

## ON THE PARTICLE CORRELATIONS IN RELATIVISTIC NUCLEAR COLLISIONS\*

ȘTEFANIA VELICA<sup>1\*</sup>, A. JIPA<sup>1</sup>, C. BESLIU<sup>1</sup>, OANA RISTEA<sup>1</sup>, C. RISTEA<sup>1</sup>,  
I. LAZANU<sup>1</sup>, V. COVLEA<sup>2</sup>, A. SCURTU<sup>1</sup>, T. EȘANU<sup>1</sup>, M. CĂLIN<sup>1</sup>, E. STAN<sup>3</sup>, I. STAN<sup>3</sup>

<sup>1</sup> University of Bucharest, Faculty of Physics, Romania,  
E-mail: stefaniavelica@gmail.com

<sup>2</sup> University of Bucharest, Faculty of Physics, Romania

<sup>3</sup> Institute of Space Science, Bucharest

Received July 29, 2011

*Abstract.* On the basis of experimental results obtained in the BRAHMS Collaboration, we studied correlations between physical quantities that can offer information on the formation of new phases of the highly excited and dense nuclear matter. The main types of correlations investigated are those between rapidity and transverse momentum, net charge and rapidity, respectively.

*Key words:* ultra-relativistic nuclear collisions, correlation, rapidity, transverse momentum, longitudinal momentum, phase transition.

### 1. INTRODUCTION

The main aim of nucleus-nucleus collisions at relativistic energies is to understand the properties of strongly interacting matter under extreme conditions of energy and baryon densities, conditions where the creation of Quark-Gluon Plasma (QGP) is expected [1-4]. Within the past several years there was an extraordinary progress on various aspects of elementary particle properties and related phenomena such as heavy ion collisions at relativistic energies [5]. The QGP could be formed at the early stage of collision, when the system is very hot and dense. During the fireball evolution, the system dilutes and cools down, and different hadrons are formed and emitted.

The size and duration of quark-gluon plasma are expected to be very small, at most a few fm in diameter, as size, and, perhaps, 5 to 10 fm/c, as duration. Furthermore, it is also important to distinguish the signals of QGP from the background emitted from the hot hadronic gas phase that follows the hadronization of the quark-gluon plasma. The QGP signals are also modified by the final state interactions in the hadronic phase.

\* Paper presented at the Annual Scientific Session of Faculty of Physics, University of Bucharest, June 17, 2011, Bucharest-Magurele, Romania.

The study of correlations and fluctuations in the relativistic nuclear collisions addresses fundamental aspects of the quantum chromodynamics (QCD) [6] and, therefore, the properties of strongly-interacting matter at extreme density and high temperature. Fluctuations have contributions of different nature. Besides the statistical fluctuations due to a finite number of particles in case of heavy ion collisions, there are also fluctuations due to finite range of the impact parameter used for particular centrality *i.e.* volume fluctuation. Both these fluctuations are trivial and add to the dynamical fluctuations which carry the real information about the properties of the system.

Fluctuations are supposed to be sensitive to the dynamics of the system, especially at the phase transition. The study of event-by-event fluctuations of various quantities in relativistic heavy ion collisions like average transverse momentum ( $\langle p_T \rangle$ ), multiplicity, and conserved quantities such as net charge, is considered as one of the main probes for quark-gluon plasma (QGP) formation [7]. Non-statistical or dynamical fluctuations, which are produced due to correlations arising in the particle production processes, have been of great interest in heavy-ion collision experiments. Fluctuations in temperature and transverse momentum,  $\langle p_T \rangle$ , are studied in connection with critical phenomena, which are relevant if the transition is close to a critical point [8]. It has been observed that the multiplicity and transverse momentum are strongly correlated, too [9]. It is proposed that the change in  $p_T$  spectra at high rapidities could be one of the possible signals of QGP formation.

In this analysis the experimental results obtained in nucleus-nucleus collisions by the BRAHMS Collaboration, at the available energies at the Relativistic Heavy Ion Collider from Brookhaven National Laboratory (USA) were used. The main results presented are for Au-Au collisions at 62.4 GeV and 200 GeV. The rapidity range  $-0.1 < y < 3.5$  was chosen in the investigation of the rapidity-transverse momentum correlation. The motivation was related to the fact that the most experimental evidences for the quark-gluon plasma formation are related to it.

These results are compared with others obtained in nucleus-nucleus collisions into a large energy range, from the JINR Dubna Synchrophasotron, up to CERN SPS, for symmetric and asymmetric collisions. Interesting anomalous states of the nuclear matter can be observed.

Before the presentation of the experimental results, a short description of the BRAHMS experiment is given.

## 2. THE BRAHMS EXPERIMENT

The BRAHMS (Broad Range Hadron Magnetic Spectrometers) Experiment is one of the experiments at Relativistic Heavy Ion Collider (RHIC) from Brookhaven National Laboratory (BNL) [10-13].

