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CONTRIBUTIONS TO THE STUDY OF SOME PHYSICAL, TECHNICAL AND BIOLOGICAL COMPLEX SYSTEMS

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Abstract. An analysis of the vast literature concerning the physical, technical and biological complex systems was accomplished. Starting from the main results of some previously published works, and using also the results obtained in the frame of this work, the present overview tries to achieve a classification of the physical, technical, and biological complex systems.

Key words: Phenomenological Universality classes (PUN), power laws, West Universality class (U2), unitary description of human body growth stages, multiplication/damping and generation/annihilation processes.

1. INTRODUCTION

The analysis of the basic features of complex systems, accomplished in the frame of the studies [1], pointed out their possible classification shown by Fig. 1.

The characteristic features of all complex systems (those of law complexity, inclusively) are [1, 2] the presence of: phase transforms, statistical behavior, fractal scaling, and power laws, to whom the medium complexity systems add the presence of self-similar functions. As it concerns the high complexity systems, they have some additional specific features specified below.

This work studies mainly the high complexity evolving systems. As it is known, one of the simplest such high complexity systems are the solitons. Their basic characteristic processes result even from the description of the solitons discoverer J. S. Russell [3], which pointed out that the basic processes characterizing the evolution of *physical* solitons are: a) *the auto-organization* (requiring the dissipation of a rather high quantity of energy), which can be substituted by a *technical organization* (by means of some specific quite sophisticated devices [4]), and the: b) *gradual structure degradation*.

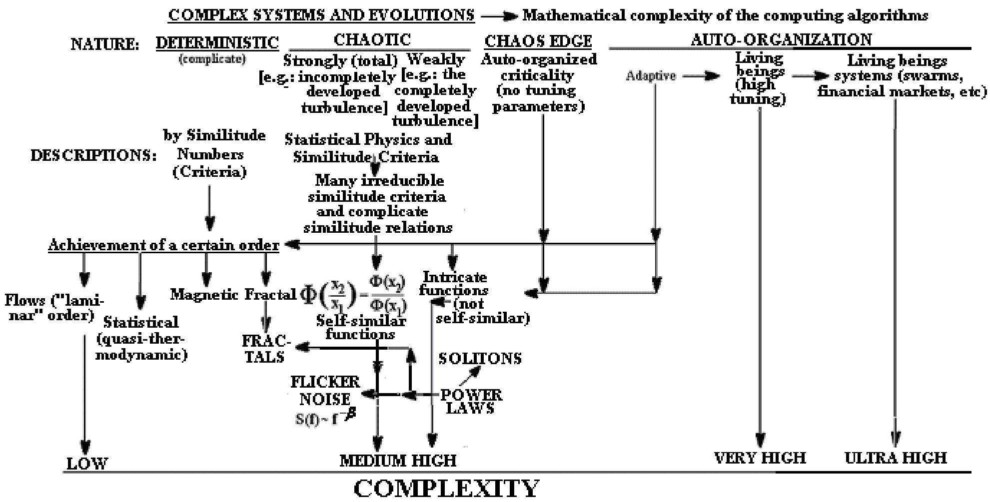

Fig. 1– Main features of the complex systems.

We have to underline also that the crucial role of these processes was confirmed also: (i) by some recent studies [5], as well as by the studies on: (ii) the optical solitons [6] (attenuation compensation by means of the Raman gain, Er^{3+} ions, etc), (iii) the morpho-genesis and growth processes led by genetic (AND) information by means of some bio-chemical reactions [7], (iv) the presence of the insurance and re-insurance agencies in the structure of financial markets [8], etc.

Of course, besides the above-indicated Complexity levels, there is that of the Composed Complexity, corresponding to the Universe, which has all Complexity features, that become active to certain specific levels.

Given being the high-complexity systems present evolutions characterized by some discrete stages (corresponding *e.g.* to the so-called circadian oscillations of the living beings growth, obviously to the financial markets, etc), the most suitable method intended to the study of these systems is that of Finite Differences (DF) [9]. The most important purposes of the FD simulation methods of simulation of the evolutions of the adaptive complex systems are: a) the optimization of the corresponding FD schemes, in order to ensure the largest possible stability radii and an optimal description accuracy [10], b) the study of the gradual pulses structure degradation, in order to understand the processes which limit these systems performances. Both aspects will be analyzed by this work, for the simplest considered adaptive complex systems – those corresponding to the solitary pulses and to some high complexity systems – the living beings.

