LASERS

A GENERAL MODEL FOR MULTI-MODE LASER PROCESSING OF SOLIDS

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Abstract. This work deals with the evaluation of the laser thermal fields in solids, when the laser source is in motion along the irradiated surface. Unlike other models, we consider in this work a continuous-wave laser source operated in a multi-mode regime. The solution to the heat equation is analytical.

Key words: laser beam in motion, heat equation, multi-mode CO₂ laser beam.

1. INTRODUCTION

High power CO₂ and Nd:YAG lasers are applied for a wide variety of engraving, cutting, welding, soldering, and 3D prototyping applications [1, 2]. Laser cutting applications [1] include metals, wood, glass, ceramics, textiles, airbags and lace among others. The cutting is very quick, accurate, and there is no edge discoloration, and clean fused edge is obtained, thus avoiding frying of the material.

Excimer lasers also play a key role in photolithography and are used to fabricate very large scale integrated circuit chips [1]. For the integrated circuit design, the wavelength range decreased in a decade from 0.35 µm to the current 193 nm. The Argon Fluoride laser lines are broad enough to prevent speckle formation yet narrow enough, less than 2 nm wavelength width, to avoid major problems with dispersion in optical imaging. The Argon fluoride (ArF) excimer laser radiation at 193 nm wavelength supports 0.04 µm design rules. Electron beams, X-rays and synchrotron radiation are next considered below the 30 nm design rules anticipated for 2015.
2. ANALYTICAL MODEL

We have used the integral transform method [3] and obtained rather simple results in both classical and quantum physics approaches [4–7]. We start with the basic ideas of the classical approach, presenting a relevant example. The heat equation in the case of laser irradiation of a homogeneous solid sample is:

\[ \frac{\partial^2 T(r, z, \varphi, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, z, \varphi, t)}{\partial r} + \frac{\partial^2 T(r, z, \varphi, t)}{\partial z^2} - \frac{1}{\gamma} \frac{\partial T(r, z, \varphi, t)}{\partial t} = -\frac{A(r, z, \varphi, t)}{k}, \]  

(1)

where: \( \gamma \) is the thermal diffusivity, \( k \) is thermal conductivity, and \( A(r, \varphi, z, t) \) is the heat rate variation per unit volume and time. \( T \) represents the temperature variation produced by laser irradiation. \((r, \varphi, z)\) are the usual polar coordinates and \( t \) is the time.

The boundary conditions are:

\[ k \frac{\partial T(r, \varphi, z, t)}{\partial r} \bigg|_{r = b} + hT(b, \varphi, z, t) = 0, \]  

(2)

\[ k \frac{\partial T(r, \varphi, z, t)}{\partial r} \bigg|_{z = 0} - hT(r, \varphi, 0, t) = 0, \]

\[ k \frac{\partial T(r, \varphi, z, t)}{\partial r} \bigg|_{z = a} + hT(r, \varphi, a, t) = 0. \]

The “periodicity” conditions are in this case as follows:

\[ T(r, 0, z, t) = T(r, 2\pi, z, t). \]  

(3)

Here \( h \) represents the heat transfer coefficient. The temperature \( T \) is a function of \((r, \varphi, z, t)\) and is defined as the temperature variation. We have in consequence:

\[ T(r, \varphi, z, 0) = 0. \]

If we consider the case of a continuous-wave (cw) CO\(_2\) laser source operated in the transversal mode \( \{m, n\} \), we have the following solution of eq. (2) [4]:

\[ T(r, \varphi, z, t) = \sum_{m,n} \sum_{\ell=0}^{\infty} \sum_{j=0}^{\infty} \tilde{f}_{\ell j}(\mu_m, \lambda_j, l) \cdot g(\mu_m, \lambda_j, \ell) \times K_s(\mu_m, r) \times K_0(2\ell, \varphi) \times K_z(\lambda_j, z) + \]

\[ + \sum_{m,n} \sum_{\ell=1}^{\infty} \sum_{j=0}^{\infty} \tilde{f}_{\ell j+1}(\mu_m, \lambda_j, l) \cdot g(\mu_m, \lambda_j, \ell) \times K_s(\mu_m, r) \times K_0(2\ell - 1, \varphi) \times K_z(\lambda_j, z), \]

