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SOLITON DYNAMICS IN FERROMAGNETIC CHAINS WITH FIRST- AND SECOND-NEIGHBOR INTERACTIONS*

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Abstract. We have considered the ferromagnetic spin chain with both first- and second-neighbor interactions. We obtained the condition for the appearance and stability of bright and dark solitons for arbitrary wave number and different anisotropies (on-site or inter-site). The complicated dependence of the dispersion and the nonlinear coefficients lead to regions in the Brillouin zone where strong second-neighbor interactions can turn the type of the soliton solution (from bright to dark or vice versa).

Key words: solitons, nonlinear Schrödinger equation, spin chains.

1. INTRODUCTION

Considerable attention has been devoted to the study of solitary waves in solids. A large amount of theoretical and experimental research has been dedicated to one-dimensional magnetic systems [1]. Models corresponding to real quasi-one-dimensional magnets are broadly investigated as first they were considered as somewhat simpler than the really interesting three-dimensional systems but then turned out to be interesting in their own right. There was the availability of rear magnetic compounds which because of extremely small coupling between neighboring chains, could be considered as reasonable realizations of magnetically one-dimensional materials. Soliton solutions for classical sine-Gordon chains [2] and Heisenberg chains with various anisotropies [3–6] were obtained and analyzed. In recent years there is a renewed interest to the topic due to their application. Spin-wave dark solitons were predicted and experimentally generated [7, 8]. Solitons in magnetic thin films [9] and in ferromagnets with biquadratic exchange [10] were investigated.

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Bright and dark solitons as exact solutions of the nonlinear Schrödinger (NLS) equation have been studied intensively [3, 4, 11]. But, the models describing the physical effects in solids are mainly discrete and the question arises how the discreteness modify the proper-ties of the nonlinear excitations [12, 13]. Another interesting topic of research with practical importance is the interaction of solitons with impurities [14–18]. The influence of the inter-action with second-neighbors [19] and third-neighbors [20] on the bright soliton dynamics has been also considered.

In the present paper we study the condition for the appearance of wide dark solitons in an anisotropic ferromagnetic chain with first- and second-neighbor interactions.

2. HAMILTONIAN OF THE SYSTEM

We consider a ferromagnetic Heisenberg chain of N spins with magnitude S described by the following Hamiltonian:

$$H=-\frac{J_1}{2}\sum_n(S_n^+ + S_{n+1}^- + S_n^- + S_{n+1}^+) - \tilde{J}_1\sum_n S_n^z S_{n+1}^z - \\ -\frac{J_2}{2}\sum_n(S_n^+ + S_{n+2}^- + S_n^- + S_{n+2}^+) - \tilde{J}_2\sum_n S_n^z S_{n+2}^z - A\sum_n\left(S_n^z\right)^2, \tag{1}$$

where both nearest-neighbor and next-nearest-neighbor exchange interactions are taken into account. The spin operators $S_n^{\pm}, S_n^z$ obey the commutation relations

$$\left[S_i^{\pm}, S_j^z\right] = \mp S_i^{\pm}\delta_{ij}, \left[S_i^+, S_j^-\right] = 2S_i^z\delta_{ij}. \tag{2}$$

$J_i > 0$ and $\tilde{J}_i > 0$ (i =1, 2) are the exchange integrals and A is the on-site anisotropy constant which can be positive (easy axis) or negative (easy plane). If $\tilde{J}_i \neq J_i$ the inter-site anisotropy is included. For $J_i = \tilde{J}_i$ and $A = 0$ the model is isotropic.

The equations of motion for the operators S_n^+ yield ($\hbar = 1$):

$$\mathrm{i}\frac{\mathrm{d}S_n^+}{\partial t} = -AS_n^+ - S_n^z\left[J_1(S_{n+1}^+ + S_{n-1}^+) + J_2(S_{n+2}^+ + S_{n-2}^+)\right] + \\ +\left[\tilde{J}_1(S_{n+1}^z + S_{n-1}^z) + \tilde{J}_2(S_{n+2}^z + S_{n-2}^z)\right]S_n^+ + 2A_n^z S_n^+. \tag{3}$$