In the particular case of KdV solitons, starting from their equation: $\dot{u} = -V_o u' - nu \cdot u' - d \cdot u'''$, it was pointed out that the basic structural parameters of the solitary solution (pulse): $u(x,t) = A \cdot \sec h^2 \left\{ \xi \left[x - \left(V_o + \Delta V \right) \cdot t \right] \right\}$, where the pseudo-vector is: $\xi = \sqrt{\frac{nA}{12d}} = \frac{2\ln(\sqrt{2}+1)}{w}$, and the additional non-linearity-dispersion velocity: $\Delta V = 4d \cdot \xi^2 = nA/3$, are the:

a) *skewness* $s = M_3 / M_2^{3/2}$, with null values for the non-deformed KdV solitons

b) *kurtosis*:

$$b_{KdV} = \frac{M_4}{M_2^2} = \int_{-\infty}^{\infty} \Phi^4 \cos h^{-2}\Phi \cdot \mathrm{d}\Phi \Bigg/ \left[\int_{-\infty}^{\infty} \Phi^2 \cosh^{-2} \Phi \cdot \mathrm{d}\Phi \right]^2 = 4.2 \text{, and its:}$$

c) *area under peak* (equivalent to the so-called "motion momentum" = prime integral of the FD evolution [11, p. 243]: $S = 2\sqrt{\frac{12d}{n}} \cdot A^{1/2}$.

2. COMPARATIVE STUDY OF THE DIFFERENT DESCRIPTION TYPES OF SOME ACCOMMODATION-DISACCOMMODATION PROCESSES

Starting from the differential equation of the accommodation (growth, particularly) of an arbitrary physical parameter $Y(t)$:

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \pi(t) \cdot Y(t), \tag{2.1}$$

where $\pi\,(t)$ is the momentary density of the accommodation (growth) probability, and introducing the similitude criteria [12]:

$$\tau = \pi(0) \cdot t,\ \ y(t) = \frac{Y(t)}{Y(0)} \ \text{and}\ \ p(t) = \frac{\pi(t)}{\pi(0)}, \tag{2.2}$$

it results the growth similitude equation:

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = p(\tau) \cdot y(\tau) \ \text{ or: }\ p(\tau) = \frac{\mathrm{d}y}{y(\tau)\mathrm{d}\tau} = \frac{\mathrm{d}z}{\mathrm{d}\tau}, \tag{2.3}$$

with: $z = \ln y$.

Assuming that the similitude variable z depends only on $p\,(\tau)$, one obtains:

$$\dot{p} = \frac{\dot{z}}{\mathrm{d}z/\mathrm{d}p} = \frac{p}{\sum_{n=0}^{\infty} \varepsilon_n p^n} = \sum_{n=1}^{\infty} \kappa_n \cdot p^n = \beta \cdot p + \gamma \cdot p^2 + \ldots \tag{2.4}$$

The growth processes can be classified in terms of the degree N of the algebraic polynomial $\dot{p}(p)$ in the following Universality classes [13]: a) *U*0 (or auto-catalytic, corresponding to a constant value of probability density $p(\tau)$), b) *U*1 (Gompertz [14]), for a *linear* dependence $\dot{p} = f(p) = \beta \cdot p$, c) *U*2 (West [15]) for a parabolic dependence $\dot{p} = f(p) = \beta \cdot p + \gamma \cdot p^2$, etc.

Table 1 presents a synthesis of the basic equations of the main phenomenological Universality equations. The works [14] pointed out that – besides these Universality equations – some power laws given by the expression: $\dot{y} = \dot{y}_o + C(y - y_o)^n$ lead to descriptions of the growth parameters of the same accuracy order as the Universality equations from Table 1. It was found also that: a) the use of the power law descriptions is preferable in the coupling regions of growth stages with different character, b) in certain conditions [$y(0) < y_\infty < y(0) \cdot \left(\sqrt{r} + 1\right)$], the predictions of the auto-catalytic growth model followed by stagnation (ACS) coincide with those of the West (U2) model.

