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PHYSICS EDUCATION

A NONLINEAR DYNAMICAL SYSTEM APPROACH OF LEARNING

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Abstract. We present an application of the theory of the complex systems to the learning processes. The mathematical model is based on the so-called "logistic map", one of the easiest ways to understand how the theory of complex system could be used in order to achieve a better educational activity. Using this model it is easier to optimize the teaching strategies if we acknowledge the fact that the learning processes behave non-linearly.

Key words: logistic model, nonlinear system, learning process.

1. INTRODUCTION

The theory of complex systems and the path to chaos could give an unconventional approach of the pupil's evolution, defined as a self-adjusting non-linear system endowed with memory. The mathematical model is based on the so-called "logistic map", one of the easiest ways to see how a complex system could be managed to a smooth educational path, or to bring that system into a chaos. In this context, we mean by chaos a sequence of educational actions that are not under the teacher's control (neither the pupils' nor of the understanding).

In a few words, a deterministic chaos model for education shows that, if you want to accomplish a good understanding of the subject, you can go either slowly, with a moderate speed, or more efficient and speedy, but that case needs some care. In the former case, the model shows that it can be done (not bringing a mess in the pupil's mind) if the feedback is quick and efficient.

Again, a computer aided teaching and learning could help [1].

There are some fears about the usefulness of the use of the computers in school. We think that is just a matter of skill, the pupils, the teachers and the curriculum must have new forms, ways and must give an elastic frame.

2. THE LOGISTIC MODEL

One of the simplest models for describing the evolution of a nonlinear system is the logistic model developed by Pierre Verhulst (1845, 1847) to describe the population's growth. It is described as a feedback machine in which x_n is the input value at stage n, and x_{n+1} is the output value after the system processed the input. The parameter λ is a control parameter that could drastically change the evolution of the system, from a classical deterministic evolution to a chaotic, unpredictable behavior, specific for the so-called chaotic dynamics. The system is dynamic because it is described by an equation of evolution and it is chaotic because it is unpredictable.

The situation is typical for an iterative process that models a feedback machine that produces successive iterated solutions (Fig. 1).

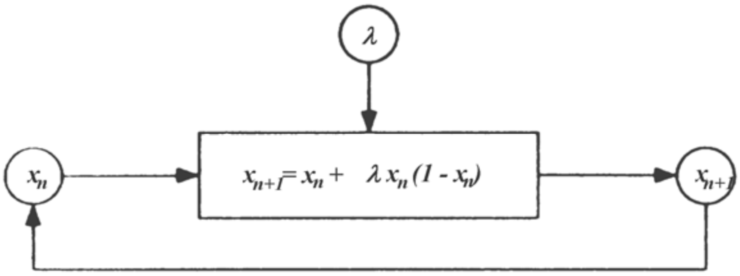

Fig. 1 – The feedback process of the logistic map [9].

In the original model, x_n was the population at the moment t, and the x_{n+1} was the population at the moment $t+1$, typically for the discreet dynamical systems [2–4]. Computer modeling of the logistic model is simple; an application of the Euler's method is described in [5]. The more general topics of nonlinear dynamics could be also followed from the very popular and understandable papers from [6–8]. The whole topic belongs also to *cybernetics,* the science of feedback systems [9]. The learning process is applied to a nonlinear system: student, teacher, or the whole school system. For example, the student learns something, and after that he will be in a different "state" than at the beginning. Just this simple observation could give us a criterion of a good lesson: if, after the lesson, the student is different from the one he/she was at the beginning, the lesson was good, useful. But if nothing happened in the knowledge of the student after the lesson, it was far from good. This gives the idea that the teacher has to emotionally "touch" the student, in order for him/her to receive the new knowledge.

Exploration of the use in physiology of cybernetics and the science of complexity can be found in [9–11].

The application of these methods gives interesting results that are summarized in Fig. 2 for different values of the control parameter, λ.

