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THE INFLUENCE OF THE RADIATION PRESSURE IN THE BINARY W UMi SYSTEM, USING THE POPOVICI MODEL

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Abstract. The model of the motion of a silicon particle under the influence of gravity and radiation pressure in the W UMi binary system is presented using the C-tin Popovici Model. We will determine the β_1 and β_2 ratios from the two luminous bodies and we will study the equilibrium points, including those outside the orbital plane.

Key words: Radiation pressure, elliptic restricted three body problem, equilibrium points, binary system.

1. INTRODUCTION

The matter of determining the equilibrium points in the photo-gravitational restricted three body problem is still a issue worth discussing. Thus V. V. Markellos, E. Perdios and P. Labropoulou [12], V. V. Markellos, E. Perdios and C. Georghiou [13] have studied the linear stability of the triangular points, A. S. Zimovshchikov and V. N. Tkhai [30] have analysed the instability of the triangular and collinear points. Furthermore, G. F. Chorny [6], using a dust particle in motion on an elliptical, parabolic and hyperbolic orbit, has studied the quasi-integrals of the photo-gravitational eccentric restricted three-body problem with Poynting-Robertson drag, B. Ishwar and A. Elipe have found secular solutions to the triangular equilibrium points in the general problem [8] and non linear stability [11], and Ming-Jiang Zhang, Chang-Yin Zhao and Yong-Qing Xiong [29] have investigated the triangular libration points in the photo-gravitational restricted three-body problem of variable mass. It is important to take into account the Poynting-Robertson effect (P-R effect), well detailed as presented by Klacka [9]. The problem for a spherical particle moving under the influence of the P-R effect in Klacka [10].

The theory of the solar radiation pressure and P-R effect was a constant concern for Romanian researchers. There were contributions to the photo-gravitational problem of the two bodies by determining the positions of the equilibrium points in the works of Mira-Cristina Anisiu [1–3], the type of state points by M. Barbosu and T. Oproiu [4], the simulation of the test particles in the photo-gravitational field of the different binary systems (for example, the movement of an ice particle in the photo-gravitational field of the RW Tauri binary done by Rodica Roman [25] or studies using McGehee variables, V. Mioc and Cristina Blaga [14, 15], made a qualitative analysis of the differential equation system and most recently Ioannis Haranas and V. Mioc [7].

We describe Constantin Popovici model within the photo-gravitational problem of two and three bodies. We begin with the photo-gravitational circular restricted problem of the two bodies, of which only one is alight, we extend the problem to the three body circular situation and we finish with the pattern in the case of the photo-gravitational elliptical restricted problem of the three bodies when the two primary bodies light. The application at the end of the paper refers to the analysis of ratio between the radiation force and the force of gravity reacting on the test particle corresponding to both primary elements in a binary system, depending on the true anomaly parameter.

The dependency of libration points of the three body restricted problem and the movement stability in their neighbourhood are searched when we take into consideration the effects of the light pressure; we consider the W UMi binary system, the test particle being a silicon grain where we determined numerically the coordinates of the collinear and triangular libration points and we established that there are no state points outside the orbital plan in this binary system.

2. BASIC EQUATIONS

At the beginning of this section we describe the photo-gravitational problem of two bodies - Constantin Popovici model. Then, we make an extension of the pattern to the case of the photo-gravitational restricted problem of the three bodies, of which only one is more luminous.

In C. Popovici model for the force projection towards the vector radius we have the expression

$$F = -\frac{A}{r^2} + \frac{R}{r^2} - \frac{R\dot{r}}{cr^2}, \tag{1}$$

where the first term $-A/r^2$ is the Newton attraction force (A is the attraction body at the unit of distance $r = 1$), R/r^2 is the force determined by the pressure of the central body light (R - the repulsion determined by the light pressure at the unit of distance), while the last one $R\dot{r}/cr^2$ introduced by Popovici [18] is an increase in the attraction determined by the light finite velocity.

The relativistic fair analysis of this problem was formulated by H. P. Robertson

[24], who proved that, up to the first order in v/c, the total force due to the light pressure is given by the expression:

$$\vec{F}_{rad} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = F_p\frac{\vec{r}}{r} - F_p\frac{\vec{v}\cdot\vec{r}}{cr}\cdot\frac{\vec{r}}{r} - F_p\frac{\vec{v}}{c}, \tag{2}$$

where F_p is the radiation pressure size, $\vec{r}$ the position vector of P - test particle, related to the radiation source S, $\vec{v}$ the corresponding velocity vector and c the velocity of light. The first term, $\vec{F}_1$, is the radiation pressure, the second term, $\vec{F}_2$, is the Doppler effect of the incidence radiation, while the third, $\vec{F}_3$, is due to the corresponding absorption and retransmission of one part of the incidence radiation. The last two terms taken together make up the Poynting-Robertson effect [20].

