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THE DETERMINATION OF TWO PHOTON THERMAL FIELDS IN LASER-TWO-LAYER SOLIDS WEAK INTERACTION USING GREEN FUNCTION METHOD

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Abstract. The goal of this paper is to improve our previous model; which considered the thermal fields for laser-periodic multilayer structures interaction. Our new model, based on the assumption of a weak interaction, takes into account non-linear effects like two photon absorption. It is assumed that the laser beam is in the IR and the interaction between the laser field and target (for example optical components) is weak, thus one can consider a small variation for the laser intensity. The Green function method, is used since it is a more adequate technique to solve the heat equations. Since the heat equation is a linear one in the sense that we have two solutions of the heat equation, than the sum of solutions is also a solution of the same heat equation, it is possible to use this property in order to "manipulate" the heat equations. Our approach is for a 1D model.

Key words: heat equation, Green function method, two-photons, multi-layer.

1. INTRODUCTION

Many practical applications require the detailed study of the thermal behavior of various systems. The difficulties arise when these systems are inhomogeneous with respect to the parameters involved in the heat diffusion process. Currently, the heat diffusion equation has no analytical solution for this case. However a wide range of methods exist to approximate the solution of the heat diffusion equation in inhomogeneous systems, starting from the numerical methods and ending with the exact analytical solution for a few particular cases, each of them in turn presenting specific advantages and disadvantages [1-6].

2. THE THEORETICAL BACKGROUND REGARDING GREEN FUNCTION METHOD

The Green function method is used to generalize the model found in the literature [7]. The case considered is that where the direction of heat transfer is perpendicular to several layers in series. We introduce the effective thermal conductivity for this multilayer medium as [7]:

$$K_{\perp} = \frac{l_1 + l_2 + \cdots + l_n}{l_1/k_1 + l_2/k_2 + \cdots + l_n/k_n} = \frac{l}{l_1/k_1 + l_2/k_2 + \cdots + l_n/k_n}, \tag{1}$$

where l is the total thickness of this medium structure.

Similarly, the effective thermal conductivity for heat flows in a parallel direction is:

$$K_{\parallel} = \frac{K_1 A_1 + K_2 A_2 + \cdots + K_n A_n}{A_1 + A_2 + \cdots + A_n} = \frac{K_1 A_1 + K_2 A_2 + \cdots + K_n A_n}{A}, \tag{2}$$

where A is the area of the near or far faces.

For most materials, the heat flux in one direction is only caused by the temperature gradient in that direction. In this case, the heat equation in equilibrium can be written as:

$$\left\{ \frac{\partial}{\partial x}[K_x \cdot \frac{\partial T}{\partial X}] + \frac{\partial}{\partial y}[K_y \cdot \frac{\partial T}{\partial y}] + \frac{\partial}{\partial z}[K_z \cdot \frac{\partial T}{\partial z}] \right\} = -A(x, y, z), \tag{3}$$

where K_x, K_y, K_z: are thermal conductivity along *x, y, z* directions. $A(x, y, z)$: is the heat source of the equation. Because we are considering, in this paper, a solid without melting, we can neglect the blackbody radiation of the sample.

We introduce the linear temperature by the expression:

$$\theta(T) = \theta(T_0) + (1/K(T_0)) \int_{T_0}^{T} K(T')\mathrm{d}T', \tag{4}$$

wherein the above equation, $\theta(T)$ and $\theta(T_0)$ are the linearized temperatures at a temperature T and at a substrate temperature T_0.

It can be observed now that the variation of the linear temperature is equal to the variation of the usual temperature when K is independent of T, an assumption which is reasonable for a weak interaction. The linear temperature obtained now for the heat equation is [7]:

$$\theta_{\perp} = \frac{P[1 - R_{\perp}(T)]}{(g)^{1/2}(\pi)^{3/2} K(T_0)} \int_0^{\infty} f_{\perp}(\xi)\mathrm{d}\xi \tag{5}$$

where $g = \frac{K_\perp}{K_{||}}$ and $K_{||} = K(T)$, $P = I/w$ is the normalized incident power, $R_\perp$ is the surface reflectivity when the incident beam is perpendicular to the layer structure of the substrate and T_0 is the original substrate temperature before the laser irradiation.