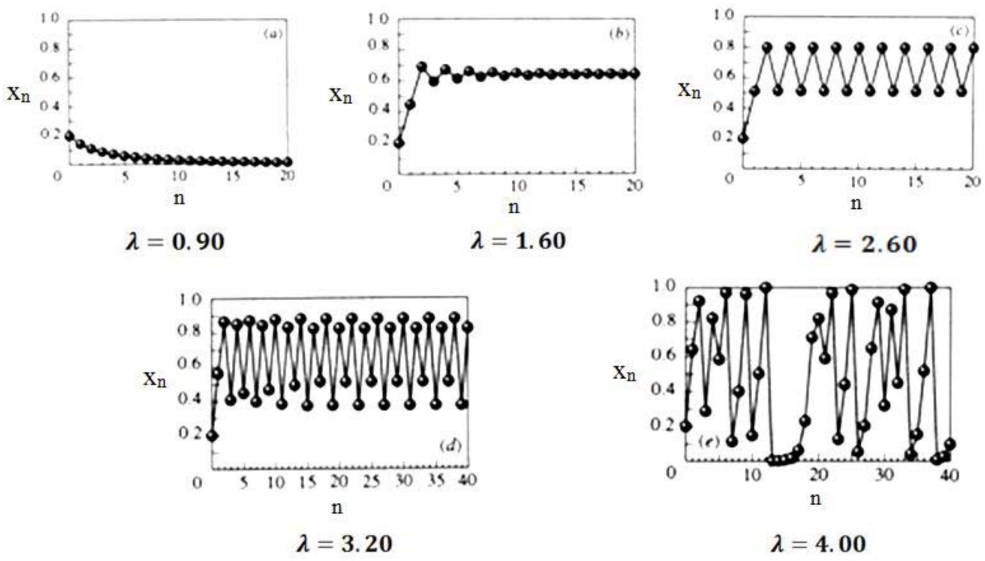

Fig. 2 – Iterated solutions for logistic equation [3].

A short cut of the computational algorithm will be:

$$\frac{x_2 - x_1}{t_2 - t_1} = \lambda x_1,$$

$$x_2 = x_1 + \lambda x_1 \tau,$$

where τ represents the unit of time for iteration.

It is simple to arrive to the following conclusions. Depending on the control parameter, λ, the system will evolve 1) to the initial state, no knowledge was assimilated, or 2) will evolve more or less quickly towards a higher steady state, or 3) will oscillate (between understanding or not understanding the items), or 4) will evolve by oscillating between "two states" ("could be this or could be that…" in the mind of the student), or 5) even it will evolve chaotically (the student will be completely confused - the last situation in Fig. 2 that correspond a "chaos" in his mind – "I do not understand nothing".

Some general trends described by this model may be generalized:

a) Even though the state of the system is described by a simple and well-defined equation (the logistic one), the evolution of this system may be (un)predictable, depending on the value of the control parameter: for values higher than a critical value the system acts chaotically. We say that we are dealing with a deterministic chaos.

b) The evolution of the system is very strongly dependant on the initial condition, especially in the region where the control parameter is higher than the critical value. This characteristic is sometimes named the *butterfly effect*, discovered by Edward Lorenz. This effect can be stated as it follows: *small variations of the initial conditions due to the wing beat of a butterfly, for instance, somewhere on the globe that can lead in the future to dramatic weather changes in another part of the globe* [12]. The effect is also known as the *effect of sensibility to initial conditions.* If such a dramatic effect can take place for a simple system, for more complicated systems this effect must be real.

c) The cause of this sensibility to initial conditions that has as a result an unpredictable acting of the system is given by the nonlinear nature of the system. Whenever a physical system is nonlinear, we can expect it to pass to a deterministic chaos in certain conditions, so that its subsequent evolution will be impossible to predict. We shall insist on the fact that the unpredictability of the system's evolution is determined neither by the system's complication, nor by not knowing the initial conditions, nor by not exactly knowing the evolution law, nor by causes of mathematical calculation. The unpredictability is of "organic" nature, *i.e.* it resides in the nature of things or of phenomena.

d) Because the majority of the real systems are nonlinear (and we are not making reference to the linear models used for the simplified study of the phenomenon), the aspects of deterministic chaos are everywhere. Only simple approximations are linear and predictable. Instead, some systems present unpredictability to a short time evolution (such as meteorological phenomena) or to a long time one (such as the cosmic ones). The result is that we need to know the moment until we can count on an acceptable predictability and the moment from which we do not have this possibility anymore.

e) From all these perspectives, the educative and learning processes must be described as a nonlinear dynamic system. That is why the research and development of the nonlinear dynamic concepts and chaos in education is an open road.

