Romanian Reports in Physics, Vol. 65, No. 4, P. 1384–1389, 2013

DETERMINATION OF ELECTRON ENERGY DISTRIBUTION FUNCTION FROM THE INTENSITY OF SPECTRAL LINES BY TIKHONOV REGULARIZATION METHOD IN LOW PRESSURE HELIUM PLASMA*

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Received September 14, 2012

Abstract. The Tikhonov regularization method FORTRAN-subroutine has been adapted for different experimental plasma diagnostics applications. In this work special attention was focused on problem of determination an electron energy distribution-EEDF in plasma from the intensity of spectral lines. Several model tasks were done to check applicability limitations of the method. Such test parameters are chosen by intention to simulate, as close as possible, low-pressure helium plasmas for future comparisons. From our numerical experiments, we found that range of the reliable results for EEDF's is started just at the energy where these lines have maximum excitation function. In the case of helium plasmas it is triplet series $2p^3P^0 - nd^3D$.

Key words: regularization method, optical emission cross section, electron energy distribution function.

1. INTRODUCTION

A spectrum of radiation emitted from plasmas serves in the long run to obtain some data about their characteristics. The distribution pattern of spectral lines in terms of wavelength reflects the composition of the plasma, but for plasma spectroscopy other spectrum characteristics are much more important – the intensity distribution patterns. These intensity distribution patterns are dependant on the conditions under which this particular plasma is produced and on the parameter values this plasma has [1]. One of the most important parameter is the plasma temperature or in more general sense-the electron energy distribution function (EEDF).

* Paper presented at the 8th General Conference of Balkan Physical Union, July 5–7, 2012, Constanța, Romania.

Obtaining the EEDF of the plasma is ill-posed problem *i.e.* little non-avoidable errors in the measured spectrum can lead to significant changes in the determining of the EEDF. In this case, regularization has to be used to get a reliable solution. The best-studied method of regularization has been developed by Tikhonov for the solution of such problems [2]. He found an effective way to transfer an ill posed problem to correct one with minimum a priory information and subjective factors.

In this work special attention was focused on problem of determination an electron energy distribution-EEDF in plasma from the intensity of spectral lines. Several model tasks were done to check applicability limitations of the method. Such test parameters are chosen by intention to simulate, as close as possible, low-pressure helium plasmas for future comparisons. Principally, there are not any differences to apply the above method to any other plasma whose optical cross-sections of their spectral lines are known, but other parameter such as electron density are needed.

2. METHOD

In cases when the population of an atom level is due to direct electron impacts and cascade's transitions, and depopulation due to spontaneous transitions, intensities of spectral lines $I(x)$, emitted in plasmas, are in connection with the EEDF trough a formula [3]:

$$I_{ji}(n) = n_0 n_e h\nu(n) \int_{E_{thr}(n)}^{\infty} Q_{ji}(n,E)\sqrt{E} f(E)\mathrm{d}E\,, \quad (1)$$

where $x-$ parameter which characterizes a given spectral line, $n_0, n_e -$ densities of atoms in ground state and electrons respectively; $h-$ Plank's constant; $\nu(x)$, $Q(x,E)$ and $E_{thr}(x)-$ frequency of transition, optical excitation function and its threshold energy, respectively of the $x_i -$ spectral line; $E-$ electron energy.

Solving equation 1, as an inverse problem and knowing all other parameters, except EEDF –*f(E)*, one can obtain information about the distribution in the range of electron energy greater than $E_{thr}(x.)$ Unfortunately, a small non-avoidable errors in the measured data can result significant errors in determination of EEDF, *i.e.*, equation 1, is an ill posed problem from the mathematical point of view [2]. There are many methods dealing with such problems and Tikhonov's method shown itself as the most effective [4]. In this method was found a way to transfer the problem to correct one by using minimal a priory information about the asked function. Usually, this a priory information is an estimation about the experimental errors.

3. THE TESTS AND PROCEDURES

To test applicability of the method for use in a real experiment, several model tasks were introduced to simulate experimental data. For the sake of simplicity and minor software modification, the emission cross sections of the spectral lines were approximated by very well-known Fabrikant's formula [3]:

$$Q_{ji}(x,E) = Q_m(x)\frac{E - E_{thr}(x)}{x_{ij} - E_{thr}(x)}\exp\left(1 - \frac{E - E_{thr}(x)}{x_{ij} - E_{thr}(x)}\right), \tag{2}$$

where is $Q_m(x)$ the maximum of the emission cross sections (see Fig.1). As one can see from the Fig. 1, relative excitation functions of a few spectral lines are obtained by simple shifting the value of x_{ij}. We assumed as well, that the threshold energies are the same for all lines. Also, all other constants $n_0, n_e, h, \nu(x), Q_m(x)$ were taken to be equal 1, because its values in the simulation do not influence the main results.