The function $f_\perp(\xi)$ is defined by:

$$f_\perp(\xi) = \frac{\exp-[[X^2(\xi^2+1)]+[Y^2/(\xi^2+1)]+(Z^2/g\xi^2)]}{\left\{(\xi^2+1)\right\}}, \tag{6}$$

with: $X = x'/w$, $Y = y'/w$ and $Z = z/w$, which are the normalized coordinates of the system. Here *w* represents the waist of the laser beam.

3. THE THEORETICAL TREATMENT FOR THE CLASSICAL AND SEMI-CLASSICAL HEAT EQUATIONS

In this chapter we will obtain a heat equation, considering the case of a double-layer structure (the layers are identical geometrically i.e., two serial parallelepipeds) which is continuously irradiated with a CO_2 laser beam. The laser beam has a Gaussian shape and we assume: $\frac{\mathrm{d}I}{I} \ll 1$. This assumption is valid for very low linear absorption coefficients; which is our case.

We will use the linearity of the heat equation in order to reach our goal which is the determination of the two photon thermal fields in during a laser-multilayer weak interaction. We consider *z* to be the direction of the laser beam propagation.

We have then:

$$A\left(x,y,z,t\right) = \left(\alpha \cdot I\left(x,y,z,t\right) + \beta I^2\left(x,y,z,t\right)\right)\left(h\left(t\right) - h\left(t-t_0\right)\right), \tag{7}$$

where *x, y, z* are the spatial coordinates, *t* is the time, t_0 is the exposure time during the irradiation and h is the step-function. The one photon absorption coefficient and the two absorption coefficient are α and β respectively.

The semi-classical heat equations for layer 1and respectively layer 2, are:

$$\begin{aligned}\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2} - \frac{1}{\gamma_1}\cdot\frac{\partial T_1}{\partial t} &= -\frac{A_1\left(x,y,z,t\right)}{K_1} = \\ &= -\frac{\left(\alpha_1 I + \beta_1 I^2\right)\left(h\left(t\right) - h\left(t-t_0\right)\right)}{K_1},\end{aligned} \tag{8.a}$$

and

$$\frac{\partial^2 T_2}{\partial x^2}+\frac{\partial^2 T_2}{\partial y^2}+\frac{\partial^2 T_2}{\partial z^2}-\frac{1}{\gamma_2}\cdot\frac{\partial T_2}{\partial t}=-\frac{A_2\left(x,y,z,t\right)}{K_2}=$$
$$=-\frac{\left(\alpha_2 I+\beta_2 I^2\right)\left(h\left(t\right)-h\left(t-t_0\right)\right)}{K_2}. \tag{8.b}$$

Here γ_1 and γ_2 are the thermal diffusivities.

If we consider that the both layers merge at $z = 0$, we have at the contact plane, the following boundary condition:

$$K_1\frac{\partial T_1}{\partial x}\Big|_{z=0}=K_2\frac{\partial T_2}{\partial x}\Big|_{z=0} \tag{9.a}$$

and

$$T_1(x,y,0,t)=T_2(x,y,0,t). \tag{9.b}$$

If we consider that: $\gamma_1\approx\gamma_2$ and $K_1\approx K_2$, the both heat equation could be replaced by one general heat equation:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}-\frac{1}{\gamma}\frac{\partial T}{\partial t}=-\frac{\left(\alpha_1 I+\beta_1 I^2+\alpha_2 I+\beta_2\cdot I^2\right)}{K}\cdot$$
$$\cdot\left(h\left(t\right)-h\left(t-t_0\right)\right) \tag{10}$$