3. LABYRINTH TEST 1

The learning process for this test has more steps. In principle, these steps suppose: a) a familiarization with the new presented piece of information with emphasis on the tasks of solving the "problem", b) a first attempt of solving the problem. If the first attempt is not successful a new attemptwill follow, (this is the stage that includes both the memorization and the creativity, using anticipation and finding better ways for solving), c) new attempts of solving the problem will follow if the problem was not solved, and that repetition of the process will lead to the optimization of the solving time (an efficiency criterion) [13].

The implementation of the proposed test (one of the studied ones in order to experience the student's particularized understanding process) is the Labyrinth Test presented below [14].

Fig. 3 – Labyrinth no. 1.

Fig. 4 – The solution of the labyrinth [14].

The tested student receives 18–30 labyrinths, numbered in ascending order (1, 2, 3, ... 30), which he/she has to solve one by one (drawing the road with a pencil). On the sheet of paper, the coming backs will be obvious, if the right path was not selected. With the following sheets, the go and back portions will be fewer, the memorizing and learning process helping the student to have a higher velocity of solving. He/she writes on the sheets the moment when starting and when ending. If a pause is taken between the two moments, it should also be written. The check of the learning process can be continuously made, without pauses between the trials, or with pauses. The difference between the two situations may be relevant for the time needed for rest when learning under stressful conditions (against time – typical for exams).

The assessor can compare the student's results with the best and most rapid solving used as a standard. After a while, the student reaches a relatively constant value of the solving time, which shows the "characteristic time" of the student and the time in which mental fatigue appears. In this way, the teacher has some objective parameters of analysis for the student's style and characteristics of learning.

An example of such an analysis on the student CCR is given in the Table 1.

Table 1

Labyrinth test 1; student CCR, 8 of July, 2012

No. test	Duration from the start of testing (the 1st test starts at 0 seconds)	Duration of solving the test [s]	Observations (analysis of the graph)
1	0	240	Starting point
2	240	120	
3	360	90	
4	450	60	
5	510	60	Fatigue appears
6	570	60	
7	630	60	
8	690	60	
9	750	60	
10	810	60	
11	870	60	
12	930	40	
13	970	45	Latency appears
14	1015	45	
15	1060	45	
16	1105	40	
17	1145	30	
18	1175	25	
19	1200	26	
20	1226	26	the "record"

If there is any solving efficiency as function of time, *i.e.* the total duration spent by the student until solving the test, we obtain a curve with the shape like in Fig. 6.