Let's use Popovici notation [5]:

$$F_p = \frac{R}{r^2}, \tag{3}$$

where $R = F_p\,|_{r=1}$ If we take into account that $\dot{r} = \vec{v}\vec{r}/r$ then eq. (2) becomes:

$$\vec{F}_{rad} = \frac{R}{r^2}\cdot\frac{\vec{r}}{r} - \frac{R}{r^2}\cdot\frac{\dot{r}}{c}\cdot\frac{\vec{r}}{r} - \frac{R}{r^2}\frac{\vec{v}}{c} = \frac{R}{r^2}\left(1-\frac{\dot{r}}{c}\right)\frac{\vec{r}}{r} - \frac{R}{r^2}\frac{\vec{v}}{c}. \tag{4}$$

Let us denote by $\vec{F}_{PR} = R/r^2\cdot\dot{r}/c\cdot\vec{r}/r + R/r^2\cdot\vec{v}/c$, the Poynting-Robertson (PR) effect, and by $\vec{F}_{CP} = R/r^2\cdot\dot{r}/c\cdot\vec{r}/r$ the Popovici's correction term (CP). We observe that:

$$\vec{F}_{CP} = \vec{F}_{PR}\,|_{\vec{v}/c=0}\,. \tag{5}$$

We note that for Popovici's model the orbital energy is not conserved. According to Stiefel and Scheifele [27], the Keplerian energy (per mass unit) is given by:

$$E_k = \frac{v^2}{2} - \frac{\mu}{r}, \tag{6}$$

where $\mu = G(M+m) \approx GM = A$, $m << M$. For the perturbed two-body problem, we know the equation:

$$\frac{dE_k}{dt} = (\vec{v},\vec{p}), \tag{7}$$

where the parenthesis on the right-hand side denote the *scalar product* of the vectors $\vec{v}$ and $\vec{p}$, and $\vec{p}$ represents forces other than central attraction which act on the particle of the mass m.

In our case $\vec{p} = \vec{F}_{rad}$, and, taking into account Eq. (2), there is obtain:

$$\begin{aligned}\frac{dE_k}{dt} &= (\vec{v},\vec{F}_{rad}) = (\vec{v},\vec{F}_1) + (\vec{v},\vec{F}_2) + (\vec{v},\vec{F}_3)\\ &= -\frac{d}{dt}\left(\frac{R}{r}\right) - \frac{R}{c}\left(\frac{\dot{r}}{r}\right)^2 - \frac{1}{c}v^2.\end{aligned} \tag{8}$$

If we denote by

$$E = E_k + \frac{R}{r} = \frac{v^2}{2} - \frac{A-R}{r} = \frac{v^2}{2} - \frac{k}{r}, \tag{9}$$

then, from Eq.(8), results:

$$\frac{dE}{dt} = -\frac{R}{c}\left(\frac{\dot{r}}{r}\right)^2 - \frac{1}{c}v^2. \tag{10}$$

If in Eq. (10), we consider $v^2/c = 0$ we recover the following Popovici's theorem [19]:

"L'energie ne se conserve plus. Cette quantite que l'on appelle dans la mecanique newtonienne l'energie, varie dans le meme sens avec le temps. La relation suivante, qui est en meme temps l'equation du mouvement:

$$\frac{dE}{dt} = -\alpha k\left(\frac{r'}{r}\right)^2, \quad r' = \frac{dr}{dt};$$

$\frac{dE}{dt} > 0$ *attraction;* $\frac{dE}{dt} < 0$ *repulsion, nous fait voir comment l'energie est depensee par le mecanisme de la propagation"*.

Eq. (10) for the variation of Keplerian energy is similar to those find by Robertson – [24], Eq. (5.10) p. 437.

3. THE ELLIPTIC RESTRICTED PHOTO-GRAVITATIONAL PROBLEM OF THE THREE BODIES C-TIN POPOVICI MODEL

A number of researchers have studied both the repulsive and the gravitational forces form the so-called *photo-gravitational force field*. It is well known, for the first time it was V.V. Radzievski who formulated "the photo-gravitational three-body problem" [21–23].