In the quasiclassical approximation, where the components of the spin operators S_n are complex amplitudes, $\alpha_n = S_n^+/S, \alpha_n^* = S_n^-/S$ and $S_n^z/S = \sqrt{1-|\alpha_n|^2}$, we have

$$\mathrm{i}\frac{\partial\alpha_n}{\partial t} = -A\alpha_n - \left[J_1 S(\alpha_{n+1} + \alpha_{n-1}) + J_2 S(\alpha_{n+2} + \alpha_{n-2})\right]\sqrt{1-|\alpha_n|^2} +$$
$$+\alpha_n\left[\tilde{J}_1 S\left(\sqrt{1-|\alpha_{n+1}|^2} + \sqrt{1-|\alpha_{n-1}|^2}\right) + \tilde{J}_2 S\left(\sqrt{1-|\alpha_{n+2}|^2} + \sqrt{1-|\alpha_{n-2}|^2}\right)\right] + \quad (4)$$
$$+2AS\alpha_n\sqrt{1-|\alpha_n|^2}.$$

The set of differential equations (4) describes our system. In the continuum limit (wide excitations), it transforms into a perturbed NLS equation, where the perturbing terms are not only due to the discreteness but also to the complicated nonlinear interactions. For $J_2 = \tilde{J}_1 = \tilde{J}_2 = A = 0$ and wide excitations, (4) turns to the discrete Ablowitz-Ladik equation which is also completely integrable and has soliton solutions. So, for narrow excitations it is necessary to investigate the whole set (4). Equations similar to (4) has been derived for exciton solitons in molecular crystals [21].

3. SOLITON SOLUTIONS

We shall look for solutions in the form of amplitude-modulated waves

$$\alpha_n(t) = \varphi_n(t)\mathrm{e}^{\mathrm{i}(kn-\omega t)}, \quad (5)$$

where k and ω are the wave number and the frequency of the carrier wave (the lattice constant equals unity) and the envelope $\varphi_n(t)$ is a slowly varying function of the position and time. In the continuum limit, when the soliton width is much larger than the lattice spacing ($L >> 1$), equation (4) transforms into the following NLS equation for the envelope:

$$\mathrm{i}\left(\frac{\partial\varphi}{\partial\tau} + v_g\frac{\partial\varphi}{\partial x}\right) = (\varepsilon - \Omega)\varphi + b_k\frac{\partial^2\varphi}{\partial x^2} + g_k|\varphi|^2\varphi \quad (6)$$

where

$$\tau = tS,\quad \Omega = \omega/S,\quad g_k = J_1\cos k - \tilde{J}_1 + J_2\cos 2k - \tilde{J}_2 - A,\quad \varepsilon = -A/S - 2g_k,$$
$$v_g = 2\,(J_1\sin k + 2J_2\sin 2k),\quad b_k = J_1\cos k + 4\,J_2\cos 2k. \quad (7)$$

v_g is the group velocity and b_k describes the group-velocity dispersion of the carrier waves. Depending on the sign of the expression $b_k g_k$ equation (6) possesses soliton solutions of different types. For negative values bright solitons exist while for positive values dark solitons are possible.

In what follows, we shall consider dark-soliton solutions which appear for

$$b_k g_k > 0 \tag{8}$$

and have nonvanishing boundary conditions, $|\varphi(x)|^2 \to$ const at $x \to \pm\infty$. Note that the condition (8) depends not only on the anisotropy constants but also on the wave number k. The dark soliton has the form

$$\varphi(x,\tau) = \varphi_0 \tanh\frac{x - v\tau}{L} + \mathrm{i}B, \tag{9}$$

where L and v are its width and velocity. B determines the value of the amplitude at the center, *i.e.* the minimum intensity of the soliton $|\varphi(0)|^2 = B^2$, while the maximum intensity is $|\varphi(\pm\infty)|^2 = \varphi_0^2 + B^2$. As independent parameters of the soliton we can choose L, k and B. Then φ_0, Ω and v are given by the relations:

$$\varphi_0^2 = \frac{2b_k}{g_k L^2}, \quad \Omega = \varepsilon + g_k(\varphi_0^2 + B^2), \quad v = v_g + v_d, \quad v \equiv B\sqrt{2b_k g_k}. \tag{10}$$

The dark soliton can be considered as an excitation of the background (carrier) wave. Its velocity consists of the group velocity v_g and a contribution v_d which depends on B. For fixed $(\varphi_0^2 + B^2)$ and k solitons moving faster have smaller amplitude φ_0.

