2}}}{\left(m+2n\right)\left(\beta-\gamma m\right)k}, \tag{24}$$

$$B=nA^{n}\sqrt{\frac{2k\left(\beta-\gamma m\right)}{\alpha m\left(m+n\right)D}}. \tag{25}$$

It is seen from (25) that the soliton will exist for

$$\alpha\left(\beta-\gamma m\right)>0 \tag{26}$$

as long as $D>0$, which is guaranted from (24).

Thus, the bright soliton solution to the generalized derivative NLS equation (2) is given by

$$q(x,t)=\frac{A}{\left[D+\cosh\left[B\left(x-vt\right)\right]\right]^{\frac{1}{2n}}}\mathrm{e}^{\mathrm{i}(-\kappa x+\omega t+\theta)}, \tag{27}$$

where the width B of the soliton is given by (25) while the velocity v is given by (18) and the wave number ω is shown in (23). Note that this solution exists provided that the conditions (17), (19) and (26) are satisfied.

3. TOPOLOGICAL SOLITON SOLUTION

In this section, we are interested by finding the exact topological soliton solution, also known as shock waves, for the considered generalized derivative

NLS equation (2). These are also known as topological optical solitons in the context of Nonlinear Optics [19]. It is known that dark optical solitons are more stable in presence of noise and spreads more slowly in presence of loss, in the optical communication systems, as compared to bright solitons [21]. Note that since to date (to our knowledge) no dark soliton solution for the considered equation has been found. For dark optical soliton solution, we introduce a solitary wave ansatz in the form

$$q(x,t)=\left[A+B\tanh\tau\right]^{p}\,\mathrm{e}^{\mathrm{i}\varphi}, \tag{28}$$

where

$$\tau=\mu\left(x-vt\right), \tag{29}$$

$$\varphi=-\kappa x+\omega t+\theta, \tag{30}$$

where μ is a free parameter, v, κ, and ω are, respectively, the velocity, the frequency, and the wave number of the soliton. θ is the phase constant of the soliton. The exponent p is also unknown and it will be determined as a function of m and n. Here A and B are constants. From the ansatz (28), we get

$$\begin{aligned}\left(q^{m}\right)_{t}=&\left\{-\mu pmB\mathrm{v}\left(1-\frac{A^{2}}{B^{2}}\right)\left[A+B\tanh\tau\right]^{pm-1}-2\mu pm\mathrm{v}\frac{A}{B}\left[A+B\tanh\tau\right]^{pm}+\right.\\&\left.+\mu pmv\frac{1}{B}\left[A+B\tanh\tau\right]^{pm+1}+im\omega\left[A+B\tanh\tau\right]^{pm}\right\}\mathrm{e}^{\mathrm{i}m\varphi},\end{aligned} \tag{31}$$

$$\begin{aligned}\left(q^{m}\right)_{xx}=&\left\{\mu^{2}B^{2}pm\left(pm-1\right)\left(1-\frac{A^{2}}{B^{2}}\right)^{2}\left[A+B\tanh\tau\right]^{pm-2}+\right.\\&+2\mu^{2}pm\left(2pm-1\right)A\left(1-\frac{A^{2}}{B^{2}}\right)\left[A+B\tanh\tau\right]^{pm-1}+\\&+2\mu^{2}p^{2}m^{2}\left(\frac{3A^{2}}{B^{2}}-1\right)\left[A+B\tanh\tau\right]^{pm}-2\mu^{2}pm\left(2pm+1\right)\frac{A}{B^{2}}\left[A+B\tanh\tau\right]^{pm+1}+\\&+\frac{\mu^{2}}{B^{2}}pm\left(pm+1\right)\left[A+B\tanh\tau\right]^{pm+2}-\\&-2\mathrm{i}m\kappa\mu pmB\left(1-\frac{A^{2}}{B^{2}}\right)\left[A+B\tanh\tau\right]^{pm-1}-4\mathrm{i}m^{2}\kappa\mu p\frac{A}{B}\left[A+B\tanh\tau\right]^{pm}+\\&\left.+2\mathrm{i}m^{2}\kappa\mu p\frac{1}{B}\left[A+B\tanh\tau\right]^{pm+1}-m^{2}\kappa^{2}\left[A+B\tanh\tau\right]^{pm}\right\}\mathrm{e}^{\mathrm{i}m\varphi},\end{aligned} \tag{32}$$