It was studied the photo-induced relaxation of the: a) permeability, b) magnetic coercive strength of some low Complexity advanced technical ferri-magnetic materials, under the irradiation by some: (i) classical (non-coherent) sources, (ii) non-stabilized lasers, (iii) stabilized lasers. Using the fitting computer code Solver of Excel, as well as some original Quick Basic codes, it was found: (i) the incompatibility of the original West model [15] with the experimental results, (ii) the compatibility (in the limits of the experimental errors, of the order of ±1%) of the Gompertz model U1 [14] and of West-Delsanto's model U2 [13] with experimental data, the last one being somewhat more accurate.

Table 1

Basic growth equations corresponding to the main classes of phenomenological Universality

Universality class	Differential growth equation	Integrated growth equation
U_0 (auto-catalytic)	p = constant	$y(\tau) = \exp\left(\tau/\tau_{ac}\right)$
Auto-catalytic followed by stagnation	$\frac{dy}{y} + r \cdot \frac{dy}{y_\infty - y} = \frac{d\tau}{\tau_{ac}}.$	$\frac{\left[y_\infty - y(\tau)\right]^r}{y(\tau)} = \frac{\left[y_\infty - y(0)\right]^r}{y(0)} \cdot \exp\left(-\frac{\tau}{\tau_{ac}}\right)$
U_1 (Gompertz)	$\dot{p} = f(p) = \beta \cdot p$	$y(\tau) = y_o \exp\left[\frac{\nu_o}{\beta}\left(\exp(\beta \cdot \tau) - 1\right)\right]$
West (original)	$\frac{dy}{d\tau} = \frac{1+b}{b} \cdot y^{1-b} - \frac{y}{b}$	$y(\tau) = \left[1 + b - b \cdot \exp(-\tau/\tau_W)\right]^{1/b}$ West original: $b = 0.25$
U_2 (West-Delsanto)	$\dot{p} = f(p) = \beta \cdot p + \gamma \cdot p^2$	$y(\tau) = y_o\left[1 + \frac{p_o \gamma}{\beta}\left(1 - \exp(\beta\tau)\right)\right]^{-1/\gamma}$

Similarly, the results concerning the description of the quasi-static compression-decompression processes of some sandstones of the types: a) Berea 1 [42], b) Berea 2 [43], c) Castlegate [44], by means of the theoretical models of Gompertz (U1), original West's model and of the West-Delsanto's model (U2). It was again found the: (i) incompatibility of the genuine West's model with the experimental results, (ii) compatibility (in the limits of experimental errors, of the magnitude order of ±2%) of the Gompertz's model (U1) and of the West-Delsanto's one (U2) with the experimental results, the West-Delsanto's model being again somewhat more accurate.

3. ANALYSIS OF THE HUMAN BODY GROWTH PLOT AND IDENTIFICATION OF POSSIBILITIES OF ITS ANALYTICAL DESCRIPTION

This work started from the conclusions referring to the main targets of the studies of the human body growth, identified as [18]: (1) the normal limits of for child's size their age, sex, population, and socio-economic group, (2) the children's rate of growth for their age, sex, etc, (3) the changes in the rate of growth of children produced by certain treatments, and that: (4) the growth processes produce without physical discontinuities, according to the studies on morpho-genesis [7].

3.1. GROWTH PROCESSES REPRESENTATION IN THE PHASES SPACE

To fulfill the requirements to be dimensionless and representative for the growth advancement degree (for the human being age, inclusively), *the uniqueness parameters* y of the representation space were chosen as logarithms of some growing parameters (length, height/stature, weight, etc) $X(t)$ divided by one of its reference values (as the initial one X_o, a certain specific measure unit, etc), *e.g.* as: $y_L = \ln[\{L\}_{0.1\text{ mm}}]$, $y_m = \ln[\{m\}_{\text{g}}]$, etc. where $\{L\}_{0.1\text{ mm}}$, $\{m\}_{\text{g}}$, etc are the numerical values of the lengths in 0.1 mm, of masses in g, etc. The second parameter of the desired growths phases space is chosen as the rate of the non-dimensional growth advancement degree, evaluated by means of a Finite Differences expression:

$$\dot{y}_L(t) = \frac{\ln\left[\left\{L(t+\Delta t)\right\}_{0.1\text{ mm}}\right] - \ln\left[\left\{L(t-\Delta t)\right\}_{0.1\text{ mm}}\right]}{2\Delta t}\text{, etc.} \tag{3.1}$$

if the time interval Δt is sufficiently small.