The bright soliton solution which exists for $b_k g_k < 0$ and $|\varphi(x)|^2 \to 0$ at $x \to \pm\infty$ has the form

$$\varphi(x,\tau) = \varphi_0 \operatorname{sech}\left(\frac{x - v\tau}{L}\right), \tag{11}$$

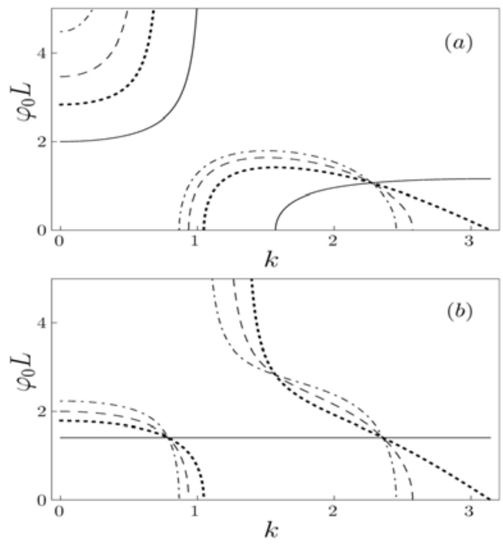

Fig. 1 – Dependence of $\varphi_0 L$ on the second-neighbor interaction for: a) on-site anisotropy ($J_i = \tilde{J}_i$, $A = 0.5$); b) inter-site anisotropy ($\tilde{J}_i = A = 0$); i=1,2. $J_2 = 0$ (solid lines), $J_2 = 0.25$ (dotted lines), $J_2 = 0.5$ (dashed lines), and $J_2 = 1$ (dashed-dotted lines).

with

$$\varphi_0^2 = -\frac{2b_k}{g_k L^2}, \quad \Omega = \varepsilon - \frac{b_k}{L^2}, \quad v = v_g. \tag{12}$$

Fig. 1 shows the dependence of the function $\varphi_0 L = \sqrt{2b_k / g_k}$. As can see from (7) the group-velocity dispersion b_k depends only on J_1 and J_2 and is not influenced by the anisotropy of the system. Without second neighbor interaction ($J_2 = 0$) b_k changes sign at the middle ($k = \pi/2$) of the Brillouin zone. The inclusion of weak second-neighbor interaction ($J_2 < 1/4$) has influence only in the moving of this point to smaller k-values. Strong second-neighbor interactions ($J_2 > 1/4$) lead to a second point where the function b_k changes again sign. The nonlinear coefficient g_k depends on the anisotropy of the system. For the isotropic case without second neighbor interaction ($J_1 = \tilde{J}_1$, $J_2 = \tilde{J}_2 = A = 0$) g_k is negative in the whole Brillouin zone. So in this case equation (6) has bright-soliton solutions for $0 < k < \pi/2$ and dark-soliton solutions exist for $\pi/2 < k < \pi$. The inclusion of positive on-site anisotropy ($A > 0$) preserves this behavior. The inclusion of negative on-site anisotropy ($A < 0$) leads to the appearance of a wave number k_c so that g_k is positive for $0 < k < k_c$ and negative for $k_c < k < \pi$. Then the Brillouin zone is divided in three regions where the solution alters from dark- to bright- and again to dark-soliton (Fig. 1a). If $k_c = \pi/2$ then the solution of equation (6) is of the dark-soliton type in the whole Brillouin zone. The inclusion of weak second-neighbor interactions has influence only on the value of k_c and the size of the regions where the different solutions exist. Strong second-neighbor interactions with $J_2 > 1/4$ will lead to the appearance of the additional region near the band edge π and consequently the solution is of the bright-soliton type (Fig. 1a). Fig. 1b shows the function $\varphi_0 L$ for the case of inter-site anisotropy ($\tilde{J}_1 = \tilde{J}_2 = A = 0$). When the second-neighbor interactions are not taken into account the solution of equation (6) is of the dark-soliton type in the whole Brillouin zone and the function $\varphi_0 L$ is constant. The inclusion of second-neighbor interactions will lead again to three (if $J_2 < 1/4$) or four (if $J_2 > 1/4$) regions where different soliton solutions exist.

