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DETUNING-DEPENDENT DYNAMICS OF MICROCAVITY PHOTON-STATISTICS

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Abstract. We investigate the resonant quantum dynamics of a moderately pumped two-level quantum dot inside a microcavity. We found that the microcavity photon-statistics depends crucially on its frequency detuning from the driving laser frequency. In particular, large photon correlations are obtained as well.

Key words: atom, photon-statistics, microcavity.

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1. INTRODUCTION

Quantum dots have received considerably attention recently also because in some cases behave as artifcial two-level atoms. Typically, the dots have nanometer sizes [1]. Interesting effects were found in such systems and some of them have potential applications in quantum-optical-information processing [2] and few-photon lasing. For example, population inversion of a driven two-level double quantum dot occurs via a structureless bath [3]. Recently, an experimental demonstration of optical quantum state inversion in a single semiconductor quantum dot using adiabatic rapid passage was performed in [4]. In atomic systems inversion happens in a more complex setup [5]. Furthermore, the resonance fluorescence spectra of a driven quantum dot placed inside a high-quality semiconductor cavity was studied in [6], while phonon-assisted damping of Rabi oscillations in semiconductor quantum dots was found in Ref. [7]. Nonmonotonic field dependence of damping and reappearance of Rabi oscillations in quantum dots was investigated in [8]. In addition, phonon-assisted transitions from quantum dot excitons to cavity photons and heat pumping with optically driven excitons were studied in [9] and [10], respectively. Inversionless gain, electromagnetically induced transparency, and refractive-index enhancement were shown to occur in a semiconductor quantum-dot structure [11]. Remarkably, superradiance from quantum dots was observed in [12]. The collective fluorescence and decoherence of a few nearly identical quantum dots and superbunched photons via a strongly pumped near-equispaced multiparticle system were analyzed in [13] and [14], respectively. Further, entanglement of two quantum dots was investigated in [15]. Moreover, a scheme that allows generation of the Bell and W-type entangled states of excitons in quantum dots confined in a single-mode microcavity was proposed, respectively, in Ref. [16]. Finally, the production of polarization-entangled photons through the biexciton cascade decay in a single semiconductor quantum dot was studied in [17].

Here, we investigate the dynamics of a quantum two-level artificial or real atom inside a microcavity. When the laser that resonantly pumps the two-level emitter is moderately intense, *i.e.* the corresponding Rabi frequency is larger than the qubit-cavity coupling strength as well as the spontaneous and cavity decay rates, we found enhanced photon-photon correlations. In particular, the photon correlations are larger than those of a thermal light source. Sensitive dependence of photon-statistics on the laser frequency detuning with respect to the cavity frequency is shown to occur.

The article is organized as follows. The analytical approach and the system of interest as well as the corresponding equations of motion are given in Section 2. Section 3 deals with discussions of the obtained results, while the Summary is given in Section 4.

2. ANALYTICAL APPROACH

The Hamiltonian describing a pumped two-level artificial (or real) single atom possessing the frequency ω_0 and embedded in a microcavity of frequency ω_c and interacting with a coherent source of frequency ω_L, in a frame rotating at ω_L, is [18–20]:

$$H = \hbar\Delta a^{\dagger}a + \hbar g(a^{\dagger}S^{-} + aS^{+}) + \hbar\Omega(S^{+} + S^{-}), \tag{1}$$

where we have assumed that $\omega_0 = \omega_L$. In the Hamiltonian (1) the first term describes the cavity free energy with $\Delta = \omega_c - \omega_L$, while the second one characterizes the interaction of the quantum dot system with the microcavity mode via the coupling g. The last term considers the interaction of the qubit subsystem with the laser with Ω being the corresponding Rabi frequency. The atomic operators $S^{+} = |2\rangle\langle 1|$ and $S^{-} = [S^{+}]^{\dagger}$ obey the commutation relations for su(2) algebra: $[S^{+}, S^{-}] = 2S_z$ and $[S_z, S^{\pm}] = \pm S^{\pm}$. Here $S_z = (|2\rangle\langle 2| - |1\rangle\langle 1|)/2$ is the bare-state inversion operator. $|2\rangle$ and $|1\rangle$ are the excited and ground state of the qubit, respectively. Further, $a^{\dagger}$ and a are the creation and the annihilation operator of the electromagnetic field (EMF), and satisfy the standard bosonic commutation relations, *i.e.*, $[a, a^{\dagger}] = 1$, and $[a, a] = [a^{\dagger}, a^{\dagger}] = 0$.