(4)

with:
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\[ \hat{f}_{2l}(\mu_{il}, \lambda_{j}, l) = \frac{1}{k_{ei}C_{i}l} \int_{0}^{\pi} \alpha \cdot e^{-az} \times K_{z}(\lambda_{j}, z)dz \times \]
\[ \times \int_{0}^{\pi/2} I_{mn}(r, \phi) \cdot r \times K_{r}(\mu_{il}, r) \times K_{q}(2l, \phi)drd\phi, \]

(5)

and

\[ \hat{f}_{2l-1}(\mu_{il}, \lambda_{j}, l) = \frac{1}{k_{ei}C_{i}l} \int_{0}^{\pi} \alpha \cdot e^{-az} \times K_{z}(\lambda_{j}, z)dz \times \]
\[ \times \int_{0}^{\pi/2} I_{mn}(r, \phi) \cdot r \times K_{r}(\mu_{il}, r) \times K_{q}(2l-1, \phi)drd\phi, \]

(6)

where \( k \) (do not be confused with \( K \)) is the thermal conductivity of the irradiated sample and \( \alpha \) is the optical absorption constant at the radiation wavelength. \( K \) represents the eigenfunction corresponding to a given eigenvalue.

In eqns. (4–6) we used the following functions:

\[ g(\mu_{il}, \lambda_{j}, t) = 1/\beta_{i}^{2} + \lambda_{j}^{2} \times [1-e^{-\gamma(t-t_{0})} - (1-e^{-\gamma(t-t_{0})})h(t-t_{0})], \]
\[ I_{mn}(x, y) = I_{0mn} \left[ H_{m} \left( \frac{\sqrt{x}}{w} \right) H_{n} \left( \frac{\sqrt{y}}{w} \right) \times \exp \left[ -\left( \frac{x^{2}+y^{2}}{w^{2}} \right) \right] \right]^{2}. \]

(7)

Here \( w \) is the width of the laser beam. Above, \( \beta_{i}^{2} = \gamma(\mu_{il}^{2} + \lambda_{j}^{2}) \) and \( h(t-t_{0}) \) is the Heaviside function and \( t_{0} \) the exposure time. The functions: \( K_{r}(\mu_{il}, r), \ K_{q}(2l, \phi), \ K_{q}(2l-1, \phi) \) and \( K_{z}(\lambda_{j}, z) \) are eigenfunctions corresponding to the eigenvalues:

\ \( \mu_{il}, 2l, 2l-1, \ \lambda_{j} \). We have \( K_{r}(\mu_{il}, r) = J_{l}(\mu_{il} \cdot r), \ K_{q}(2l, \phi) = \cos(l\phi), \ K_{q}(2l-1, \phi) = \sin(l\phi) \) and \( K_{z}(\lambda_{j}, z) = \cos(\lambda_{j} \cdot z) + (h/k\lambda_{j}) \cdot \sin(\lambda_{j}z) \). Here \( i, l, j \)

are usual integer index parameters which goes for \( l \) from 0 to infinity, and for \( i \) and \( j \) from 1 to infinity.

We computed the thermal field generated in a Cu sample by cw CO2 laser irradiation with eqs. (4–7). The results we obtained for a cw-CO2 laser source operated in the TEM03 transversal mode, beam waist 3mm at \( P = 10W \), and an irradiation time of 10s and shown in Fig 1.

We observed two symmetric maxima on the \( y \) axis on the two sides of the origin \( y = 0 \).
Fig. 1 – The thermal field generated in bulk Cu by a cw CO2 laser source operating in TEM$_{03}$ transversal mode at 10 W, stationary (v = 0m/s) on the sample surface. The irradiation time was 10s.