Let us consider the restricted circular three-body problem: P_1 (with the mass m_1), P_2 (with the mass m_2) and a third body P, with the mass m, negligible with respect to the mass of the first two. The P_1 and P_2 bodies, named "primaries", are moving along circular orbits around their own centre of mass, under the effect of gravitation. The body with a negligible mass is submitted to the photo-gravitational action generated by both primary. The equations of motion of the third body, in a vectorial form, can be written as (Fig. 1):

$$m\ddot{\vec{r}} = \vec{F}_1 + \vec{F}_2, \tag{11}$$

where

$$\vec{F}_i = \vec{F}_g + \vec{F}_{rad} = -G\frac{mm_i}{r_i^2}\frac{\vec{r}_i}{r_i} + \frac{L_i}{4\pi c r_i^2}a\left(1 - \frac{\dot{r}_i}{c}\right)\frac{\vec{r}_i}{r_i} \tag{12}$$

is the photo-gravitational action due to body P_i, where L_i - the luminosity of body P_i $(i = 1,2)$, c - the speed of light and a - the area of the section on the P_1P direction.

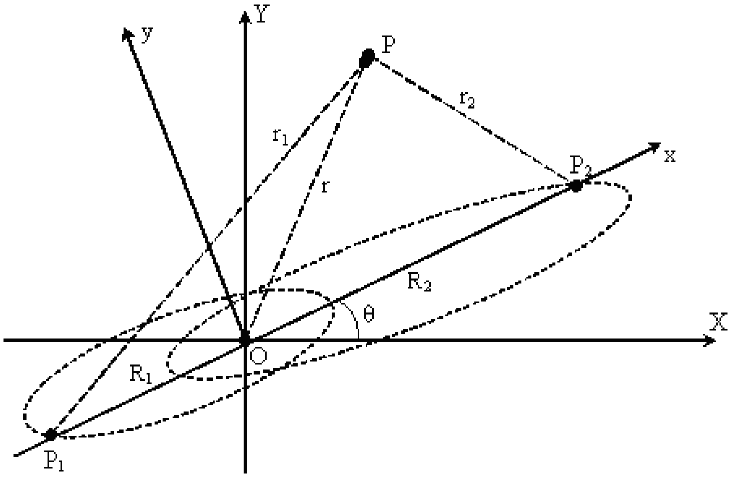

Fig. 1 – Systems of coordinates.

System of equations that we obtain after calculations (using the notation: $R_{CP1} = \frac{L_1}{4\pi c}$ and $R_{CP2} = \frac{L_2}{4\pi c}$, $q_1 = Gm_1 - \frac{R_{PC1}}{\sigma}$, $q_2 = Gm_2 - \frac{R_{PC2}}{\sigma}$, $\sigma = \frac{1}{a}$) is [5]:

$$\begin{cases} \ddot{x} - 2n\dot{y} = \dot{n}y + n^2x - \dfrac{q_1}{r_1^3}(x+R_1) - \dfrac{R_{CP1}}{\sigma r_1^3}\dfrac{\dot{r_1}}{c}(x+R_1) \\ -\dfrac{q_2}{r_2^3}(x-R_2) - \dfrac{R_{CP2}}{\sigma r_2^3}\dfrac{\dot{r_2}}{c}(x-R_2), \\ \ddot{y} + 2n\dot{x} = -\dot{n}x + n^2y - \dfrac{q_1}{r_1^3}y - \dfrac{R_{CP1}}{\sigma r_1^3}\dfrac{\dot{r_1}}{c}y - \dfrac{q_2}{r_2^3}y - \dfrac{R_{CP2}}{\sigma r_2^3}\dfrac{\dot{r_2}}{c}y. \end{cases} \tag{13}$$

Next, we will tackle the elliptical case problem. The important change appearing in elliptic case is R_1 and R_2, so the distances between the mass point and the primary bodies are no longer constant:

$$R_1 = \frac{m_2}{m_1+m_2}\cdot\frac{a(1-e^2)}{1+ecos\theta},\ R_2 = \frac{m_1}{m_1+m_2}\cdot\frac{a(1-e^2)}{1+ecos\theta}, \tag{14}$$

where R_1 and R_2 are time variables by means of θ, while a is the major semi axis and e is the eccentricity of the P_1 body orbit. We will analyse which of the elements that appear in the movement equations are influenced by the real anomaly variation θ.