The procedures of the test and the regularization algorithm are following: the excitation functions of the hypothetical spectral lines, described by eq. 2, were chosen and together, with a given EEDF, the spectral line intensities were calculated according to eq. 1 *i.e.*, the forward problem was solved. Then, artificial random noises, were added to the calculated intensities of the spectral lines to simulate a "measured" specter with experimental errors. Finally, Tikhonov's regularization method was used to solve the inverse problem, *i.e.*, from such "measured" specter and known optical emissions cross sections, the EEDF was determined and compared with the given EEDF.

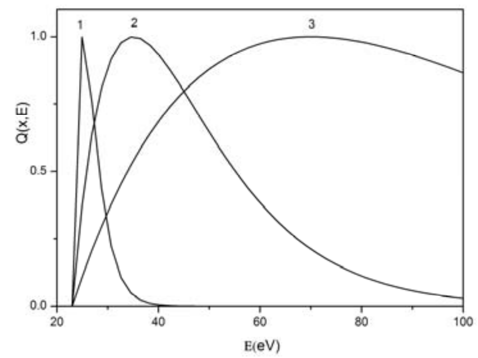

Fig. 1 – The relative excitation functions of three spectral lines; E_{thr}(x) = 23 eV for the all lines; x_1 = 25 eV, x_2 = 35eV, x_3 = 70eV.

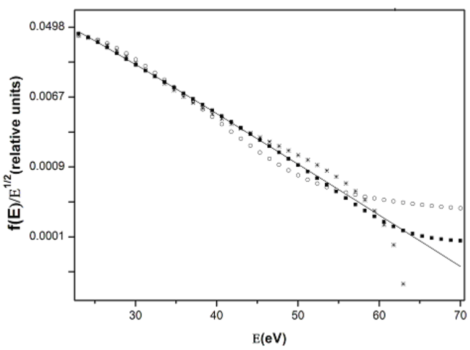

Fig. 2 – The EEDFs obtained by the regularization method, the exact EEDF is Maxwellian T_e = 5eV (solid line), (squares) 0.1% errors, (stars) 5 % errors, (circles) 10 % errors.

4. THE RESULTS

Obtaining the given EEDF from the intensity of spectral lines depends of many parameters: a shape of the EEDF, number of lines used for the EEDF determination, its excitation functions and experimental errors.

Example I. A hypothetical ideal case was chosen just to prove usefulness of the Tikhonov's method and the experimental error's influence on the results. EEDFs were chosen to be simple Maxwellian with temperature T_e=5eV and emission cross section was calculated according the Eq.2, $Q_m(x)$ is taken to be 1. Result obtained by regularization method for different experimental error are presented on Fig. 2. We took higher the electron temperatures in order to avoid small values of the EEDFs in this electron energy range.

Example II. Case study-helium low pressure plasmas: Such test parameters are chosen by intention to simulate, as close as possible, low-pressure helium plasmas for future comparisons. Helium plasmas were chosen due to several reasons: first, there are a lot of experimental data (spectrum, EEDF); second, there are a lot of atomic data (oscillator strengths, cross sections); and third, the intensities of spectral lines are highly sensitive in respect to the EEDF. In the work [5], the emission cross sections for electron-impact excitation have been measured for 73 transitions $(\lambda = 260 - 800\,\text{nm})$ belonging to the six series of He I spectrum:

singlet series (1) $2p\,^1P_1^0 - ns\,^1S_0$; (2) $2p\,^1P_1^0 - nd\,^1D_2$; (3) $2s\,^1S_0 - np\,^1P_1^0$;
triplet series (4) $2p\,^3P^0 - ns\,^3S$; (5) $2p\,^3P^0 - nd\,^3D$; (6) $2s\,^3S - np\,^3P^0$.

As it was shown in [5], the relative excitation functions for all lines belonging to one series are almost the same. In our method, as it is described above, are relevant only lines with different relative excitation functions and thus, one can use a suitable (easy measured) spectral line or a few lines per a series.