In BRAHMS experiment two different regimes can be studied, namely: (a) baryon poor region with a high energy density, created at mid-rapidity; (b) a region at high rapidities, near the initial nuclei, very rich in baryons at relatively high temperature. The BRAHMS experiment well identified charged hadrons –  $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$ ,  $p$  and  $\bar{p}$  - over a wide range of rapidity and transverse momenta at several energies and beams available at RHIC can be measured.

Taking into account the accelerator system structure and the size of the experimental halls the BRAHMS spectrometers are small solid angle devices. They provide semi-inclusive measurements in very different experimental conditions.

The experiment was designed with two moveable magnetic spectrometers, namely: (i) Forward Spectrometer (FS); (ii) Mid Rapidity Spectrometer (MRS) [10-13].

The Forward Spectrometer covers the angular region  $2.3^\circ < \theta < 30^\circ$ , and the Mid Rapidity Spectrometer covers the angular region  $30^\circ < \theta < 95^\circ$ . The pseudorapidity ranges covered by the two spectrometers are:  $1.3 < \eta < 4.0$ ,  $0.1 < \eta < 1.3$ , respectively. To the two magnetic spectrometers three event characterization detector systems have been added; they provide global information. The three detectors systems are the following: Beam-Beam Counters (BBC), multiplicity detector and Zero Degree Calorimeters (ZDC). Together, the three detector systems will provide centrality coverage in the mid rapidity region, namely:  $-2.2 < \eta < 2.2$ , in the region  $3.2 < |\eta| < 4.3$ , as well as at  $0^\circ$ . A top view of the BRAHMS experiment is shown in Fig. 1.

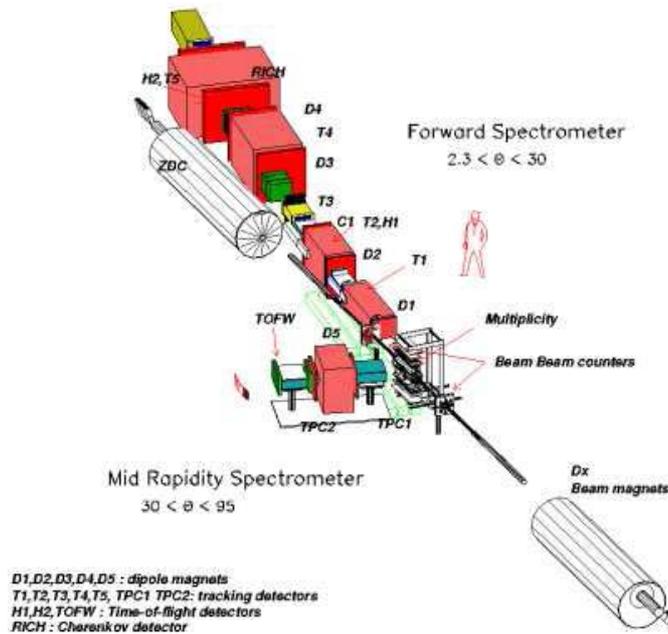


Fig. 1 – A top view of the BRAHMS experiment.

### 3. RAPIDITY – TRANSVERSE MOMENTUM CORRELATIONS

Dynamical fluctuations could be analyzed in the relativistic nuclear collisions by using the correlations between different parameters with dynamic significance.

A possible observation of dynamical fluctuations associated with phase transition will provide direct information on the order of the transition and the effective degrees of freedom in the previous phase. In the BRAHMS experiment from RHIC-BNL, for the Au-Au collisions at 200 A GeV, in the center of mass system, the hadron multiplicities produced in these collisions and the correlations between them are observable that can provide information on the nature, composition and size of their medium.

Statistical interpretation of the particle production in these collisions at very high energies is a good approach because of the fact that a large multiplicity of particles is obtained. Therefore, to describe the multiplicity and particle production ratios are used statistical models (they consider that the nuclear matter created in these collisions form an ideal gas composed of hadrons and resonances, whose composition is governed by statistical laws).