In what follows, we are interested in the laser dominated regime where $\Omega \gg$

$\{g,\gamma,\kappa\}$ (here γ and κ are the spontaneous and cavity decay rates, respectively) and shall describe our system using the dressed-states formalism [19, 20]:

$$|1\rangle = \frac{1}{\sqrt{2}}(|\bar{1}\rangle + |\bar{2}\rangle), \quad |2\rangle = \frac{1}{\sqrt{2}}(|\bar{2}\rangle - |\bar{1}\rangle). \tag{2}$$

Restricting ourselves to values of $\Delta \ll \Omega$ and secular approximation, one then arrives at the following master equation describing our system:

$$\begin{aligned}\frac{d}{dt}\rho(t) + i[H_0,\rho] = -\Gamma_0[R_z, R_z\rho] - \Gamma\{[R^+, R^-\rho] \\ + [R^-, R^+\rho]\} - \kappa[a^\dagger, a\rho] + h.c..\end{aligned} \tag{3}$$

Here

$$H_0 = \Delta a^\dagger a + g_0 R_z(a^\dagger + a),$$

where $g_0 = g/2$. Further, $\Gamma_0 = \gamma/4$ and $\Gamma = (\gamma + \gamma_d)/4$ with 2γ being the single-qubit spontaneous decay rate, while γ_d is the quantum dot dephasing rate. The new quasispin operators, *i.e.* $R^+ = |\bar{2}\rangle\langle\bar{1}|$, $R^- = [R^+]^\dagger$ and $R_z = (|\bar{2}\rangle\langle\bar{2}| - |\bar{1}\rangle\langle\bar{1}|)$ are operating in the dressed-state picture. They obey the following commutation relations: $[R^+, R^-] = R_z$ and $[R_z, R^\pm] = \pm 2R^\pm$.

In the next subsection, we shall obtain the equations of motion of variables of interest in order to calculate the second-order microcavity photon correlation function: $g^{(2)}(0) = \langle a^\dagger a^\dagger a a\rangle/(\langle a^\dagger a\rangle)^2$. Values of $g^{(2)}(0)$ smaller than unity describe the sub-Poissonian photon statistics and it is a quantum effect. The Poissonian photon-statistics has $g^{(2)}(0) = 1$. $g^{(2)}(0) > 1$ characterizes the super-Poissonian photon statistics. In particular, for thermal light one has $g^{(2)}(0) = 2$ and, therefore, we are interested in correlations larger than two, *i.e.* $g^{(2)}(0) > 2$.

2.1. EQUATIONS OF MOTION

Using Eq. (3) one can obtain the following equations of motion in order to calculate the microcavity photon intensity and their second-order photon-photon correlations:

$$\frac{d}{dt}\langle a^\dagger a\rangle = ig_0(\langle R_z a\rangle - \langle R_z a^\dagger\rangle) - 2\kappa\langle a^\dagger a\rangle, \tag{4a}$$

$$\frac{d}{dt}\langle R_z a\rangle = -(4\Gamma + \kappa + i\Delta)\langle R_z a\rangle - ig_0, \tag{4b}$$

$$\frac{d}{dt}\langle a^{\dagger 2} a^2\rangle = 2ig_0(\langle R_z a^\dagger a^2\rangle - \langle R_z a^{\dagger 2} a\rangle) - 4\kappa\langle a^{\dagger 2} a^2\rangle, \tag{4c}$$

$$\frac{d}{dt}\langle R_z a^\dagger a^2\rangle = ig_0\langle a^2\rangle - 2ig_0\langle a^\dagger a\rangle - (3\kappa + 4\Gamma + i\Delta)\langle R_z a^\dagger a^2\rangle, \tag{4d}$$

$$\frac{d}{dt}\langle a^2\rangle = -2ig_0\langle R_z a\rangle - 2(\kappa + i\Delta)\langle a^2\rangle. \tag{4e}$$

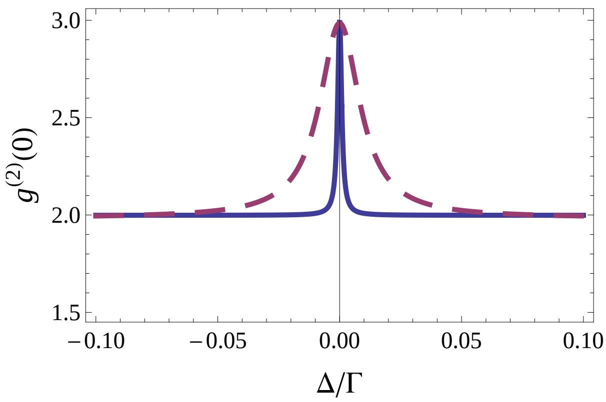

Fig. 1 – The steady-state dependence of the microcavity second-order photon correlation function $g^{(2)}(0)$ as a function of Δ/Γ. The solid line is for $\kappa/\Gamma = 0.001$ while the long-dashed line corresponds to $\kappa/\Gamma = 0.01$.