In the elliptical restricted problem of the three bodies, the equilibrium points cannot be found in the meaning of the restricted circular problem of the three bodies. The state points in the elliptical problem will oscillate round an average value of the position in the circular problem [17].

We are going to develop in this pattern some considerations on the position of the libration collinear points related to the rotating coordination system. We find these points in the system when $\ddot{y} = \dot{y} = y = 0$:

$$\begin{cases} n^2 x - \frac{q_1 \cdot (x+R_1)}{r_1^3} - \frac{q_2 \cdot (x-R_2)}{r_2^3} - \frac{R_{CP1}}{\sigma r_1^3}\frac{\dot{r_1}}{c}(x+R_1) - \frac{R_{CP2}}{\sigma r_2^3}\frac{\dot{r_2}}{c}(x-R_2) = 0, \\ 2n\dot{x} = -\dot{n}x. \end{cases} \tag{15}$$

We make replacements in the relations from (15) and we introduce the terms r_1 and r_2 and we find the equation by replacing $\dot{R}_1$ and $\dot{R}_2$ and the values of the terms $\dot{r_1}$ and $\dot{r_2}$):

$$\begin{aligned} &\frac{C^2}{p^4}(1+ecos\theta)^4 x - \frac{q_1(x+R_1)}{|x+R_1|^3} - \frac{q_2(x-R_2)}{|x-R_2|^3} - \\ &\quad - \frac{R_{CP1}}{\sigma c}\frac{(x+R_1)^2}{|x+R_1|^4}\left(\frac{esin\theta}{1+ecos\theta}x + \frac{m_2}{m_1+m_2}\frac{\sqrt{\mu esin\theta}}{\sqrt{a(1-e^2)}}\right) - \\ &\quad - \frac{R_{CP2}}{\sigma c}\frac{(x-R_2)^2}{|x-R_2|^4}\left(\frac{esin\theta}{1+ecos\theta}x + \frac{m_1}{m_1+m_2}\frac{\sqrt{\mu esin\theta}}{\sqrt{a(1-e^2)}}\right) = 0. \end{aligned} \tag{16}$$

This equation can be solved on the intervals determined by the explanation of modules using the techniques from the numerical analysis whose parameter is the true anomaly. The variation of the ratio β_1, and similar for β_2, for the forces which are determined by the first primary body during the orbital movement can be calculated as follows ($i = 1,2$):

$$\beta_i = \frac{F_{Ri}}{F_{Gi}} = \frac{L_i}{4\pi\sigma c G m_i}\left(1 - \frac{\dot{r_i}}{c}\right). \tag{17}$$

This ratio is no longer constant, as in the case of the circular photo-gravitational problem of three bodies, because the term $\dot{r_i}$ appears and depends on the true anomaly. Starting from the relation (17), we may search the variation of ratio between the radiation force and the force of gravity reacting on the test particle from both stars of the binary system, depending on the true anomaly, knowing the particle radius and the material which makes it up.

4. THE INFLUENCE OF THE DIRECT RADIATION PRESSURE IN THE CASE OF W UMi BINARY SYSTEM

Let there be a spherical silicon particle in the binary W UMi system. In this section we will try to solve the following problems:

The trajectory of the particle in an inertial system, considering the radiation pressure. The trajectory of the particle in a rotating system, considering the radiation pressure. The particles equilibrium points.

We will use as an example the elliptical binary system W UMi of $e = 0.09$ eccentricity. We used the conditions found in the Svechnikov catalogue [26] and we got the necessary parameters within the calculations for the binary system W UMi [16].

The mass of the first star, $m_1 = 5.76839 \cdot 10^{30}$ kg, the mass of the second star, $m_2 = 2.486375 \cdot 10^{30}$ kg, the distance between the system components, $a = 67.11575 \cdot 10^8$ m, the system period, $P = 147134.9816$ s, the luminosity of the first star, $L_1 = 2.528333 \cdot 10^{28}$ w, the luminosity of the second star, $L_2 = 2.102719 \cdot 10^{27}$ w, the silicon particle density, $\rho = 2330$ kg/m^3.

The units of measurement that we will use thenceforth are: for mass $m_1 + m_2 = 1$, for lengths $a = 1$, for time $P = 2\pi$. From the third Kepler's law results the fact that $G = 1$. That is the meter, kilogram and second become: 1 m $= 1.48996 \cdot 10^{-10}$ units of length; 1 kg $= 0.12114215 \cdot 10^{-30}$ units of mass; 1 s $= 4.2681896 \cdot 10^{-5}$ units of time.