$$\left\{|q|^{2n} q^m\right\}_x = \left\{\mu Bp\left(2n+m\right)\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{p(2n+m)-1} + \right.$$
$$+\frac{2\mu p\left(2n+m\right)A}{B}\left[A+B\tanh\tau\right]^{p(2n+m)} - \frac{\mu p\left(2n+m\right)}{B}\left[A+B\tanh\tau\right]^{p(2n+m)+1} - \tag{33}$$
$$\left.-\mathrm{i}\kappa m\left[A+B\tanh\tau\right]^{p(2n+m)}\right\}\mathrm{e}^{\mathrm{i}m\varphi},$$

$$|q|^{2n}\left(q^m\right)_x = \left\{\mu Bpm\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{p(2n+m)-1} + \frac{2\mu pmA}{B}\left[A+B\tanh\tau\right]^{p(2n+m)} - \right.$$
$$\left.-\frac{\mu pm}{B}\left[A+B\tanh\tau\right]^{p(2n+m)+1} - \mathrm{i}\kappa m\left[A+B\tanh\tau\right]^{p(2n+m)}\right\}\mathrm{e}^{\mathrm{i}m\varphi}. \tag{34}$$

Substituting (28–34) into (2), removing the exponential term, and then separating the real and imaginary parts, we can obtain the following pair of equations:

$$-\mu pmBv\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{pm-1} - 2\mu pmv\frac{A}{B}\left[A+B\tanh\tau\right]^{pm} +$$
$$+\mu pmv\frac{1}{B}\left[A+B\tanh\tau\right]^{pm+1} - 2\alpha m\kappa\mu pmB\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{pm-1} -$$
$$-4\alpha m^2\kappa\mu p\frac{A}{B}\left[A+B\tanh\tau\right]^{pm} + 2\alpha m^2\kappa\mu p\frac{1}{B}\left[A+B\tanh\tau\right]^{pm+1} +$$
$$+\beta\mu Bp\left(2n+m\right)\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{p(2n+m)-1} + \tag{35}$$
$$+\beta\frac{2\mu p\left(2n+m\right)A}{B}\left[A+B\tanh\tau\right]^{p(2n+m)} - \beta\frac{\mu p\left(2n+m\right)}{B}\left[A+B\tanh\tau\right]^{p(2n+m)+1} =$$
$$= \gamma\mu Bpm\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{p(2n+m)-1} +$$
$$+\frac{2\gamma\mu pmA}{B}\left[A+B\tanh\tau\right]^{p(2n+m)} - \frac{\gamma\mu pm}{B}\left[A+B\tanh\tau\right]^{p(2n+m)+1},$$

and

$$
\begin{aligned}
&-m\omega\left[A+B\tanh\tau\right]^{pm}+\alpha\mu^2B^2pm\left(pm-1\right)\left(1-\frac{A^2}{B^2}\right)^2\left[A+B\tanh\tau\right]^{pm-2}+\\
&+2\alpha\mu^2pm\left(2pm-1\right)A\left(1-\frac{A^2}{B^2}\right)\left[A+B\tanh\tau\right]^{pm-1}+\\
&+2\alpha\mu^2p^2m^2\left(\frac{3A^2}{B^2}-1\right)\left[A+B\tanh\tau\right]^{pm}-2\alpha\mu^2pm\left(2pm+1\right)\frac{A}{B^2}\left[A+B\tanh\tau\right]^{pm+1}+\\
&+\alpha\frac{\mu^2}{B^2}pm\left(pm+1\right)\left[A+B\tanh\tau\right]^{pm+2}-\alpha m^2\kappa^2\left[A+B\tanh\tau\right]^{pm}+\\
&+\beta\kappa m\left[A+B\tanh\tau\right]^{p(2n+m)}=\gamma\kappa m\left[A+B\tanh\tau\right]^{p(2n+m)}+\delta\left[A+B\tanh\tau\right]^{p(4n+m)}
\end{aligned}
\tag{36}
$$