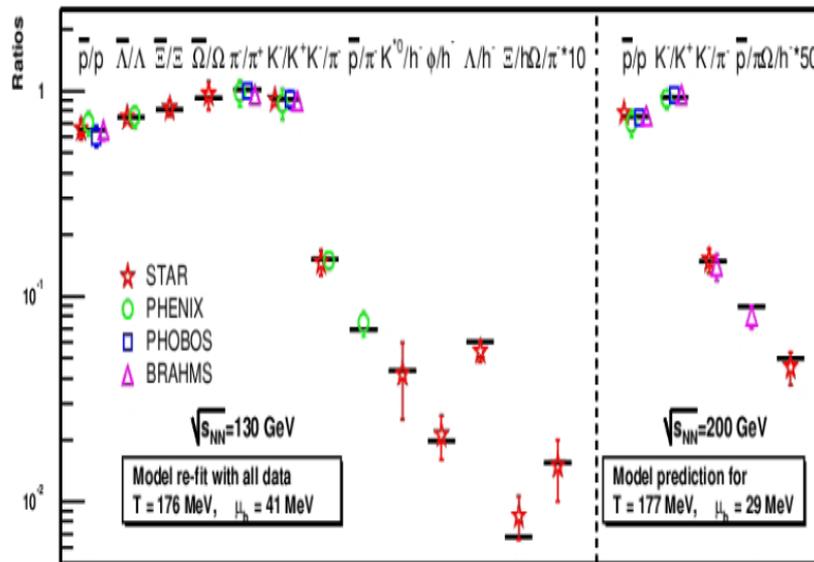


Fig. 2 – Ratios of particles measured (symbols) and statistical model calculations (lines) for collisions Au-Au at 130 A GeV (left) and 200 A GeV (right), in center of mass system. These ratios were measured at midrapidity [14].

Figure 2 shows a good description of experimental data obtained by experiments at RHIC. Baryonic chemical potential  $\mu_B$  decreases with increasing energy, whereas for chemical freeze-out temperature it is not observed a significant

change. The low value of the baryonic chemical potential obtained in Au-Au at 200 A GeV, in center of mass system, indicates a very small net baryonic density of the environment and an increase of the transparency of the collision.

From Fig. 3 is observed that ratios of the kaons and the protons are strongly correlated in the rapidity range from 0 to 3. The blue line from the Figure 3 represent the Becattini statistical model [14] calculations and is observed a good agreement over a wide range of energy and centrality, being an indication of the fact that the system is in chemical equilibrium in the ranges of energy,  $\sqrt{s}$ , and rapidity,  $y$ , considered.

Main emphasis of the present work is to study the correlations between rapidity-transverse momentum and longitudinal momentum-transverse momentum, in a large energy range *i.e.* the work is primarily directed to understand the formation of exotic matter under extreme conditions of density and temperature (study the nature of phase transition) through presence of the correlations between different physical quantities which are one of the various predicted signatures of the phase transition from hadronic phase to QGP phase in heavy ion collisions. These signatures are expected to provide information about the formation of a deconfined state of matter in high-energy heavy ion collisions.

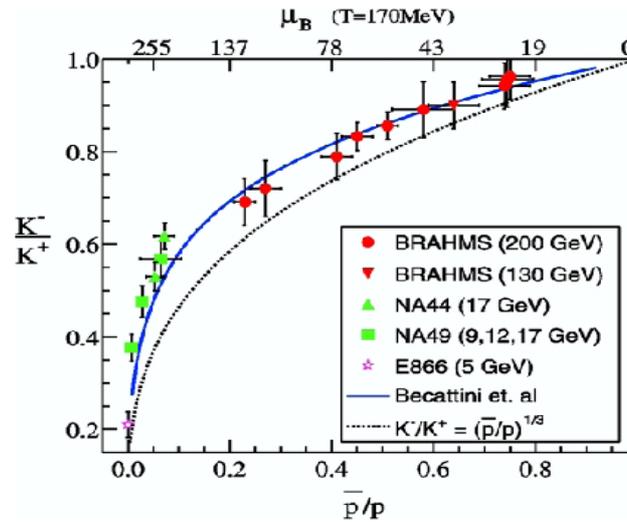


Fig. 3 – The correlation between ratios of production of the kaons and the protons. Lines represent the predictions of the statistical model Becattini. Scale from the top for baryonic chemical potential is in MeV. Also, are presented and similar ratios produced in heavy ions collisions at AGS and SPS [14].

In a central ultrarelativistic collision (Au-Au) at RHIC are expected to produce about 10000 particles per collision, and thus present the remarkable opportunity to analyze, on an event-by-event basis, fluctuations in physical observables such particle multiplicities, transverse momenta, correlations and

ratios. From these global observables (e.g. multiplicity of photons, charged particles and transverse energy), one understands the dynamics of particle production and evolution of the system.

The study of correlations is very informative for relativistic nuclear collisions [15–19]. Many correlation types can be considered. For proving the existence of anomalous states in nuclear matter, unusual correlations between rapidity and transverse momentum should be observed [19, 20].

Using the usual methods from statistics and probability theory [16, 21, 22], we introduce a linear correlation coefficient for the rapidity and transverse momentum (BRAHMS collaboration) and for the longitudinal and transverse momentum components (SKM 200 collaboration) for all charged particles.

The linear correlation coefficient between two measure,  $x$  and  $y$ , is defined by the following relationship [16, 21, 22]:

$$r = C_{xy} = \frac{\text{cov}(\sigma)}{\sigma_x \sigma_y}, \quad (1)$$

where  $\text{cov}(\sigma)$  is the covariance,  $\sigma_x$  is the standard deviation for the measure  $x$ , and  $\sigma_y$  is the standard deviation for the measure  $y$ .