If we use now the linearity of the heat equation, we can split the above equation into:

$$\frac{\partial^2 T_\alpha}{\partial x^2}+\frac{\partial^2 T_\alpha}{\partial y^2}+\frac{\partial^2 T_\alpha}{\partial z^2}-\frac{1}{\gamma}\frac{\partial T_\alpha}{\partial t}=-\frac{(\alpha_1 I+\alpha_2 I)}{K}\cdot\left(h\left(t\right)-h\left(t-t_0\right)\right) \tag{11.a}$$

and

$$\frac{\partial^2 T_\beta}{\partial x^2}+\frac{\partial^2 T_\beta}{\partial y^2}+\frac{\partial^2 T_\beta}{\partial z^2}-\frac{1}{\gamma}\frac{\partial T_\beta}{\partial t}=-\frac{(\beta_1 I^2+\beta_2 I^2)}{K}\cdot\left(h\left(t\right)-h\left(t-t_0\right)\right), \tag{11.b}$$

where:

$$T=T_\alpha+T_\beta. \tag{12}$$

Now we make two reasonable physical assumptions: i) we are interested only in the steady state solution and in consequence in equations (8), (10) and (11) we can neglect the temperature derivative with respect to time; and ii) the thermal conductivities along the *x, y ,z* axes are equal. For these conditions we have a perfect compatibility between equations (3) and (8), (10), (11). Thus using these conditions, we linked between the theories from chapter 2 and the theories from chapter 3, and may use formula (5) for solving the heat equation.

4. SIMULATIONS

We consider a system consisting of equal pieces of GaAs (the first bulk sample which interact with the laser beam) and ZnTe with geometrics of: $l/2 = l_1 = l_2 = 4\text{mm}$ and: $A = A_1 = A_2 = 4\text{mm}^2$. In this case the following results are obtained. From the bulk thermal conductivity, one can calculate $K_\perp$ and $K_\parallel$ using the relations [7]:

$$K_\perp = {}^{2K_{GaAs}K_{ZnTe}}\!\big/_{(K_{GaAs}+K_{ZnTe})}\,, \qquad K_\parallel = {}^{(K_{GaAs}+K_{ZnTe})}\!\big/_{2}\,.$$

All the results are considered when the steady state is reached. The source we consider is a Gaussian cwCO_2 laser at $10.6\mu\text{m}$ (waist of $w = w_x = w_y = 1\text{mm}$), and intensity of $I = 10^8 \text{W/cm}^2$.

We will use the values from [8-10]: for GaAs, $\alpha_1 = 0.00001\text{mm}^{-1}$, $\beta_1 = 0.35\text{cm/MW}$, $k_{GaAs} = 0.04\text{W/(mmK)}$ and for ZnTe: $\alpha_2 = 0.00003\text{mm}^{-1}$, $\beta_2 = 0.5\text{cm/MW}$, $k_{ZnTe} = 0.02\text{W/(mmK)}$; We suppose for simplicity that $R_\perp = 0$. We consider in using equation(5) that the power is given for the case of two photon interaction by: $P = -(\alpha I + \beta I^2)$.

We consider also, following the technique from reference [11] that:

$$\alpha = \alpha_{\text{GaAs}} \times \text{Tanh}[z] \cdot H[-z] + \alpha_{\text{ZnTe}} \times \text{Tanh}[z] \cdot H[z] + 0.00002\,, \qquad (13)$$

and

$$\beta = \beta_{\text{GaAs}} \times \text{Tanh}[z] \cdot H[-z] + \beta_{\text{ZnTe}} \times \text{Tanh}[z] \cdot H[z] + 0.45. \qquad (14)$$

In reference [11], we give a full discussion regarding the use of Tanh[z] for making a smooth transition of the absorber coefficients at $z = 0$). Here H represents the step function. The one- and two- absorption coefficient we have plot the figures 1 and 2. To solve the heat equations we have used formula (5).

We present in Fig. 3, the thermal field produced by just one-photon absorption.

