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ASYMMETRIC TELECLONING USING NON-MAXIMALLY ENTANGLED CHANNELS

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Abstract. We discuss the asymmetric telecloning of a qubit using non-maximally entangled channels. The information of a qubit is transmitted to two spatially separated observers. We analyze the efficiency of this protocol by using the average fidelity.

Key words: quantum teleportation, quantum cloning, fidelity.

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1. INTRODUCTION

The *no-cloning* theorem forbids the perfect copy of an unknown quantum state [1]. Therefore the imperfect cloning was considered, this process being characterized as symmetric [2, 3], when the copies are identical or asymmetric if the copies are not the same [4–6].

An interesting task in quantum information theory is the transmission of the two imperfect clones to two spatially separated observers. The optimal protocol is called telecloning [7] and is constructed as a combination of quantum teleportation [8] and cloning. Quantum telecloning has found important applications: telebroadcasting of an entangled state [6], telecloning of a bipartite two-level entangled state [9], quantum processors, which are obtained with the help of telecloning and programmable quantum gate arrays [10, 11].

The quantum teleportation was generalized with the help of non-maximally entangled channels and modified Bell basis measurements [12]. The idea of this protocol was recently used by us for imperfect many-to-many teleportation [13] and generalized telebroadcasting of entanglement [14].

In addition, the non-maximally entangled channels were the key element of the generalization of the quantum symmetric telecloning [15]. Further we proposed the non-optimal asymmetric telecloning, a process that uses quantum channels constructed with given specific outputs from an asymmetric cloning machine [16].

The structure of the paper is as follows. In Sec. 2 we describe the imperfect one-to-many teleportation, where we use non-maximally entangled channels shared between the sender and receivers. Sec. 3 presents the main result: we apply the imperfect one-to-many teleportation to the case when the quantum channels are states obtained by the Pauli optimal asymmetric cloning machine. The two asymmetric clones transmitted to Bob 1 and Bob 2 are evaluated by considering the average fidelity. The conclusions are given in Sec. 4.

2. IMPERFECT ONE-TO-MANY TELEPORTATION

In this section we present the imperfect one-to-many teleportation, a process that is based on a non-maximally entangled channel shared by the sender and receivers.

The initial state of a spin-1/2 particle of Alice is:

$$|\psi\rangle_A = \alpha|0\rangle + \beta|1\rangle, \tag{1}$$

where α and β satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. The purpose of this protocol is to transmit the information of this state to M spatially separated receivers, *i.e.* the final state shared by the receivers to be

$$|\eta\rangle_{B_1 B_2 \ldots B_M} = \alpha|\phi_0\rangle + \beta|\phi_1\rangle. \tag{2}$$

The quantum channel shared by Alice and the M receivers B_1, B_2,..., B_M is a non-maximally entangled one, that depends on a real parameter n:

$$|\xi\rangle_{AB_1B_2\ldots B_M} = N\left(|0\rangle_A|\phi_0\rangle_{B_1\ldots B_M} + n|1\rangle_A|\phi_1\rangle_{B_1\ldots B_M}\right) \tag{3}$$

with N having the expression $N = 1/\sqrt{1+n^2}$. For $n = 1$, we obtain the standard one-to-many teleportation scheme [17].

The Bell basis of two qubits is defined as follows:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle);$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle);$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle);$$
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

The total $M+2$-particle state of the whole system becomes:

$$|\psi\rangle|\xi\rangle = \frac{N}{\sqrt{2}}\Big[|\Phi^+\rangle\Big(\alpha|\phi_0\rangle + \beta n|\phi_1\rangle\Big) + |\Phi^-\rangle\Big(\alpha|\phi_0\rangle - \beta n|\phi_1\rangle\Big) + |\Psi^+\rangle\Big(\alpha n|\phi_1\rangle + \beta|\phi_0\rangle\Big) + |\Psi^-\rangle\Big(\alpha n|\phi_1\rangle - \beta|\phi_0\rangle\Big)\Big].$$