The covariance can be obtained using the following definition:

$$\text{cov}(\sigma) = \frac{1}{n} \sum_{j=1}^n (x_j - \langle x \rangle)(y_j - \langle y \rangle), \quad (2)$$

with  $\langle x \rangle = \frac{1}{n} \sum_{k=1}^n x_k$  the average of the values of the measure  $x$ , and  $\langle y \rangle = \frac{1}{n} \sum_{k=1}^n p_{Tk}$  the average of the values of the measure  $y$ . Finally, we can write:

$$r = C_{xy} = \frac{\sum_{i=1}^n (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\sqrt{\sum_{i=1}^n (x_i - \langle x \rangle)^2 \sum_{i=1}^n (y_i - \langle y \rangle)^2}}. \quad (3)$$

For the experimental values of the correlation coefficient the standard deviation can be estimated using the following relationship:

$$\sigma_r = \frac{1-r^2}{n-1}. \quad (4)$$

The connection between longitudinal momentum and transverse momentum are reflected by the experimental values of the correlation coefficient. The values are in the range  $[-1, +1]$ . The positive values reflect the direct correlations, and the negative values reflect the anti-correlation between the two momentum components; if the value of the correlation coefficient is 0 there is no correlation between the longitudinal momentum and transverse momentum. The correlation is stronger for experimental values of the correlation in the proximity of +1, and the anti-correlation is stronger for experimental values of the same coefficient in the proximity of -1.

In the SKM experiment, at JINR Dubna, for nucleus-nucleus collisions at the incident momentum of 4.5 A GeV/c, were studied linear correlations between  $p_L$  and  $p_T$  momentum components for negative pions, establishing linear correlations between these parameters using the correlation coefficient formula from the relation (3).

The establishment of direct dependencies between the charged hadrons momentum components from an event of a relativistic nuclear collisions is very important because it shows us the conversion mode of the longitudinal momentum into transverse momentum, and highlights a collective behavior of the nuclear matter and, also, of the new nuclear states in nuclear matter at high energies. The observation of a rapidly conversion of the longitudinal momentum in the transverse momentum, resulting from the study of the linear correlations between  $p_L$  and  $p_T$ , is an important result relating to the hypothesis of the existence of a local equilibrium in the relativistic nuclear collisions studied.

#### 4. RESULTS

There are several methods of analyzing fluctuations in order to extract the most interesting dynamical part.

First of them consists of studying global characteristic of produced particles (such as transverse momentum, rapidity, azimuthal angle) and looking for non-statistical, local peaks or intermittency [23] in such distributions.

The second method, so called event by event analysis is based on the comparison of a given event characteristic (*e.g.* mean transverse momentum per event or  $K/\pi$  ratio) and looking how it changes from event-to-event. Event-by-event fluctuations are usually pictured as a superposition of statistical fluctuations (connected with the fact that the number of particles in the system is finite) and dynamical (non-statistical) fluctuations – the subject of interest. However, it appeared that other sources can be a possible origin of non-zero dynamical fluctuation, for example two particle correlations. Therefore, many fluctuations measures used in the literature are also called correlations measure due to the fact that they can measure the inter-particle correlations, are not the only source of dynamical fluctuations.

Table 1

Correlation coefficient between the longitudinal and transverse momentum components for negative pions obtained in the central nuclear collisions C-C and C-Cu at the incident momentum of 4.5 A GeV/c in the SKM 200 Experiment

Pioni negativi	$C_{p_L p_T}$
C-C	-0.62
C-Cu	0.42

Observing the values of the correlation coefficient in the Table 1, for the central nuclear collisions C-C and C-Cu, we can say that the existence of the positive and negative linear correlation for the experimental events show that the negative pions from each event have a collective behavior, thus suggesting the existence of the pions sources in the participating region.

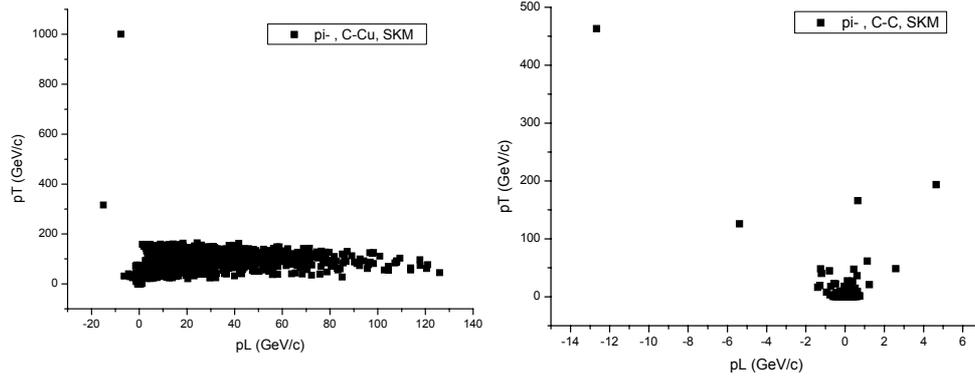


Fig. 3 – The  $(p_L, p_T)$  plane for the central collisions C-C and C-Cu for the SKM 200 collaboration.