The system of equations (4) is not complete. Additional equations can be obtained via hermit conjugation of $\langle R_z a\rangle$, $\langle R_z a^\dagger a^2\rangle$ and $\langle a^2\rangle$.

In the following Section, we shall discuss the microcavity photon statistics.

3. RESULTS AND DISCUSSION

It is easy to show that the steady-state mean-photon number in the microcavity mode is given by the expression:

$$\langle a^\dagger a\rangle = \frac{(\kappa+4\Gamma)g_0^2}{[(\kappa+4\Gamma)^2+\Delta^2]\kappa}. \tag{5}$$

The general expression for the second-order photon correlation function in the steady-state is a little bit too cumbersome and it is represented as follows:

$$g^{(2)}(0) = \frac{\big((\kappa+4\Gamma)^2+\Delta^2\big)\big(3\kappa^2(\kappa+4\Gamma)(3\kappa+4\Gamma)+(\kappa^2+24\kappa\Gamma+32\Gamma^2)\Delta^2\big)}{(\kappa+4\Gamma)^2(\kappa^2+\Delta^2)\big((3\kappa+4\Gamma)^2+\Delta^2\big)}. \tag{6}$$

One can observe here that the second-order correlation function does not depend on microcavity-dot coupling strength g_0. For $\Delta = 0$ one arrive at a simpler expression for the photon correlation function:

$$g^{(2)}(0) = \frac{3(\kappa+4\Gamma)}{3\kappa+4\Gamma} = 3 - \frac{6\kappa}{3\kappa+4\Gamma}. \tag{7}$$

To understand the steady-state behaviors of the correlation function for lower values of κ/Γ, in Fig. 1, we plot the dependence of $g^{(2)}(0)$ versus Δ/Γ. Small variations of Δ/Γ lead to pronounced modifications of microcavity photon statistics.

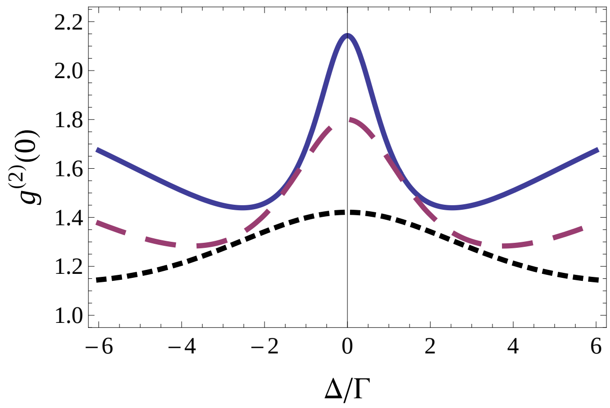

Fig. 2 – The steady-state dependence of the microcavity second-order photon correlation function $g^{(2)}(0)$ as a function of Δ/Γ. The solid line is for $\kappa/\Gamma = 1$, the dashed one corresponds to $\kappa/\Gamma = 2$, while the dotted line to $\kappa/\Gamma = 5$.

In particular, enhanced photon correlations characterized by the super-Poissonian photon statistics are observed at resonance. Figure 2 depicts the photon statistics for larger values of κ/Γ. For these particular cases one can observe lower values of $g^{(2)}(0)$ (compare Fig. 1 and Fig. 2). The second-order correlation function is symmetrical with respect to Δ/Γ and shows a small dip (see Fig. 2). For larger values of κ/Γ the photon statistics exhibits coherent features, *i.e.* $g^{(2)}(0)$ approaches unity.

Finally, we note that the above results were obtained for $\delta = \omega_0 - \omega_L = 0$. In this case the steady-state dressed-state inversion is zero, *i.e.* $\langle R_z \rangle = 0$. However, in Figures 1, 2, we changed the laser frequency meaning that $\delta \neq 0$ for some laser frequencies because we can not modify dynamically the atomic frequency ω_0 to be on resonance. Therefore, we estimated the steady-state dressed-state inversion to first order in the small parameter $\delta/\Omega \ll 1$, namely:

$$\langle R_z \rangle = -\frac{2\gamma\delta}{(\gamma + \gamma_d)\Omega}. \tag{8}$$

One can see that $\langle R_z \rangle \to 0$, when $\Omega \gg |\delta|$.

4. SUMMARY

In summary, we have investigated the quantum dynamics of a driven two-level quantum-dot system in a microcavity. Particularly, we focused on microcavity photon statistics. Large photon-photon correlations were shown to occur and sensitive dependence on cavity- pumping-laser detuning was demonstrated.

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