We present in Fig. 4, the thermal field produced by just two-photons absorption. One may observe that the thermal field produced by just two photon absorption is clearly detectable in this case. In Fig. 5, we plot the thermal field produced by one- and two-photons absorption mechanisms.

It is obviously the fact that one-photon absorption is more important in the heat equation than the effects produces by only two-photon fields.

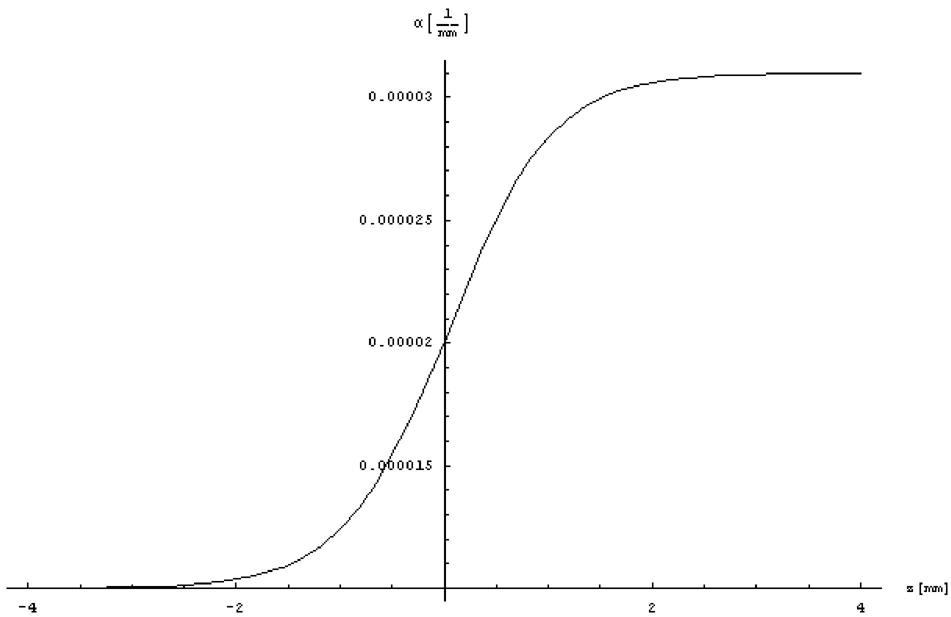

Fig.1 – The "global" one-photon absorption coefficient ([–4, 0] interval is located the GaAs sample, and from [0, 4] interval the ZnTe sample; the GaAs sample is the first heated by the laser beam).