As shown in Fig. 4, the  $\langle p_T \rangle$  values are fairly constant in both rapidity and centrality for charged pions.

Figure 5 presents the inverse slope parameters *versus* centrality, for charged pions, and it is clear that she increase with centrality for both PHENIX and SKM 200 experiments.

In Fig. 6 it is shown that the  $\langle p_T \rangle$  increases with particle mass and with centrality, for the charged mesons, in central Au-Au collisions at  $\sqrt{s_{NN}} = 62.4$  GeV, for different rapidities in the BRAHMS experiment.

In Fig. 7 the average  $p_T$  values show a decrease with rapidity, demonstrating that these observables are strongly correlated.

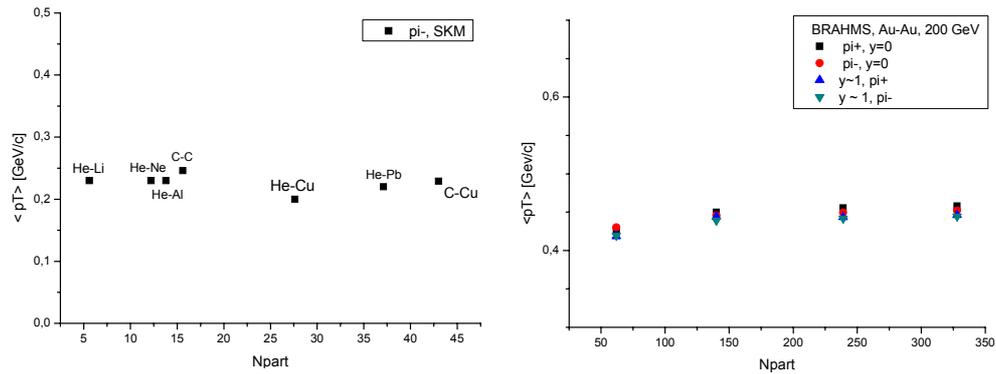


Fig. 4 – Average transverse momentum  $\langle p_T \rangle$  as a function of the number of participants for negative pions (left), the SKM 200 experiment and for  $\pi^\pm$  (right) at the BRAHMS experiment.

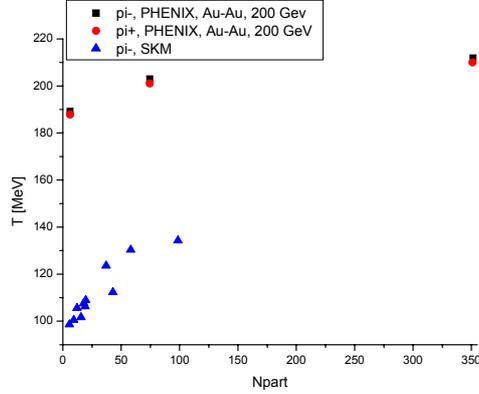


Fig. 5 – The inverse slope parameters *versus* centrality for charged pions, for the PHENIX and SKM 200 experiments.

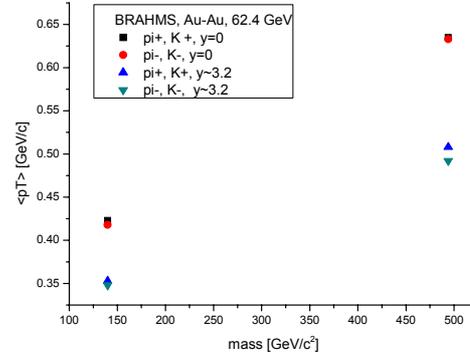


Fig. 6 – Average transverse momentum  $\langle p_T \rangle$  as a function of the mass of for the charged mesons in central Au-Au collisions at 62.4 GeV, in the BRAHMS experiment.