The first two particles belong to Alice, who performs a measurement in the Bell basis. Alice obtains four possible outcomes that represent the elements of the Bell basis and communicates the result to the receivers B_1, B_2,..., B_M. The M receivers have to apply local unitary operations and will get the following two outcomes:

$$\begin{aligned} |\Lambda_0\rangle &= \frac{1}{\sqrt{|\alpha|^2+|\beta|^2 n^2}}\,(\alpha|\phi_0\rangle + \beta n|\phi_1\rangle); \\ |\Lambda_1\rangle &= \frac{1}{\sqrt{|\alpha|^2 n^2+|\beta|^2}}\,(\alpha n|\phi_0\rangle + \beta|\phi_1\rangle). \end{aligned} \tag{4}$$

The probabilities of obtaining the states $|\Lambda_0\rangle$ and $|\Lambda_1\rangle$, respectively are

$$\begin{aligned} p_0 &= \frac{1}{1+n^2}\,(|\alpha|^2+|\beta|^2 n^2); \\ p_1 &= \frac{1}{1+n^2}\,(|\alpha|^2 n^2+|\beta|^2). \end{aligned} \tag{5}$$

We can estimate the efficiency of the one-to-many teleportation with the help of the average fidelity:

$$\overline{F} = p_0|\langle\eta|\Lambda_0\rangle|^2 + p_1|\langle\eta|\Lambda_1\rangle|^2 = 1 - 2|\alpha|^2|\beta|^2 + \frac{4n|\alpha|^2|\beta|^2}{1+n^2}.$$

3. ASYMMETRIC TELECLONIG OF QUBITS USING NON-MAXIMALLY ENTANGLED CHANNELS

Further we analyze the possibility to transmit the two clones obtained by the Pauli optimal asymmetric cloning machine to two distant receivers Bob 1 and Bob 2. We propose to consider telecloning of qubits, a process that combines imperfect one-to-many teleportation described in Sec. 2 and asymmetric cloning.

The qubit to be telecloned is given by Eq. (1). We apply the method of Sec. 2, by considering the states $|\phi_0\rangle$ and $|\phi_1\rangle$ of the quantum channel (2) to be the following three-particle states:

$$\begin{aligned} |\phi_0\rangle_{B_1B_2B_3} &= \frac{1}{\sqrt{1+p^2+q^2}}\,(|000\rangle + p|011\rangle + q|101\rangle) \\ |\phi_1\rangle_{B_1B_2B_3} &= \frac{1}{\sqrt{1+p^2+q^2}}\,(|111\rangle + p|100\rangle + q|010\rangle), \end{aligned}$$

where p and q satisfy $p+q=1$. These states are obtained by the Pauli optimal asymmetric cloning machine [4]. Let us denote by $|\Upsilon_0\rangle$ and $|\Upsilon_1\rangle$ the one-particle states

$$|\Upsilon_0\rangle = \frac{1}{\sqrt{|\alpha|^2+|\beta|^2 n^2}}\,(\alpha|0\rangle+\beta n|1\rangle);$$
$$|\Upsilon_1\rangle = \frac{1}{\sqrt{|\alpha|^2 n^2+|\beta|^2}}\,(\alpha n|0\rangle+\beta|1\rangle).$$

3.1. THE OUTCOME $|\Lambda_0\rangle$ OF THE ONE-TO-MANY TELEPORTATION

According to the Sec. 2, the observers B_1, B_2, and B_3 will obtain the state $|\Lambda_0\rangle$ of Eq. (4) with the probability p_0. The third observer B_3 has an ancillary particle and we will compute the reduced density operators of the observers B_1 and B_2. The reduced state of the observer B_1 is

$$\rho_0^{B_1} = \mathrm{Tr}_{B_2,B_3}|\Lambda_0\rangle\langle\Lambda_0| = \frac{1}{1+p^2+q^2}\left[(1-q^2+p^2)|\Upsilon_0\rangle\langle\Upsilon_0|+q^2 I\right]. \tag{6}$$

The fidelity of this state and the initial state $|\psi\rangle$ has the expression:

$$F_0^{B_1} = \langle\psi|\rho_0^{B_1}|\psi\rangle = \frac{1}{1+p^2+q^2}\left[(1-q^2+p^2)\frac{(|\alpha|^2+|\beta|^2 n)^2}{|\alpha|^2+|\beta|^2 n^2}+q^2\right]. \tag{7}$$