Other consequences: $m_1 = 0.6988$ units of mass, $m_2 = 0.3012$ units of mass, $a = 1$, $L_1 = 8.7447939 \cdot 10^{-10}$ units of luminosity, $L_2 = 7.2727149 \cdot 10^{-11}$ units of luminosity, $c = 1.0472 \cdot 10^3$ units of velocity, $\rho = 85.334$ units of density, $\omega_k = 1$ Keplerian angular velocity.

We can find out the variation of the ratios β_1 and β_2 in a relatively simplified way, when the infinitesimal point is in a collinear state point ($y = 0$).

$$\begin{cases} \beta_1 = \dfrac{L_1}{4\pi\sigma c G m_1} \cdot \left[1 - \dfrac{1}{c}\left(\dfrac{esin\theta}{1+ecos\theta}x + \dfrac{m_2}{m_1+m_2}\dfrac{\sqrt{\mu esin\theta}}{\sqrt{a(1-e^2)}}\right)\right], \\ \beta_2 = \dfrac{L_2}{4\pi\sigma c G m_2} \cdot \left[1 - \dfrac{1}{c}\left(\dfrac{esin\theta}{1+ecos\theta}x - \dfrac{m_1}{m_1+m_2}\dfrac{\sqrt{\mu esin\theta}}{\sqrt{a(1-e^2)}}\right)\right]. \end{cases} \tag{18}$$

In order to calculate the values of β_1 and β_2 we will use the relation [25]:

$$\beta_i = \frac{s^3 \cdot L_i}{4 \cdot \pi \cdot \sigma \cdot c \cdot G \cdot m_3 \cdot m_i}, \tag{19}$$

where the mass of the silicon particle is expressed by the relation $m_3 = \frac{4 \cdot \pi \cdot s^3}{3} \cdot \rho$ – s is the radius of the infinitesimal particle and ρ the silicon's density. Thus, we will obtain:

$$\beta_1 = \frac{L_1}{16 \cdot \pi \cdot \sigma \cdot c \cdot G \cdot m_1}, \beta_2 = \frac{L_2}{16 \cdot \pi \cdot \sigma \cdot c \cdot G \cdot m_2}. \tag{20}$$

That is, according to the calculations, in the new system of units:

$$\beta_1 = 0.8361 \cdot 10^{-15} \cdot \frac{1}{s}, \beta_2 = 0.1613 \cdot 10^{-15} \cdot \frac{1}{s}. \tag{21}$$

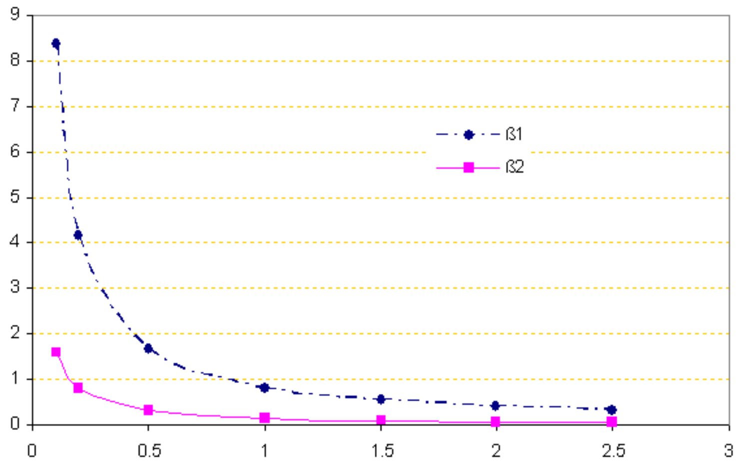

Fig. 2 – The variation of the parameters β_1 and β_2 according to the test particle's radius $s * 10^{-15}$.

We notice the fact that the two parameters depend on the test particle's radius which will be used as follows: when the radius increases the value of the β_1 and β_2 parameters decreases (Fig. 2), a fact that leads to a low influence of the radiation pressure.

The equations of the equipotential surfaces are:

$$U(x,y,z) = \frac{\omega_k(x^2+y^2)}{2} + \frac{(1-\beta_1)Gm_1}{r_1} + \frac{(1-\beta_2)Gm_2}{r_2} = const.$$

If the test particle's radius $s = 1.55 \cdot 10^{-15}$ units of length, we obtain three collinear equilibrium points: $x_{L1} = 1.2031$, $x_{L2} = 0.19966$, $x_{L3} = -0.9261$, we obtain two triangular equilibrium points $x_{L4} = 0.16019$, $y_{L4} = 0.831805$; $x_{L5} = 0.16019$, $y_{L5} = -0.831805$.