The accomplished study [16] pointed out that: a) the most suitable representation space is $(\dot{y}, y)$, b) the main types of growth stages are those of: (i) burst-U2, (ii) inflation-U2, (iii) auto-catalytic (U0), (iv) Gompertz (U1), (v) slowing down (U2), (vi) growth stop (U2) (see Fig. 5 and Table 1 of [16] inside the indicated web page), c) the accuracy of descriptions given by the methods of phenomenological Universality and of the power laws are of the same magnitude order, the use of the power law descriptions being preferable in the coupling regions of growth stages with different character.

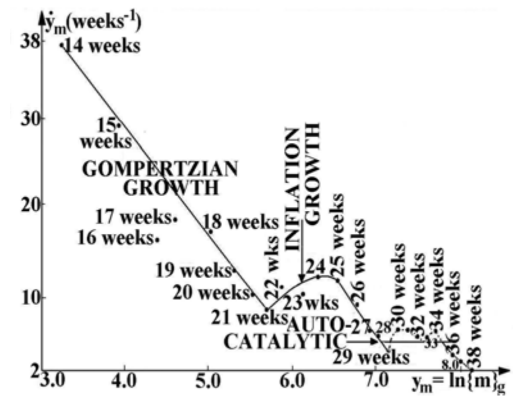

Fig. 2 – Basic stages of the human fetus growth.

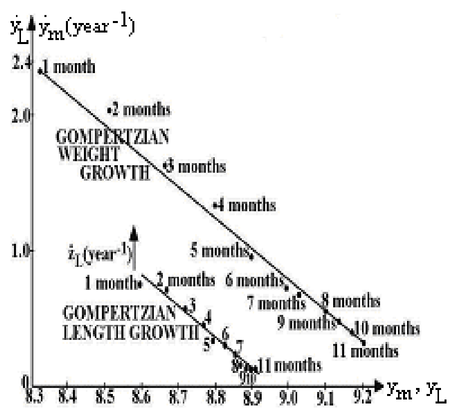

Fig. 3 – Plots of the boy and teenager growth.

3.2. IDENTIFICATION OF THE BASIC STAGES OF THE HUMAN BEING GROWTH PROCESS MACROSTRUCTURE, STARTING FROM THE DATA OF THE LOW VOLUME STATISTICS

To identify the basic stages of the human being growth macrostructure, the data provided by the low volume statistics [21–24] were used. According to the results of our work [16] (see also the corresponding web page), the basic stages of the growth process macrostructure are: a) the fetus growth, b) the babies growth, c) the childhood development, and the: d) adolescence growth (Fig. 2, 3). The work [16] presents more details about other growth stages.

3.3. EVALUATION OF THE PARAMETERS OF HUMAN BODY GROWTH STAGES, STARTING FROM THE HIGH VOLUME STATISTICS DATA

To achieve quantitative descriptions of some specific fragments of the growth plot, we introduced and studied [17] some new versions of the West (U2) phenomenological universality equation, and of the corresponding power law, namely:

$$\dot{y} = \left(\dot{y}_o + \frac{\beta}{\gamma} \right) \mathrm{e}^{\gamma(y - y_o)} - \frac{\beta}{\gamma}$$

$$\dot{y} = \dot{y}_o + C(y - y_o)^n , \tag{3.3}$$

where y_o, $\dot{y}_o$ are the values of the used dynamic variables, at the beginning of the considered growth stage. Given being the existing high-volume statistics growth data [25,26], characterized also by a considerably better [the possibility to know not only the average values, but additionally the probability distribution function $f_t(P)$ (hence the percentile values and plots) for all growth times t and different growing parameters x, a considerably higher confidence level of all numerical values, etc.], the analysis of the high-volume statistics growth data is necessary.

3.3.1. General Features of the High-Volume Statistics Data

We studied both the continuity of the first $\dot{y}$ and of the second derivative $\ddot{y}$ of the non-dimensional growth parameter $y = \ln\left(x/x_o\right)$ and of the slope of the $\dot{y} = f(y)$ plot: $\mathrm{d}\dot{y}/\mathrm{d}y = \ddot{y}/\dot{y}$.

The usefulness of this last parameter $\mathrm{d}\dot{y}/\mathrm{d}y = \ddot{y}/\dot{y}$ is pointed out by its possible applications:

a) to uncover some discontinuities (and perhaps some imperfections) of the existing data sets, even of those corresponding to high-volume statistics,

b) to establish the limits of the main stages of the human body growth.