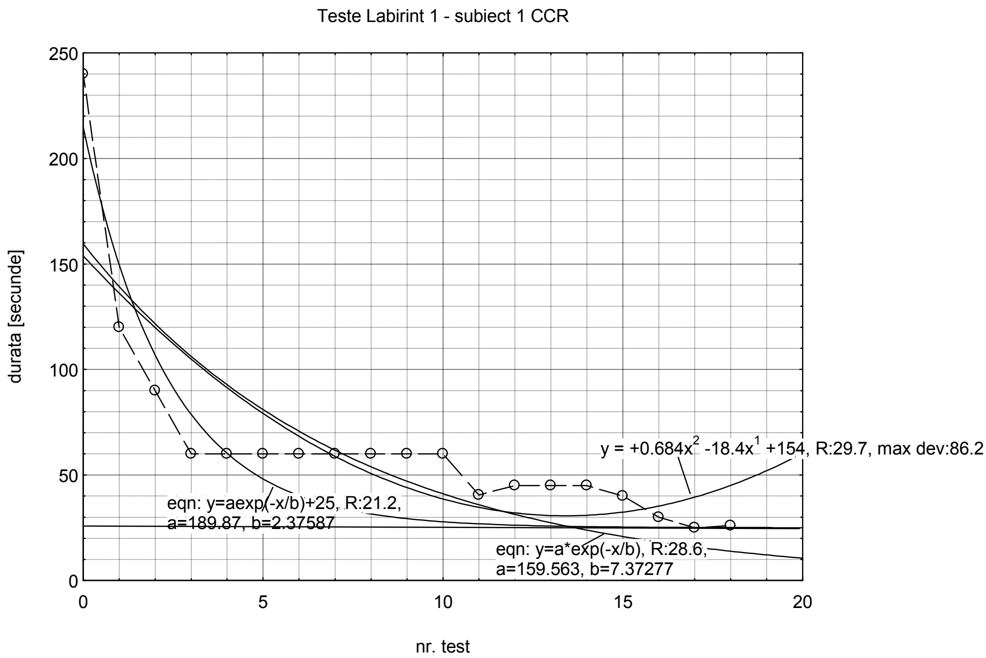

Fig. 5 – Graph of the data from Table 1, with different function for a mathematical fit.

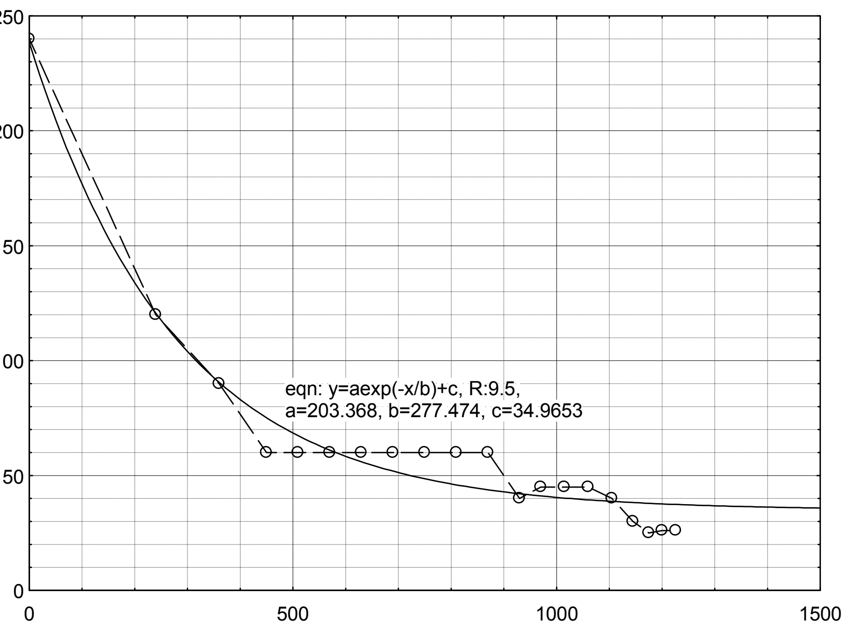

Fig. 6 – The best fit of the experimental data.

The time variation does not put into light anymore the fatigue moment. The time representation is more real, because it takes into consideration the necessary time to solve the labyrinth for the *n*-th time.

If we consider that there is a saturation (which here appears somewhere at second 25 – minimum time of solving), we can also represent the learning curve which is one of saturation, *i.e.* (250–*f*(*t*)) (Fig. 7).

In order to show the saturation process we used a new function, $z(t) = 250 - f(t)$, derived from the first one $f(t)$.