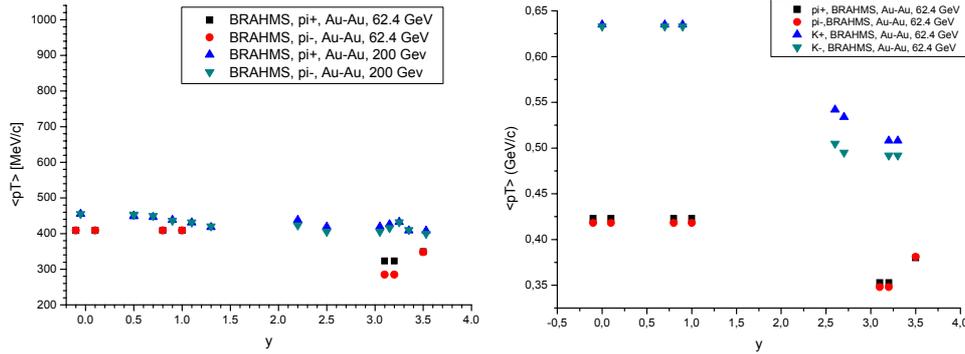


Fig. 7 – Average transverse momentum  $\langle p_T \rangle$  as a function of rapidity for the charged pions at different energies from the BRAHMS experiment (left) and  $\langle p_T \rangle$  as function of rapidity for the charged pions and kaons at 62.4 GeV, within the BRAHMS experiment (right).

To have a picture of the results obtained in the study of correlations between longitudinal momentum and transverse momentum of charged hadrons, in Figs. 9, 10, 11 is presented the distribution of the linear correlation coefficient  $C_{p_L p_T}$  for central collisions C-C, C-Cu and He-Cu at incident momentum 4.5 A GeV/c for the experimental events, in the SKM 200 experiment.

From the distribution of the linear correlation coefficient  $C_{p_L p_T}$  it is observed that the number of events correlated positively with  $C_{p_L p_T} \geq 0$  is greater than the number of events correlated negatively with  $C_{p_L p_T} < 0$ .

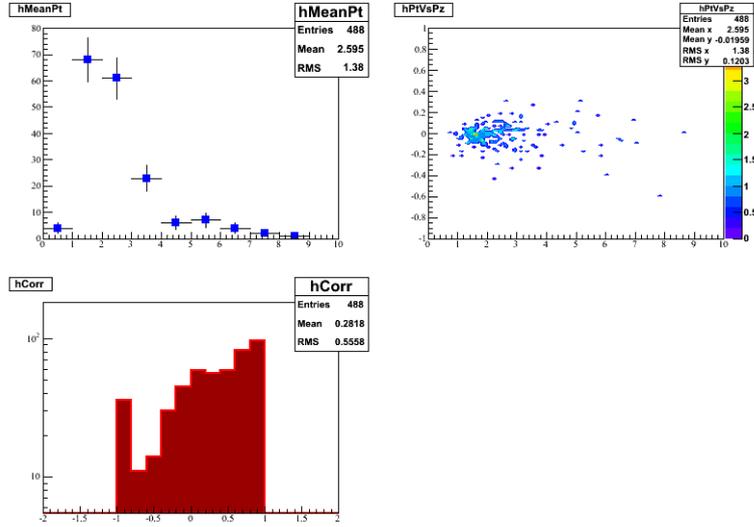


Fig. 8 – Distribution of the average transverse momentum of the event (left picture on the top) for central C-Cu collisions at incident momentum 4.5 A GeV/c, the plane  $(p_L, p_T)$  (right picture on the top) for the central C-Cu collisions, distribution of the linear correlation coefficient  $C_{p_L p_T}$  (left picture on the bottom) in central C-Cu collisions, at the SKM 200 collaboration.

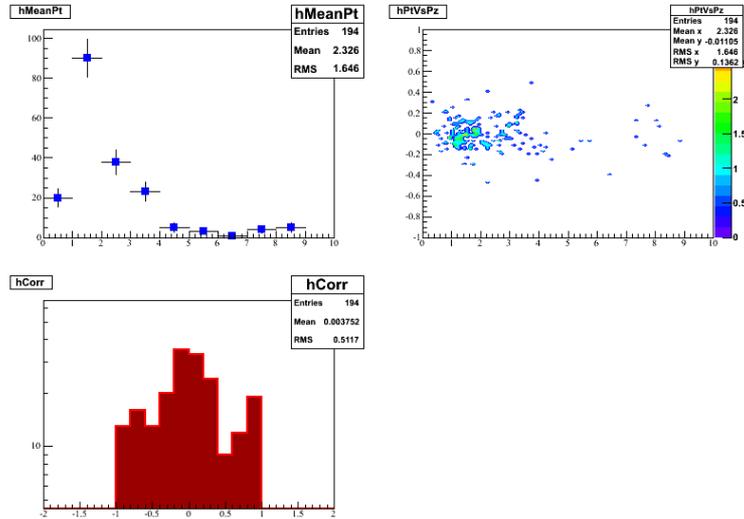


Fig. 9 – Distribution of the average transverse momentum of the event (left picture on the top) for central C-C collisions at incident momentum 4.5 A GeV/c, the plane  $(p_L, p_T)$  (right picture on the top) for the central C-C collisions, distribution of the linear correlation coefficient  $C_{p_L p_T}$  (left picture on the bottom) in central C-C collisions, at the SKM 200 collaboration.