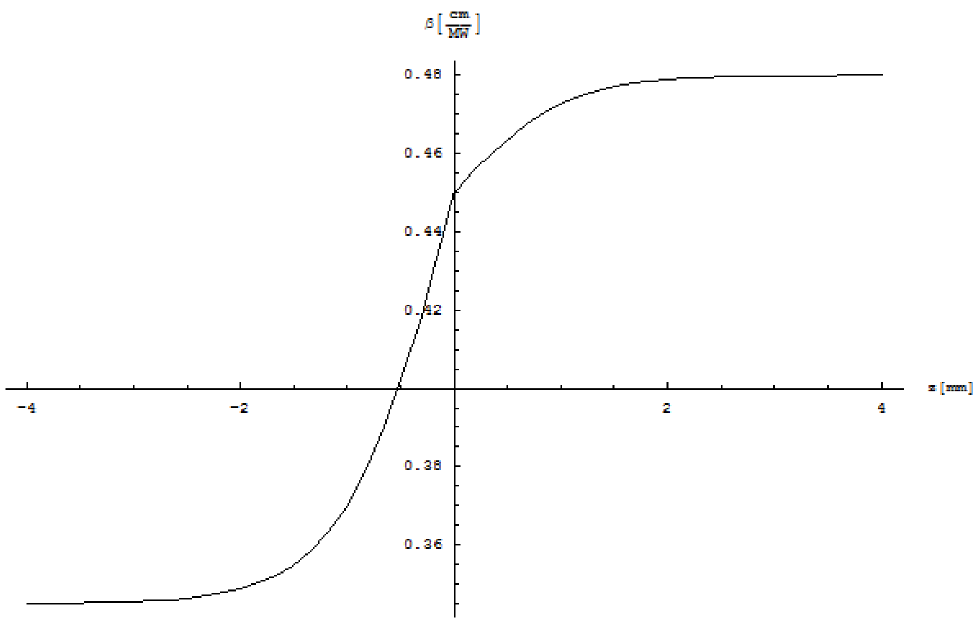

Fig. 2 – The "global" two-photon absorption coefficient. ([-4, 0] interval is located the GaAs sample, and from [0, 4] interval the ZnTe sample; the GaAs sample is the first heated by the laser beam).

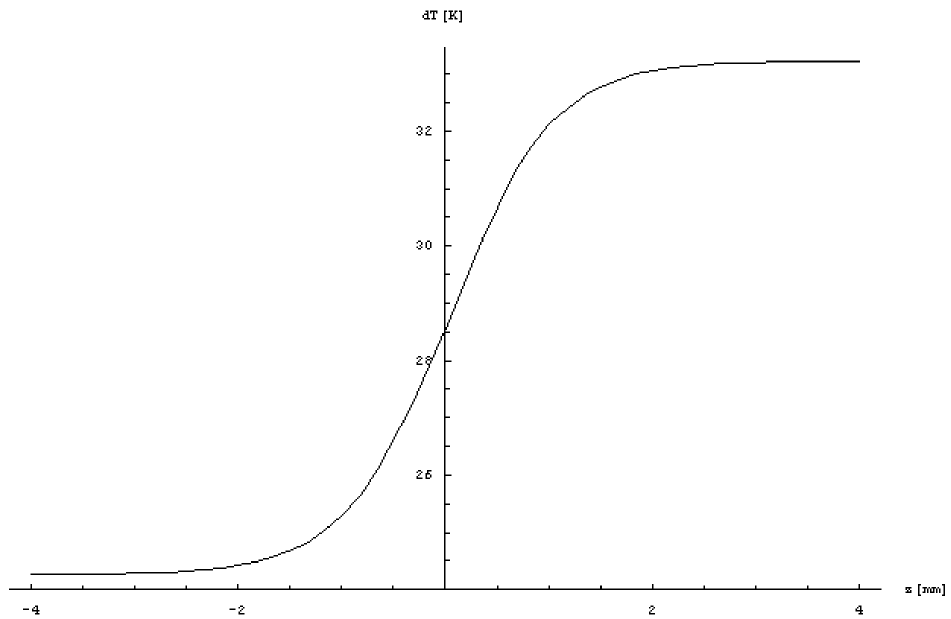

Fig. 3 – The thermal field produced by just one-photon absorption.