We like to make some remarks about the influence of the next-nearest-neighbor interaction on the form and velocity of the soliton solution. The dependence of the velocity on k is shown on Fig. 2. The group velocity v_g is defined in the whole Brillouin zone and coincides with the velocity of the bright solitons and of the dark solitons for $B = 0$ (Fig. 2a). It is independent of the anisotropy of the model. The small second neighbor interactions change only the value of the velocity, while strong second neighbor interactions ($J_2 > 1/4$) can change also its sign. The velocity of the dark soliton itself v_d for $B = 0.5$ is shown on (Fig. 2b). It is nonequal zero in the regions where the dark solitons exist ($b_k g_k > 0$) and its sign is determined by B as given in (10). The gaps in the total dark soliton velocity v coincide with regions where $b_k g_k < 0$ (due to the presence of A and second neighbor interactions) and the solution of equation (6) is of a bright-soliton type (Fig. 2c).

We performed numerical simulations based on the general discrete equation (4) and we investigate mainly the dark soliton solution. As initial function we put for $t = 0$ the form

$$\alpha_n(0) = \left(\frac{\sqrt{2b_k / g_k}}{L} \tanh \frac{n}{L} + \mathrm{i}B \right) \mathrm{e}^{\mathrm{i}kn}, \tag{13}$$

which is exact solution of the quasicontinuum equation, and observe the evolution for different anisotropic cases. For wide solitons ($L >> 1$) this solution remain stable and its form and velocity are preserved. When the soliton width decreases the discreteness effects of equation (4) become important and the parameters of the dark soliton change.

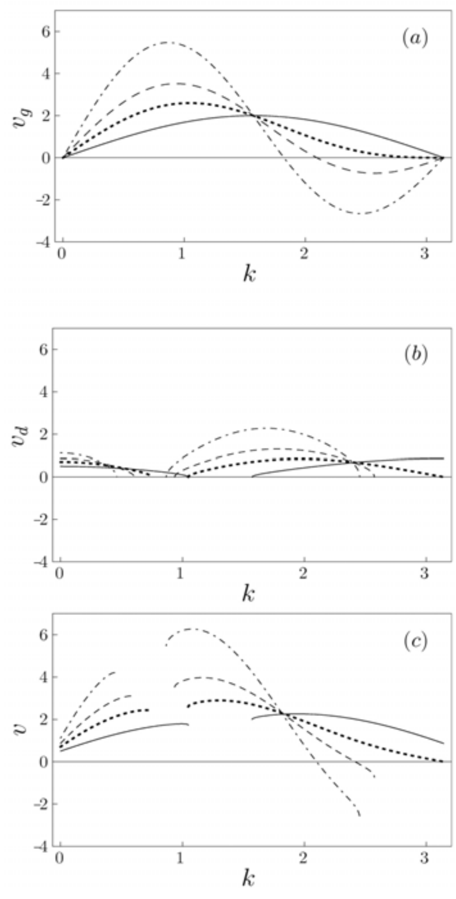

Fig. 2 – k-dependence of the group velocity v_g (a), the dark-soliton velocity v_d for $B = 0.5$ (b) and the total soliton velocity v (c) for on-site anisotropy $J_1 = \tilde{J}_1 = 1$, $A = 0.5$. $J_2 = \tilde{J}_2 = 0$ (solid lines), $J_2 = \tilde{J}_2 = 0.25$ (dotted lines), $J_2 = \tilde{J}_2 = 0.5$ (dashed lines), and $J_2 = \tilde{J}_2 = 1$ (dashed-dotted lines).

In what follows we have $L = 10$ and $J_1 = 1$. Fig. 3 shows the propagation of the dark solitons in the case of on-side anisotropy $\tilde{J}_i = J_i$, $i = 1,2$ and $A = -0.5$ for two k-values ($k_1 = 0.0534$ and $k_2 = \pi/2 - k_1$). When the second neighbor interaction is absent ($J_2 = 0$) the solitons propagate with the same velocity and different amplitudes (Fig. 3a, a').