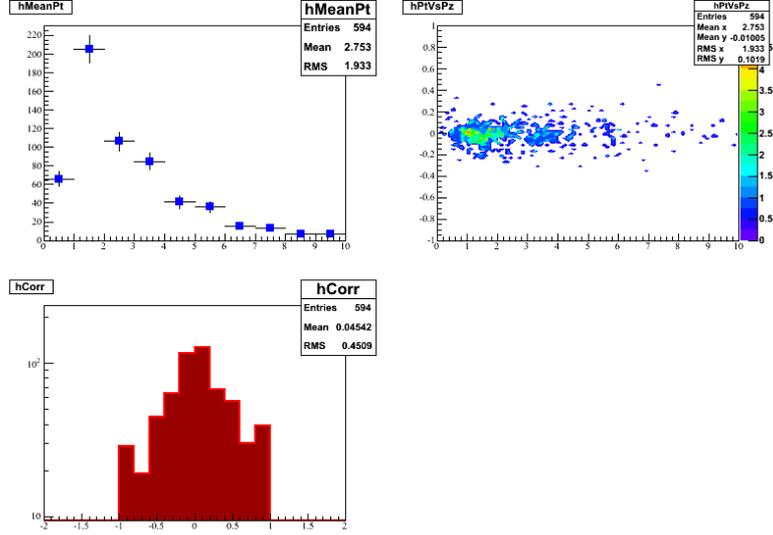


Fig.10 – Distribution of the average transverse momentum of the event (left picture on the top) for central He-Cu collisions at incident momentum 4.5 A GeV/c, the plane  $(p_L, p_T)$  (right picture on the top) for the central He-Cu collisions, distribution of the linear correlation coefficient  $C_{p_L p_T}$  (left picture on the bottom) in central He-Cu collisions, at the SKM 200 collaboration.

Also note that, for the  $C_{p_L p_T} \geq 0$ , i.e.  $C_{p_L p_T} < 0$ , there are a large number of experimental events with the linear correlation coefficient  $C_{p_L p_T} \geq 0.8$ , i.e.  $C_{p_L p_T} < -0.8$ , indicating that there was a significant linear correlation between the longitudinal and the transverse momentum components of the charge hadrons.

Experimental events with the linear correlation coefficient between the longitudinal momentum and the transverse momentum of the charged hadrons  $C_{p_L p_T} \geq 0.8$ , are positively correlated, which means that with increase of the longitudinal momentum, there is and an increase of the transverse momentum and vice versa.

Experimental events with the linear correlation coefficient between the longitudinal momentum and the transverse momentum of the charged hadrons  $C_{p_L p_T} < -0.8$ , are negatively correlated, which means that with increase of the longitudinal momentum, there is a decrease of the transverse momentum and vice versa.

## 5. CONCLUSIONS

The study of correlations between transverse momentum and longitudinal momentum did not indicate a significant change in the behavior from the classical expected anti-correlation. In the analysis of the correlation between rapidity and

average transverse momentum, for the  $\pi^\pm$  and  $K^\pm$  mesons, the particles detected with the BRAHMS experimental set-up, a regularity in the behaviour of these mesons was observed, under 1.5 GeV/c. It is important to see the modification of the  $\langle p_T \rangle$  for rapidities higher than 3. This behaviour could be associated with the behaviour of the nuclear modification factor in the rapidity ranges:  $y < 2$  and  $y > 2$ . These are determined by the suppression of high transverse momentum. These results confirm the importance of correlations and fluctuations studies for characterization of the different possible phases of the nuclear matter. The obtained results are in good agreement with other experimental results.

Within the SKM 200 collaboration, it has been concluded that the selection of the events which are negatively correlated, from different nuclear collisions, could be an important method to highlight the high-temperature states of the nuclear matter at high energies.

The existence of a large number of particles with correlations between longitudinal and transverse momentum support the hypothesis regarding the presence of the non-equilibrium and pre-equilibrium production mechanisms for some types of particles.

For the positive correlations, the fact that the longitudinal momentum increases with transverse momentum and vice versa, is not normal from the point of view of the kinematics of a nuclear collision, suggesting in this case the existence of the abnormal states in the participating region, and also the existence of non-equilibrium states in the nuclear matter formed at high energies. In the case of the negative correlations,  $p_L$  increase with the decrease of  $p_T$  and vice versa, thus highlighting of some states of pre-equilibrium/equilibrium, taking into account the fact that  $p_L$  and  $p_T$  are the momentum components for each charged hadron from an event.

The number of positively correlated events is greater than the number of negatively correlated events and therefore, we can say that, in the case of nuclear collisions at incident momentum of 4.5 GeV/c, non-equilibrium states predominate in relation to the pre-equilibrium states of nuclear matter formed in these collisions.

If different mechanisms of particle production, relating to the various degrees of equilibrium, can coexist then is possible the direct observation of the linear correlation between the two components of momentum.