The reduced density operator of the observer B_2 is as follows

$$\rho_0^{B_2} = \mathrm{Tr}_{B_1,B_3}|\Lambda_0\rangle\langle\Lambda_0| = \frac{1}{1+p^2+q^2}\left[(1+q^2-p^2)|\Upsilon_0\rangle\langle\Upsilon_0|+p^2 I\right]. \tag{8}$$

The fidelity of the mixed state $\rho_0^{B_2}$ and the initial state $|\psi\rangle$ has the expression:

$$F_0^{B_2} = \langle\psi|\rho_0^{B_2}|\psi\rangle = \frac{1}{1+p^2+q^2}\left[(1+q^2-p^2)\frac{(|\alpha|^2+|\beta|^2 n)^2}{|\alpha|^2+|\beta|^2 n^2}+p^2\right]. \tag{9}$$

3.2. THE OUTCOME $|\Lambda_1\rangle$ OF THE ONE-TO-MANY TELEPORTATION

Following the one-to-many teleportation, the observers B_1, B_2, and B_3 will obtain the state $|\Lambda_1\rangle$ of Eq. (4) with the probability p_1. The observer B_1 will have the mixed state:

$$\rho_1^{B_1} = \mathrm{Tr}_{B_2,B_3}|\Lambda_1\rangle\langle\Lambda_1| = \frac{1}{1+p^2+q^2}\left[(1-q^2+p^2)|\Upsilon_1\rangle\langle\Upsilon_1|+q^2 I\right]. \tag{10}$$

The fidelity of this state and the initial state $|\psi\rangle$ has the expression:

$$F_1^{B_1} = \langle\psi|\rho_1^{B_1}|\psi\rangle = \frac{1}{1+p^2+q^2}\left[(1-q^2+p^2)\frac{(|\alpha|^2 n+|\beta|^2)^2}{|\alpha|^2 n^2+|\beta|^2}+q^2\right]. \tag{11}$$

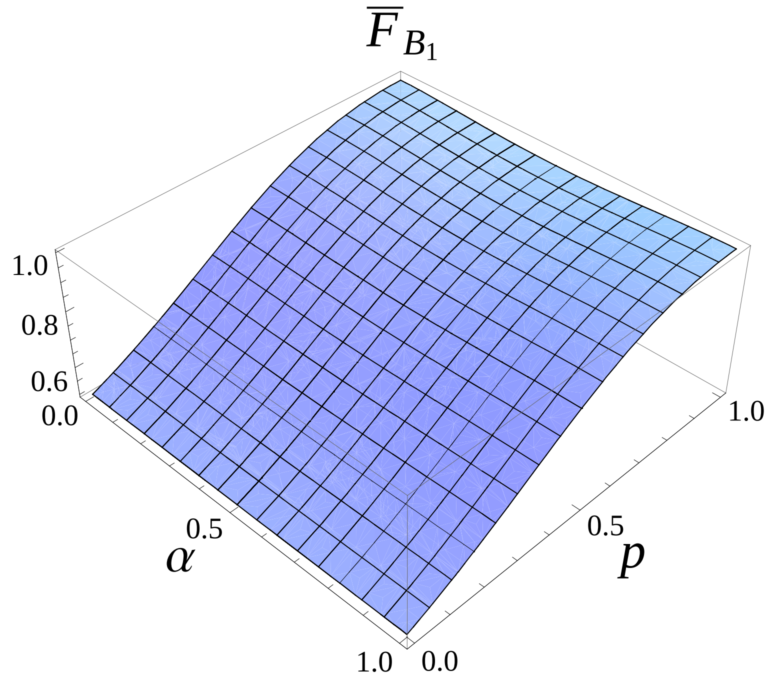

Fig. 1 – The average fidelity of the clone of the observer B_1 for $n = 1/2$.