When the second neighbor interactions are present the solitons have different velocities, too (Fig. 3b, b' for $J_2 = 0.1$). As can be seen from the k-dependence of φ_0 (Fig. 1a) and v (Fig. 2c), the amplitude and the velocity decrease with the increase of k. The increase of the second neighbor interaction leads to the turning of the solution for k_2 to a bright soliton type (Fig. 3c, c' for $J_2 = 0.3$).

We like to point out that Eq. (4) is invariant with respect to the transformation $\alpha_n \to \alpha_n^* e^{i\pi n}$ when $J_2 \to -J_2$, $J_1 \to -\tilde{J}_1$, $J_2 \to -\tilde{J}_2$, and $A \to -A$. So the results on Fig. 3b,c can be obtained also for k_2 provided that $A = 0.5$, $\tilde{J}_1 = -1$ and $J_2 = \tilde{J}_2$ equal -0.1 and -0.3, respectively.

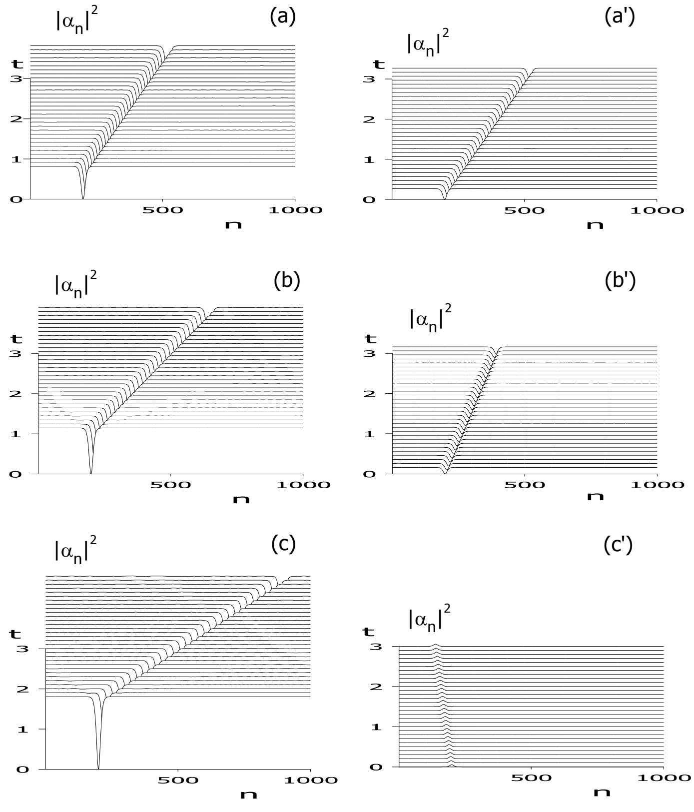

Fig. 3 – Time evolution of dark solitons for on-site anisotropy $J_i = \tilde{J}_i = 1$, $i = 1, 2$; $A = -0.5$ and $k_1 = 0.0534$ [(a), (b), (c)], $k_2 = \pi - k_1$ [(a'), (b'), (c')]. (a),(a') $J_2 = \tilde{J}_2 = 0$; (b),(b') $J_2 = \tilde{J}_2 = 0.1$; (c),(c') $J_2 = \tilde{J}_2 = 0.3$. The time is in units of $10^3/J_1S$.

Similar results have been obtained for the case of inter-site anisotropy $\tilde{J}_1 = \tilde{J}_2 = A = 0$. When the second neighbor interaction is absent ($J_2 = 0$) the solitons propagate with the same velocity and amplitude. If second neighbor interaction is present the solitons have different amplitudes and velocities and when its value increases the solution for k_2 alters from dark- to bright-soliton type.

4. CONCLUSION

We have investigated the existence and stability of dark solitons in a discrete ferromagnetic chain with both inter-site and on-site anisotropy. We have considered the influence of the second-neighbor interactions on the soliton dynamics in the whole Brillouin zone. Strong second-neighbor interactions can turn the type of the soliton solution from bright to dark or vice versa. This is due to the complicated nonlinear interactions of the system.

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