So to highlight the states of the nuclear matter at high energies, with very high temperatures, it is necessary the study of the central nuclear collisions and, especially, of the negatively correlated events from these collisions.

*Acknowledgements.* We gratefully acknowledge support from:

The work of Stefania Velica was supported by the Project POSDRU/6/1.5/S/10, "Projected development and performance in doctoral research of interdisciplinary type".

The work of Oana Ristea and Catalin Ristea was supported by the strategic grant POSDRU/89/1.5/S /58852, Project „Postdoctoral programme for training scientific researchers”, co-financed by the European Social Found within the Sectorial Operational Program Human Resources Development 2007-2013.

We thank the BRAHMS and the SKM 200 Collaborations for their excellent and dedicated work to acquire and process the unique experimental data and their support to our group.

## REFERENCES

1. A. Jipa and C. Besliu, *Elemente de Fizică Nucleară Relativistă. Note de curs*, Editura Universității, București, 2002.
2. J. Collins and M. Perry, *Phys. Rev. Lett.*, **34**, 1353 (1975);  
E. Shuryak, *Phys. Lett.*, **A78**, 150 (1978).
3. O. Kalashnikov and V. Klimov, *Phys. Lett.*, **B88**, 328 (1979).
4. J. Kapusta, *Nucl. Phys.*, **B148**, 461 (1979).
5. M. Petrovici *et al.*, *Rom J. Phys.*, **56**, 654 (2011);  
M. Petris *et al.*, *Rom J. Phys.*, **56**, 349 (2011); A. Saftoiu *et al.*, *Rom. J. Phys.* **56**, 664 (2011);  
A. M. Apostu *et al.*, *Rom. Rep. Phys.* **63**, 220 (2011);  
G. Toma *et al.*, *Rom. Rep. Phys.* **63**, 383 (2011);  
B. Mitrica *et al.*, *Rom. Rep. Phys.* **62**, 750 (2010);  
M. I. Cherciu and A. Jipa, *Rom. Rep. Phys.* **62**, 731 (2010);  
M. Petris *et al.*, *Rom J. Phys.* **55**, 324 (2010).
6. M. Visinescu, *Teoria cuantică a câmpului* (partea III și partea VIII), Editura Institutului Central de Fizică, 1986.
7. Gordon Baym and Henning Heiselberg, *Phys. Lett.*, **B 469**, 7 (1999).
8. M. Stephanov, K. Rajagopal, and E.V. Shuryak, *Event-by-event fluctuations in heavy ion collisions and the QCD critical point*, *Phys. Rev.*, **D60**, 114028 (1999).
9. M.Gazdzicki and St. Mrowczynski, *A method to study "equilibration" in nucleus-nucleus collisions*, *Z. Phys. C54*, **127** (1992).
10. D.Beavis *et al.*, *BRAHMS Conceptual Design Report*, BNL-62018, October 1994.
11. Fl.Videbaek, Talk at Workshop UIC, June 14-17, 1998, World Scientific, Singapore, 1999.
12. F.Videbaek for the BRAHMS Collaboration, International Conference "Quark Matter 2001", Stony Brook and Brookhaven, January 2001, USA, published in *Nuclear Physics*, **A698**, 29c-38c (2002).
13. M.Adameczyk *et al.* (BRAHMS Collaboration), *Nucl. Inst. Meth. Phys. Res.*, **A499**, 437-468 (2003).
14. F. Becattini *et al.*, *Phys. Rev.*, **C64**, 024901 (2001).
15. M. Plümer, S. Raha, R.M. Weiner (Eds.), *International Workshop on Correlations and Multiparticle Production*, Marburg, Germany, May 14-16, 1990, World Scientific, Singapore, New Jersey, London, Hong Kong, 1991.
16. P. Carruthers, C.C. Shih, *Int. J. Mod. Phys.*, **A2**, 1447 (1987).
17. S.A. Bass, C. Hartnack, H. Stoecker, W. Greiner, *Phys. Rev.*, **C51**, R12 (1995).
18. G. Baym, B. Blaettel, L.L. Frankfurt, H. Heiselberg, M. Strikman, *Phys. Rev.*, **C52**, 1604 (1995).
19. P. Boezek, M. Ploszajczak, R. Botet, *Phys. Rep.*, **252**, 101 (1995).
20. Al. Jipa, C. Besliu, M. Iosif, R. Zaharia, *Nuovo Cimento*, **A 112**, 179-203 (1999).
21. B.R. Martin, *Statistics for Physicists*, Plenum Press, London, New York, 1971.
22. Gh. Mihoc, V. Craiu, *Tratat de Statistică Matematică*, Vol. 4, Editura Academiei Române, București, Romania, 1981.
23. K. Haglin and D. Seibert, *Scaled factorial moments and split bin correlation function: a thermodynamic model comparison*, *Phys.Lett.*, **B273**, 211 (1991).