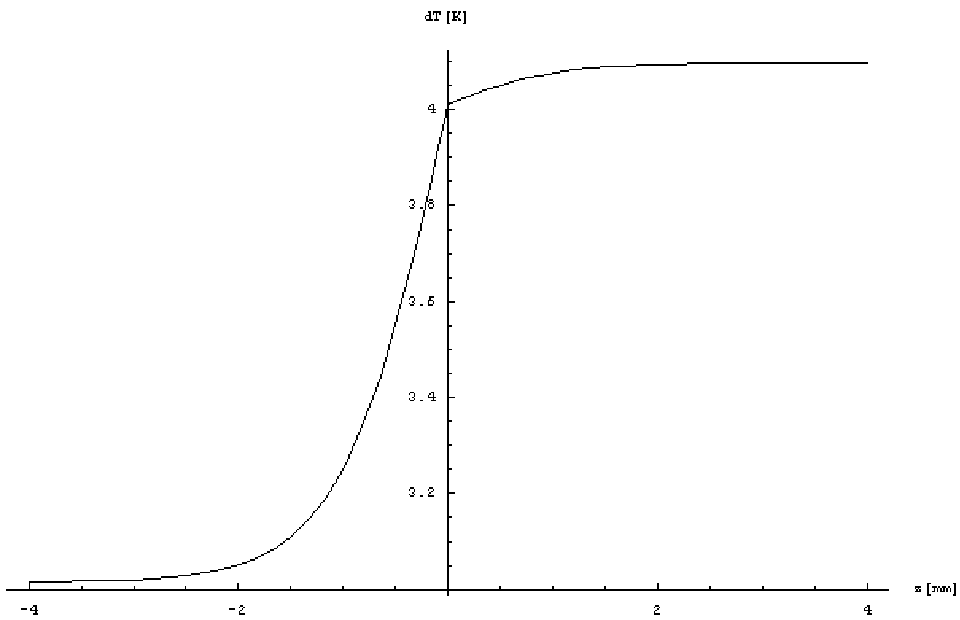

Fig. 4 – The thermal field produced by just two-photon absorption.

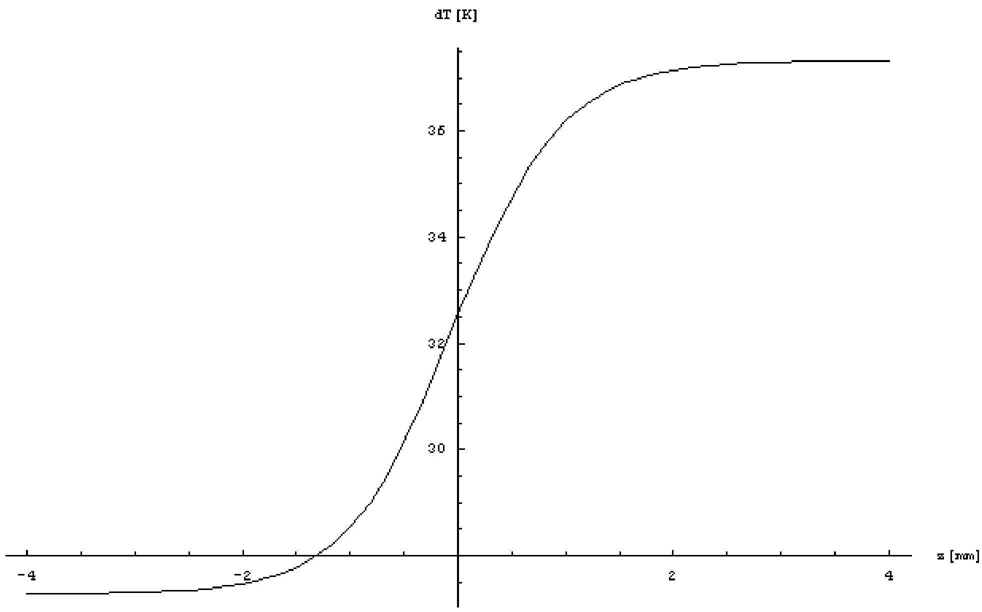

Fig. 5 – The thermal field produced by one and two-photon absorption.

5. CONCLUSIONS

We have studied the bulk properties of ZnTe and GaAs during laser irradiation deriving a model which assumes a weak laser-target interaction.

In this paper, the model has been used to analyze the temperature increase upon laser irradiation of two serial pieces of different materials and also determine if the two photons absorption mechanism can produce a detectable temperature variation. The calculation showed that it is possible to thermal detects the two photons interaction even in the case when the laser-solid interaction is weak.

To solve the heat equations we use two major mathematical tools: i) the Green function method and ii) the linearity of the heat equation obtaining thus an analytical solution.

We make also a thermal analysis as a function of contact interface length between GaAs and ZnTe. Our simulations indicate [12] that a shorter interface length do not affect the general thermal fields but change in an observable measure the thermal fields in the proximity of interface ($z = 0$).

In our model we consider that we have a steady state situation and also that the exchange between target and its surrounding media is null, which means that we have the conditions of very high vacuum.

A comparison between the Green function method and the integral transform may be found reference [13].

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