3.3.2. Identification of the Discontinuities of Growth Plots

The strongest tool for the identification of discontinuities is given by the values of the slope $\mathrm{d}\dot{y}/\mathrm{d}y = \ddot{y}/\dot{y}$ of these plots. This criterion allowed us to find some discontinuities of the *median* plots (probability 50% to find the growth representative points under the theoretical plot) referring to the: a) baby boys length growth around 2 years, b) boys height/stature growth around of 3 years, and finally find that the differences of the monthly averages of the boys lengths and statures, respectively, for all 12 months between the 24th and the 35th one appear as exactly equal to 0.80000 cm (see the web pages ZLENAGEINF and ZSTATAGE [26]), which represents an artifact.

3.3.3. Evaluation of the limits of the basic growth stages

The extreme values of the slope $\mathrm{d}\dot{y}/\mathrm{d}y = \ddot{y}/\dot{y}$ indicate the inflexion points of the $\dot{y} = f(y)$ plots, hence the limits of some growth domains. *E.g.*, the maximum value (corresponding to the middle of the 145th month) of the slope $\mathrm{d}\dot{y}/\mathrm{d}y$ of the median plot inside the interval 127…159 months of adolescents stature growth indicates that the burst growth acts between 127 and 145 months, while the following inflation stage is located between 145 and 159 months.

3.3.4. Study of the compatibility of the theoretical models with the experimental data on the human body growth

To study the compatibility of some theoretical results $u_i \equiv \dot{y}_{theor,i}$, ($i$ = 1,…, N) relative to certain human body growth experimental data, it was used the compatibility criterion λ: $\lambda = \frac{V(r)}{(1+|r|)^2}$, defined by means of the variance $V(r)$.

Quantitative features of the main height growth stages of human beings of the correlation coefficient r: $r = \frac{\mathrm{Cov}(t,u)}{\sqrt{V(t)\cdot V(u)}}$.

The detailed expressions of the compatibility criterion λ for different types of weights are presented in the frame of our work [17].

3.3.5. Numerical results corresponding to the High-Volume Statistics Data and Comparison with those of the Low-Volume Statistics

The iterative procedure of the classical gradient method [27–30] was used for the evaluation of the specific parameters of the different growth stages.

Taking into account that the low-volume statistics growth data [21–22], distinguish only the burst length growth stage in adolescence instead of the 2 successive burst and inflation phases, we have introduced in Table 2 the obtained numerical values of the growth parameters of the burst phase [18] between those corresponding to the burst growth stage and to the inflation one, respectively.

The most important findings of the study accomplished in this chapter are:

a) the qualitative conclusions resulted by means of the analysis of the low-volume statistics growth data [21, 22, 31, 32], are confirmed by the study of the high-volume statistics data,

b) due to the considerably better accuracy of the high-volume statistics growth data, the discrimination power of these data is sensibly higher; in consequence, the number of post-natal growth stages distinguished by the high-volume statistics is somewhat larger: 5 post-natal length/height growth stages instead of only 4 given by the low-volume statistics (the “overall” burst phase observed for the low-volume statistics data being split into a burst phase and an inflation one; see Table 2),

Table 2

Accuracy of the main descriptions of the basic height growth stages of human beings