The optical and thermal parameters used in the calculations are given in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>$k$ [W/cm K]</th>
<th>$\gamma$ [cm$^2$/s]</th>
<th>$\alpha$ [cm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>3.95</td>
<td>1.14</td>
<td>$7.7 \times 10^5$</td>
</tr>
</tbody>
</table>

3. CW LASER BEAM IN MOTION ACROSS SAMPLE SURFACE

We considered the case when the laser beam is scanned over a Cu sample surface with a velocity $v$. The components of this velocity along the two axes are $v_x$ and $v_y$, respectively. We then introduced a new reference system with the coordinates $x = x_0 + v_x \cdot t$ and $y = y_0 + v_y \cdot t$ and adapted correspondingly the source term.

We performed calculations for $v_x = 10$ m/s, $v_y = 0$, with $t = 10$s (Fig. 2) and $t = 20$s (Fig. 3). In the case presented in Fig. 1, the cw CO$_2$ laser beam was operated in the TEM$_{03}$ transversal mode at an output power of 10 W.
Fig. 2 – The thermal field generated in bulk Cu by a cw CO₂ laser source operating in TEM₀₃ transversal mode at 10 W, scanning the sample surface along the x axis with \( v = 10 \text{m/s} \). The irradiation time was 10s.

Fig. 3 – The thermal field generated in bulk Cu by a cw CO₂ laser source operating in TEM₀₃ transversal mode at 10 W, scanning the sample surface along the x axis with \( v = 10 \text{m/s} \). The irradiation time was 20s.
Fig. 4 – The thermal field generated in bulk Cu by a cw CO$_2$ laser source operating in TEM$_{01}$ transversal mode at 10 W, stationary ($v = 0$m/s) on the sample surface. The irradiation time was 10s.

Fig. 5 – The thermal field generated in bulk Cu by a cw CO$_2$ laser source operating in TEM$_{01}$ transversal mode at 10 W, scanning the sample surface along the x axis with $v = 10$m/s. The irradiation time was 10s.
Fig. 6 – The thermal field generated in bulk Cu by a cw CO$_2$ laser source operating in TEM$_{01}$ transversal mode at 10 W, scanning the sample surface along the $x$ axis with $v = 10$ m/s. The irradiation time was 20s.

By comparing Fig.1 to Figs. 2, 3, one notices that two temperature maxima are present in all cases. The main difference is related to the depth of the minimum between the two peaks, which is decreasing from Fig. 1 to Fig. 3. The differences between the minimum and maximum temperatures are also diminishing and the plateau between the two maxima moves to higher temperatures. This observation is congruent with a more uniform heating of the sample due to the laser beam movement. This effect is more pronounced in the case of a higher velocity of the laser beam.

Calculations were performed for various laser beam scan velocities and times i.e., $v_x = 0$ m/s, $t = 10$ s (Fig. 4), $v_x = 10$ m/s, $t = 10$ s (Fig. 5) and $v_x = 20$ m/s, $t = 20$ s (Fig. 6). In Figs. 4, 5, 6 the calculated thermal fields for the mode TEM$_{01}$ (10 W) are shown.

The heat equation was solved for a laser beam either stationary $v_x = 0$ m/s or in motion, when irradiating a solid (Cu) sample with a CW CO$_2$ laser source.

4. DISCUSSION

There are many methods for evaluating the thermal fields in laser-matter interaction but most of them require a complex mathematical apparatus.
This paper presents a direct and powerful mathematical approach to compute the thermal field. The solving procedure is based on applying the integral transform technique which was developed in 1960, by the Russian school of theoretical physics [4]. The comparison with data from the literature is in good agreement with this model [5, 6, 7, 8]. The disadvantages though of this model is that it requires very accurate values of the thermal and optical parameters, and also that it cannot take into account the variation with temperature of all these parameters. In consequence the model should be regarded as a first approximation of the thermal field.