By equating the exponents of $\left[A+B\tanh\tau\right]^{mp+1}$ and $\left[A+B\tanh\tau\right]^{p(m+2)}$ terms in Eq. (35), one gets

$$pm+1=p\left(2n+m\right). \tag{37}$$

That gives the following value of p:

$$p=\frac{1}{2n}. \tag{38}$$

Again this same value of p is obtained on equating the exponents pm and $p\left(2n+m\right)-1$ and also the exponents $pm+2$ and $p\left(4n+m\right)$. By collecting the coefficients of functions of the same exponent of $\left[A+B\tanh\tau\right]^{pm+j}$ for $j=0,\ \pm1,\ \pm2$ in Eq. (35), respectively, where each has to vanish, we obtain the following system of algebraic equations:

$$-\mu pmBv\left(1-\frac{A^2}{B^2}\right)-2\alpha m\kappa\mu pmB\left(1-\frac{A^2}{B^2}\right)=0, \tag{39}$$

$$-2\mu pmv\frac{A}{B}-4\alpha m^2\kappa\mu p\frac{A}{B}+\beta\mu Bp\left(2n+m\right)\left(1-\frac{A^2}{B^2}\right)=\gamma\mu Bpm\left(1-\frac{A^2}{B^2}\right), \tag{40}$$

$$\mu pmv\frac{1}{B}+2\alpha m^2\kappa\mu p\frac{1}{B}+\frac{2\beta\mu p\left(2n+m\right)A}{B}=\frac{2\gamma\mu pmA}{B}, \tag{41}$$

$$\frac{\beta\mu p\left(2n+m\right)}{B}=\frac{\gamma\mu pm}{B}. \tag{42}$$

Solving the above equations yields

$$v = -2\alpha m\kappa, \tag{43}$$

$$\frac{\gamma}{\beta} = \frac{2n+m}{m}. \tag{44}$$

By collecting the coefficients of functions of the same exponent of $\left[A + B\tanh\tau\right]^{pm+j}$ for $j = 0, \pm1, \pm2$ in Eq. (36), respectively, where each has to vanish, yields

$$\omega = \frac{4B^2\delta - \alpha m\left(m+2n\right)\kappa^2}{\left(m+2n\right)}, \tag{45}$$

$$A = B, \tag{46}$$

$$\kappa = \frac{4\delta B^2\left(m+n\right)}{m\left(m+2n\right)\left(\beta-\gamma\right)}, \tag{47}$$

$$\mu = 2nB\sqrt{\frac{\delta}{\alpha m\left(m+2n\right)}}. \tag{48}$$

Eq. (48) shows that the soliton will exist for

$$\alpha\delta > 0. \tag{49}$$

Hence, finally, the dark soliton solution of the generalized derivative nonlinear Schrödinger equation (2) is given by

$$q(x,t) = \left\{A + B\tanh\left[\mu\left(x - vt\right)\right]\right\}^{\frac{1}{2n}} \mathrm{e}^{\mathrm{i}\left(-\kappa x+\omega t+\theta\right)}, \tag{50}$$

where the velocity v is given by (43) and the wave number ω is shown in (45). The inverse width μ of the soliton is given by (48), while the frequency κ is given by (47). It is worth noting that the existence of the dark soliton solution (50) depends on the characteristics of the nonlinear medium, which satisfy the conditions (44) and (49).

4. CONCLUSIONS

This paper analyzed and studied a generalized form of the derivative nonlinear Schrödinger equation, describing the propagation of Alfven waves in plasmas with perturbation terms. The additional terms, besides the pure generalized derivative nonlinear Schrödinger equation that are taken into consideration are the dissipation and higher order nonlinearity effects. In presence of these perturbation terms, both topological (dark) as well as non-topological (bright) solitons are obtained by using the solitary wave ansatz method. All the physical parameters in the soliton are obtained as functions of the dependent model coefficients. The conditions of the existence of the derived soliton solutions are presented. A more general model having higher-order effects and time-dependent coefficients will be studied is the future works.