Table 1

No	λ (nm)	Transition	Threshold energy (eV)	x (eV)	Q_{max} 10^{20}cm^2	Q_{100eV} 10^{20}cm^2
1.	388.865	$2s\,^3S_1$-$3p^3P^0_{2,1,0}$	23.01	30	45.2	7.91
2.	396.473	$2s^1S_0$-$4p^1P^0_1$	23.74	100	3.10	3.10
3.	412.082 412.099	$2p^3P^0_{1,2}$-$5s^2S_1$ $2p^3P^0_0$-$5s^2S_1$	23.97	27	3.19	0.43
4.	414.376	$2p^1P^0_1$-$6d^1D_2$	24.21	40	2.35	1.63
5.	504.774	$2p^1P^0_1$-$4s^1S_0$	22.72	33	11.0	6.40

In the Table 1 is given a set of five spectral lines and their absolute excitation cross sections used in our numerical experiments [5]. More lines and their excitation cross sections one can find in [6]. Our assumption that the threshold energies are the same for all lines is fulfilled (the lower levels of transitions of neutral helium), because they are actually dispersed in a small energy interval, compared to energy interval where EEDF is searched.

The results of our simulation by using five spectral lines (presented on Fig. 3) from the Table I is shown on the Fig. 4 (added experimental error was 0.1%).

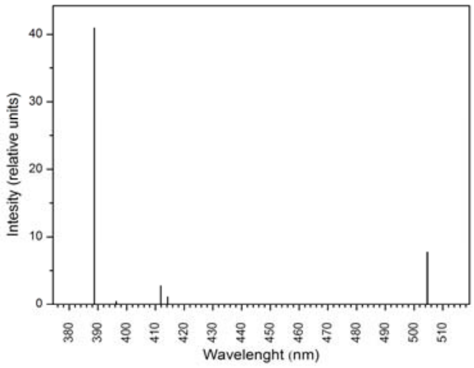

Fig. 3 – Relative intensity of five helium spectral lines.

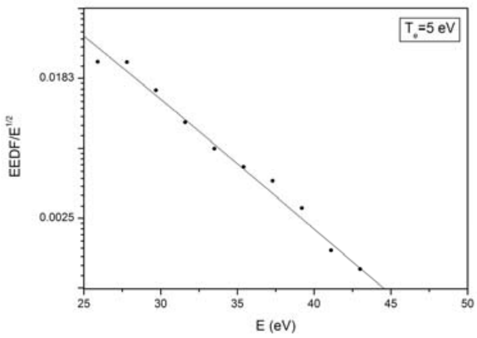

Fig. 4 – EEDF obtained by regularization method from 5 helium lines. The exact EEDF is Maxwellian – T_e=5 eV (solid line), EEDF obtained from five helium lines by regularization method (black circle).

5. DISCUSSION AND CONCLUSION

Determination of the “tail” of an EEDF is very important from the point of view of calculating the rates of threshold’s processes in plasmas. The specters of spectral lines, emitted by plasma carry out information of its EEDF. In the cases where EEDFs are close to Maxwellian shape the electron temperatures can be obtained. The temperature could be extracted either directly by measuring the ratio of the intensities of two appropriate spectral lines, or by using highly-advanced Tikhonov’s mathematical regularization procedure.

The range of the reliable results for EEDF’s obtained by regularization procedure is started just at the energy where spectral line has maximum excitation function. In the case of helium plasmas it is triplet series $2p\,^3P^0 - nd\,^3D$.

Advantages to use this method are: absence of the condition that EEDFs are close to be Maxwellian; application in fast changing plasma processes; and this is non-contact method. Disadvantages are: mostly determination only of “fast” electron distribution; and limitation to the plasma model described above.

The experimental conditions under which is possible the application of this method should be checked carefully. One should take in mind that the model of plasma described above does not take into account step excitation processes from the lower laying electronics levels which could contribute significantly. So, an effective way to examine the applicability of this method is changing the EEDF by changing the discharge current and the working gas pressure and measure EEDF by an electrical probe. Then the EEDF is compared with the one obtained based on the regularization method for different line spectra taken into consideration.

In absence of the electrical probes the easiest way to check the plasma model is following the changes in the intensity distribution patterns of the spectrum in terms of wavelength by increasing the discharge current. The intensities of the spectral lines must be a linear dependence on the current. Otherwise, only qualitative information about the EEDFs can be obtained. The verification of our method should be done in a real helium low pressure plasma.

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