An important result for computing the eigenvalues \( \lambda_j \) and \( \mu_{il} \) is that present an important periodicity:

\[
\lambda_j \approx \frac{j \pi}{a}, \quad j > 3 \quad \text{and} \quad \mu_{il} \approx \frac{i \pi}{b}, \quad i > 3, \quad (\forall) \ l.
\]  

This periodicity is due to the direct analytical calculation of the eigenvalues, using the boundary condition given in equation (2), where the temperature parameters is replace by eigenfunctions, according to the integral transform technique [4]. The “result” is that the first three eigenvalues, are very small, and for indices factors: \( j > 3 \) and \( i > 3 \), the eigenvalues increase with a precise and important periodicity; according to equations (8). On the other hand, if we take a look at the “global analytical structure” of the heat equation solution (Eqns. 4, 5, 6, 7), we observe that the lower the eigenvalues the higher the increase the sample’s temperature; and the higher the eigenvalues the lower the increase the sample’s temperature. This behavior produce a very big advantage thus is that the solution of eqn. 4 is a series which converges very rapidly. For typical calculations, after 10 iterations the precision of the value for the thermal field has a value of \( 10^{-2} \)K, acceptable for many experiments.

The above mathematical argument has been check in real experiments, for: \( v_x = v_y = 0 \) m/s [9].

Although we can limit the series to 10 iterations we obtain in fact an analytical solution rather than a semi-analytical one. In conclusion equation (4) can be written as follows:

\[
T(r, \varphi, z, t) = \sum_{m, n} \left[ \sum_{i=1}^{10} \sum_{j=1}^{10} \hat{f}_{2i}(m, n, i, j, l) \cdot g(m, n, l) \times K_{2i}(m, n, r) \times K_{\varphi}(2i, \varphi) \times K_{z}(l, z) + \right. \\
+ \left. \sum_{m, n} \left[ \sum_{i=1}^{10} \sum_{j=1}^{10} \hat{f}_{2i-1}(m, n, i, j, l) \cdot g(m, n, l) \times K_{2i-1}(m, n, r) \times K_{\varphi}(2i-1, \varphi) \times K_{z}(l, z) \right].
\]  

(9)

For a typical single transversal mode \( \{m, n\} \), the temperature is a sum of 2 200 functions. In consequence we can calculate easily the thermal fields using a regular desktop PC.
Even for a large surface absorption for one photon absorption the model is flexible enough to take into account the situation. Denoting with \( r_\delta \) the surface absorption coefficient (which varies in general between 0.1 and 0.01), eqns. (5) and (6) are given by:

\[
\hat{f}_{2l}(\mu_l, \lambda, l) = \frac{1}{k \pi C_d C_j} \int_0^a (\alpha \cdot e^{-\alpha x} (1 - r_\delta) + r_\delta \cdot \delta(z)) \times K_z (\lambda, z) dz \times \\
\times \int_0^{2\pi} \int_0^b I_{mn}(r, \varphi) \cdot r \times K_r (\mu_l, r) \times K_\varphi(2l, \varphi) dr d\varphi
\]

and

\[
\hat{f}_{2l-1}(\mu_l, \lambda, l) = \frac{1}{k \pi C_d C_j} \int_0^a (\alpha \cdot e^{-\alpha x} (1 - r_\delta) + r_\delta \cdot \delta(z)) \times K_z (\lambda, z) dz \times \\
\times \int_0^{2\pi} \int_0^b I_{mn}(r, \varphi) \cdot r \times K_r (\mu_l, r) \times K_\varphi(2l - 1, \varphi) dr d\varphi.
\]

One may consider some very special situations when one-photon absorption is forbidden due to the rules of quantum mechanics or when two-photon absorption is very high. The model however, can take into consideration these situations as well as described in reference [5].

### 5. CONCLUSIONS

A more uniform heating of the metal was observed when the laser beam was moving across surface, and two clear maxima were evident in all temperature fields. One may note that the approach is quite general, because the irradiating source can be a laser beam but also electron or particle beam. Moreover, the typical analysis used here can be extended to any material with known thermal properties.

One may be aware though that there are other models competitive with the model described here. These models are however more complicated, but can used in conjunction with our model to extract more information about the [10].

For example in reference [10] it avoids the Fourier differential heat equation and it uses the Duhamel’s principle; which leads to a convolution integrated analytically by using a Taylor series approximation.

We conclude that the model is a direct and powerful method providing rapidly to a first approximation the temperature fields for the laser-material interaction.
REFERENCES