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REFERENCES

1. A. Biswas, Physics of Plasmas, **12**, 022306 (2005).
2. A. Biswas, Physics Letters A, **372**, 5941–5943 (2008).
3. A. Biswas, K. Porsezian, Communications in Nonlinear Science and Numerical Simulation, **12**, 886–903 (2007).
4. A. Biswas, Physics Letters A, **372**, 4601–4602 (2008).
5. A. Biswas, D. Milovic, Communications in Nonlinear Science and Numerical Simulations, **15**, 1473–1484 (2010).
6. S. Chen, Physical Review E, **78**, 025601 (2008).
7. M. Du, A.K. Chan, C.K. Chui, IEEE Journal of Quantum Electronics, **31**, 177–182 (1995).
8. W.P. Hong, Optics Communications, **194**, 217–223 (2001).
9. X. Li, M. Wang, Physics Letters A, **361**, 115–118 (2007).
10. W. Malfliet, American Journal of Physics, **60**, 650–654 (1992).
11. D. Mihalache, N. Truta, N.C. Panoiu, D.M. Baboiu, Physical Review A, **47**, 3190–3194 (1993);
   D. Mihalache, L. Torner, F. Moldoveanu, N.C. Panoiu, N. Truta, Physical Review E, **48**, 4699–4709 (1993);
   D. Mihalache, D. Mazilu, L. Torner, Physical Review Letters, **81**, 4353–4356 (1998).
12. M.S. Ismail, Applied Mathematics and Computation, **202**, 520–531 (2008).
13. E. Kengne, A. Lakhssassi, T.Nguyen-Ba, R. Vaillancourt, Canadian Journal of Physics, **88**, 55–66 (2010).
14. I.V. Melnikov, D. Mihalache, F. Moldoveanu, N.C. Panoiu, Physical Review A, **56**, 1569–1576 (1997);
   H. Leblond, D. Mihalache, Physical Review A, **79**, 063835 (2009);
   H. Leblond, D. Mihalache, Romanian Reports in Physics, **63**, 1254–1266 (2011).
15. S.L. Palacios, Chaos, Solitons, Fractals, **19**, 203–207 (2004).
16. S.L. Palacios, J.M. Fernandez-Diaz, Optics Communications, **178**, 457–460 (2000).
17. S. L. Palacios, Journal of Optics A: Pure and Applied Optics, **5**, 180–182 (2003).
18. S.L. Palacios, A. Guinea, J. M. Fernandez-Diaz, R.D. Crespo, Physical Review E, **60**, R45 (1999).
19. M. Saha, A.K. Sarma, A. Biswas, Physics Letters A, **373**, 4438–4441 (2009).
20. M.M. Scott, M.P. Kostylev, B.A. Kalinikos, C.E. Patton, Physical Review B, **71**, 174440 (2005).
21. H. Triki, A.M. Wazwaz, Physics Letters A, **373**, 2162–2165 (2009).
22. H. Triki, A.M. Wazwaz, Applied Mathematics and Computation, **214**, 370–373 (2009).

23. S.A. El-Wakil, M.A. Abdou, A. Elhanbaly, Physics Letters A, **353**, 40–47 (2006).
24. A.M. Wazwaz, Physica Scripta, **81**, 035005 (2010).
25. A.M. Wazwaz, Communications in Nonlinear Science and Numerical Simulations, **12**, 1172–1180 (2007).
26. A.T. Grecu, D. Grecu, A. Visinescu, Romanian Journal of Physics, **56**, 339–348 (2011).
27. D. Grecu, A. Visinescu, R. Fedele, S. De Nicola, Romanian Journal of Physics, **55**, 585–600 (2010).
28. D. Grecu, R. Fedele, S. De Nicola, A. T. Grecu, A. Visinescu, Romanian Journal of Physics, **55**, 980–994 (2010).
29. D. Grecu, A. T. Grecu, Romanian Reports in Physics, **63**, 1302–1315 (2011).