Remark:

In this case the equilibrium points outside the orbital plane do not exist.

We can calculate different constants for the equipotential surfaces that pass through the equilibrium points: for L_1 point $C_{L_1} = 1.472757578$; for L_2 point $C_{L_2} =$

0.1218154732; for L_3 point $C_{L_3} = 0.77776919$; for L_4 and L_5 similar point $C_{L_4} = 1.286824204$.

5. CONCLUSIONS

Starting from the motion of an infinitesimal particle in the gravitational and photo-gravitational fields of a star, we have deduced that a bigger influence of the term introduced by C-tin Popovici will appear when the particle moves within a system formed by two luminous bodies. In the W UMi system, binary system with $e = 0,09$ eccentricity, we have determined that there are no equilibrium points outside the orbital plane, we have determined the collinear libration points and the equations of the equipotential areas. We have discovered that for the silicon particle, the β_1 and β_2 ratios are influenced by the true anomaly and by the radius of the particle.

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REFERENCES

1. M.C. Anisiu, Ann. St. Univ. Al.I. Cuza Iasi **16**, 115 (1995).
2. M.C. Anisiu, Rom. Astron. J. **5**, 49 (1995).
3. M.C. Anisiu, Rom. Astron. J. **13**, 171 (2003).
4. M. Barbosu, T. Oproiu, General Mathematics **12**(2), Univ. ”Lucian Blaga”, Sibiu, **19** (2004).
5. C. Chiruta, Rom. Astron. J. **18**(Suppl.), 177 (2008).
6. G.F. Chorny, Celest. Mech. **97**(4), 229-248 (2007).
7. Ioannis Haranas, V. Mioc, Astrophys. Space. Sci. **331**(1), 289-294 (2011).
8. B. Ishwar, A. Elipe, Astrophys. Space. Sci. **277**(3), 437-446 (2001).
9. J. Klacka, Earth, Moon, and Planets **59**(1), 41-59 (1992).
10. J. Klacka, Earth, Moon, and Planets **61**(2), 119-124 (1993).
11. B.S. Kushvah, J.P. Sharma, B. Ishwar, Astrophys. Space. Sci. **312**(3-4), 279-293 (2007).
12. V.V. Markellos, E. Perdios, P. Labropoulou, Astrophys. Space. Sci. **194**(2), 207-213 (1992).
13. V.V. Markellos, E. Perdios, C. Georghiou, Astrophys. Space. Sci. **199**(1), 23-33 (1993).
14. V. Mioc, C. Blaga, Rom. Astron. J. **11**, 45 (2001).
15. V. Mioc, C. Blaga, Serb. Astron. J. **165**, 9 (2002).
16. Y. Nakamura, K. Asada, R. Sato, Inf. Bull. Var. Stars, no. **4647** (1998).
17. H.J. Pernicka, M.Sc. Thesis, Purdue University, West Lafayette (1986).
18. C. Popovici, Bull. Astron. **3**, 257 (1923).
19. C. Popovici, C. R. Acad. Sci. Paris **208**, 2052 (1939).
20. J.H. Poynting, Proc. Phys. Soc. London **19**, 475 (1903) .
21. V.V. Radzievski, Astron. J. **27**, 249 (in russian) (1950).
22. V.V. Radzievski, Astron Zh. **30**(3), 265 (in russian) (1953).
23. V.V. Radzievski, Astron Zh. **27**, 178 (in russian) (1966).
24. H.P. Robertson, Mon. Not. Roy. Astron. Soc. **97**, 423 (1937).

25. R. Roman, Publ. Astron. Dept. Eotvos Univ. **19**, 141 (2007).
26. M.A. Svechnikov, *Catalogue of Orbital Elements, Masses and Luminosities of Close Binary Stars* (in Russian, Univ. Irkutsk Publ. House, Irkutsk, 1986).
27. E.L. Stiefel, G. Scheifele, *Linear and Regular Celestial Mechanics* (Springer Verlag, Berlin, 1971).
28. C.G. Zagouras, Celest. Mech. **51**(4), 331 (1991).
29. Ming-Jiang Zhang, Chang-Yin Zhao, Yong-Qing Xiong, Astrophys. Space. Sci. **337**(1), 107-113 (2012).
30. A.S. Zimovshchikov, V.N. Tkhai, Solar System Res. **38**(2), 155-164 (2004).