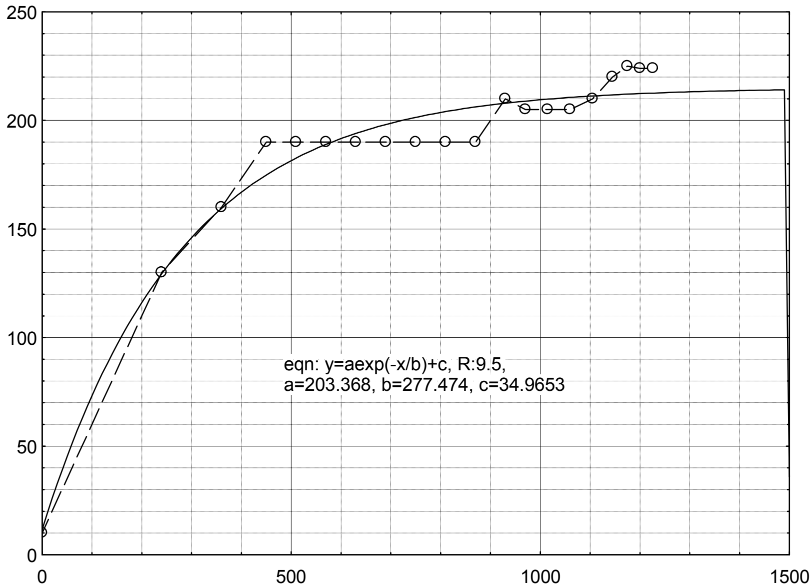

Fig. 7 – The saturation process (tendency to a limitation of the acquirement of knowledge).

In order to directly use it in experiments, the image of the labyrinth may be multiplied and it can be used with the same student or with different ones. The experiment has shown that it is possible for the test to require more than 18 trials.

4. PROCESSING DATA

For processing the data, it is necessary to note the time needed to solve the labyrinth through the *x* variable. The variation of this duration, $y = x_2 - x_1$, is the equivalent of the "growth slope" of the learning process, proportional with the learning efficiency.

In the theory of the nonlinear systems, there are various procedures of analysis. One of them is the so-called graph *return map that* has the purpose to follow the advance of the learning process. Another graph is the one of an image in the space of phases for the investigated nonlinear dynamic system, *i.e.* $y = x_2 - x_1$ (variation – slope, time function). Both processing are presented in Table 2.

Table 2

Successive duration of solving the test and the difference between them

x_1	x_2	$y = x_2 - x_1$
0	240	240
240	360	120
360	450	90
450	510	60
510	570	60
570	630	60
630	690	60
690	750	60
750	810	60
810	870	60
870	930	60
930	970	40
970	1015	45
1015	1060	45
1060	1105	45
1105	1145	40
1145	1175	30
1175	1200	25
1200	1226	26
1226	1245	19

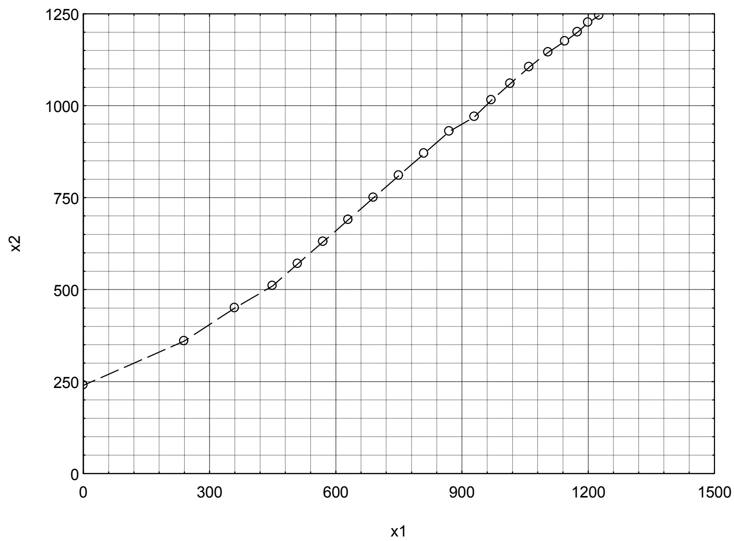

Fig. 8 – The return map.

5. CONCLUSION

The application of the nonlinear dynamics models to the learning process is beneficial and it can bring useful pieces of information to the teacher, in order to lead the learning process in an optimized structure.

In the present paper, we have shown some of the nonlinear dynamics ideas with learning potential. The mathematical model used in the paper was based on the "logistic map", one of the easiest ways to understand how the theory of complex system could be used in order to achieve a better educational activity. Using this model we have shown that is easier to optimize the teaching strategies, if we acknowledge the fact that the learning processes behave non-linearly.

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