The reduced density operator of the observer B_2 is as follows

$$\rho_1^{B_2} = \mathrm{Tr}_{B_1,B_3} |\,\Lambda_1\,\rangle\langle\,\Lambda_1\,| = \frac{1}{1+p^2+q^2}\left[(1+q^2-p^2)\,|\,\Upsilon_1\,\rangle\langle\,\Upsilon_1\,| + p^2\, I\right]. \quad (12)$$

The fidelity of the mixed state $\rho_1^{B_2}$ and the initial state $|\,\psi\,\rangle$ has the expression:

$$F_1^{B_2} = \langle\,\psi\,|\rho_1^{B_2}|\,\psi\,\rangle = \frac{1}{1+p^2+q^2}\left[(1+q^2-p^2)\,\frac{(|\alpha|^2 n + |\beta|^2)^2}{|\alpha|^2 n^2 + |\beta|^2} + p^2\right]. \quad (13)$$

3.3. THE AVERAGE FIDELITY OF THE TWO ASYMMETRIC CLONES

We evaluate the average fidelity of the clone obtained by the observer B_1:

$$\overline{F}_{B_1} = p_0\, F_0^{B_1} + p_1\, F_1^{B_1} = \frac{1-q^2+p^2}{1+p^2+q^2}\,(1-2|\alpha|^2|\beta|^2)$$
$$+ \frac{1}{1+n^2}\,\frac{1-q^2+p^2}{1+p^2+q^2}\,4\,|\alpha|^2\,|\beta|^2 n + \frac{1}{1+p^2+q^2}\,q^2.$$

We plot in Fig. 1 the average fidelity of the clone of the observer B_1 for $n = 1/2$.

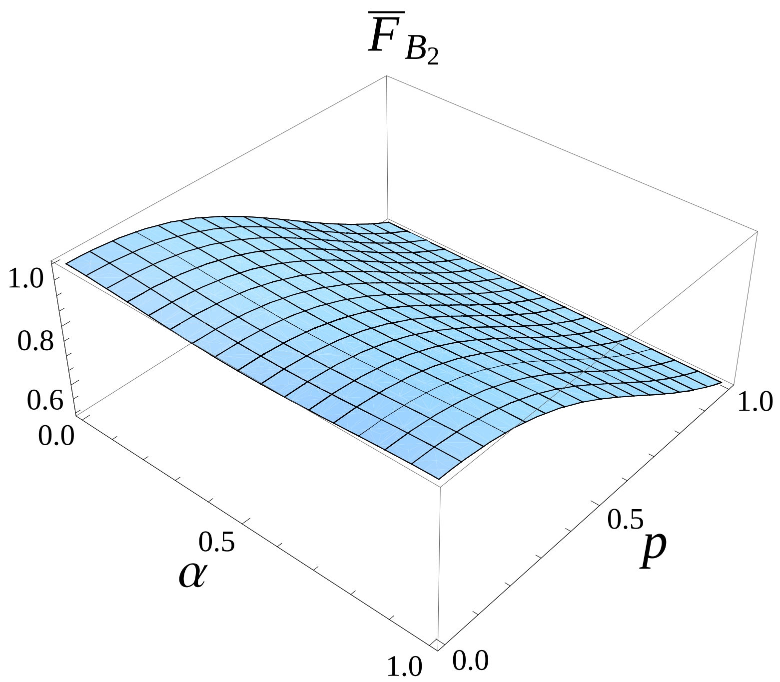

Fig. 2 – The average fidelity of the clone of the observer B_2 for $n = 1/2$.

The average fidelity of the clone obtained by the observer B_2:

$$\overline{F}_{B_2} = p_0\, F_0^{B_2} + p_1\, F_1^{B_2} = \frac{1+q^2-p^2}{1+p^2+q^2}\,(1-2|\alpha|^2|\beta|^2) + \frac{1}{1+n^2}\,\frac{1+q^2-p^2}{1+p^2+q^2}\,4\,|\alpha|^2\,|\beta|^2 n + \frac{1}{1+p^2+q^2}\,p^2.$$

The average fidelity of the clone of the observer B_2 for $n = 1/2$ is shown in Fig. 2.

4. CONCLUSIONS

We have analyzed the asymmetric telecloning of a qubit using non-maximally entangled channels. The quantum channel is constructed with the help of two states obtained using the Pauli optimal asymmetric cloning machine. The efficiency of this process is measured by the average fidelity of the outcome of each receiver.

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