Correlation type	Source	CDC [26]	CDC [26]	CDC [26]	Hartung [21]	CDC [26]	CDC [26]
	Param	Baby lengt slowing down	Boy stature slowing down	Adol. burst growth stage	Adol. burst growth stage	Adol. infl.growth stage	Stature growth stop
West	Time interv	0÷33 months	2÷8 years	127÷145 months	11÷15 years	145÷157 months	159÷225 months
	Numb of data	11 (quarterly)	12 (biannual)	9 (bi-monthly)	5 (annual)	12 (monthly)	12 (biannual)
	β, %/yr	0.0362	0.2122	-1.035	2.909	1.326	-0.6538
	γ	-3.776	-5.278	31.86	2.825	-24.713	7.713
	Stand.dev.	0.8%	1.6%	0.4%	2.9%	0.2%	4.6%
West linear $\dot{y} = f(\dot{y}_{calc})$	Slope	0.9985	0.9789	1.017	1.035	1.011	1.042
	Cross.coord.	$7.4\cdot10^{-5}$	$1.2\cdot10^{-3}$	$-6.8\cdot10^{-4}$	-0.155	$-5.3\cdot10^{-4}$	$-1.5\cdot10^{-4}$
	Std. dev.	0.8%	1.7%	0.4%	2.0%	0.2%	5.2%
	r	0.9999	0.9983	0.9984	0.9794	0.9993	0.9988
	λ	131.597	4.070	4.197	4.374	4.189	4.395
Power Law	Time interv.	0÷3 years	2÷10 years	127÷144 months	11÷15 years	145÷159 months	159÷239 months
	Numb of data	35 (monthly)	95 (monthly)	18 (monthly)	5 (annual)	14 (monthly)	80 (monthly)
	C, %/yr	-0.947	-0.1097	1.191	21.88	0.056	-1.548
	n	0.3395	0.5622	1.697	1.169	0.6794	1.567
	Std. dev.	7.1%	3.1%	0.1%	2.7%	0.7%	6.4%
Power Law linear $\dot{y} = f(\dot{y}_{calc})$	Slope	1.093	1.0037	1.0005	1.011	1.0182	1.0382
	Cross.coord.	-0.0110	$-1.1\cdot10^{-4}$	$-1.97\cdot10^{-5}$	-0.0497	$-8.8\cdot10^{-4}$	$-5.4\cdot10^{-5}$
	Std. dev	14.6%	9.5%	0.1%	1.9%	0.8%	6.3%
	r	0.98717	0.99470	0.99985	0.98040	0.98980	0.99927
	λ	4.759	4.051	3.344	4.170	4.221	3.961

c) both the West's type expressions and the power law ones are compatible with the studied experimental growth data,

d) as it was to be expected, there are some significant differences between the basic parameters of the high-volume (World) statistics and the ones for the low-volume (regional) statistics; as a unique example, the maximum value of the non-dimensional length growth rate $\dot{y}$ is reached after 157 months (approximately 13 years) for the CDC statistics, and after 15 years according to the low-volume (regional) statistics given by the treatise [21],

e) the results seem to be valid also (at least, qualitatively) for the growth of all living organisms [33], of tumors [34–37], of some economic activities [36], of the population dynamics [38], and even for the qualitative description of the Universe evolution [39, 40,45].

4. UNITARY DESCRIPTION OF HUMAN BODY WEIGHT GROWTH

The first unitary description (a qualitative-empirical one) of the human body mass growth was achieved in 1926 by C. B. Davenport, in the frame of his work "Human growth curve" [41], by means of 3 additive auto-catalytic – stagnation (ACS) growth plots, two rather "abrupt" corresponding to the intervals: a) fetus – baby growth, and: b) adolescence, and a third one more flat, for the childhood interval (approximately 2-12 years). This work achieved also (Table 3) a detailed comparison of the PUN functions and of the main semiempirical functions (of the Davenport-Robertson's ones, particularly) used for the description of the human body growth.

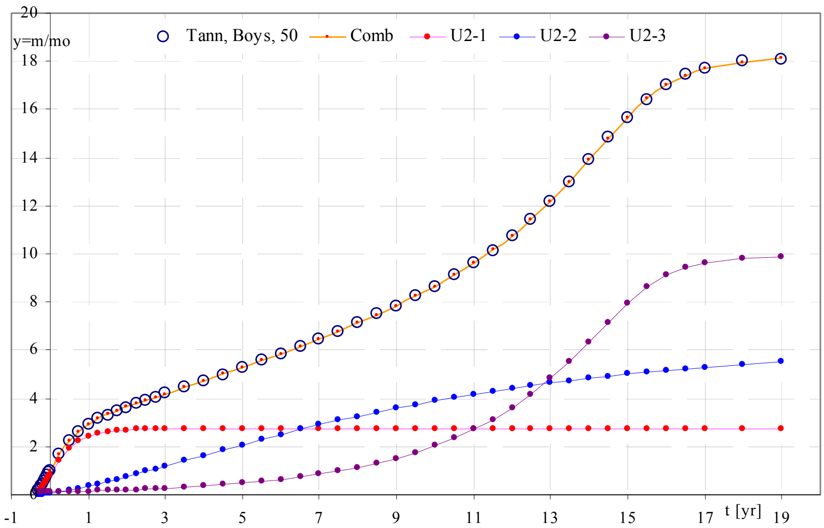

Fig. 4 – The tree components U_2 of the growth plot, represented separately, their resultant and the observed data (the used data: prenatal – Gardossi [20], postnatal – Tanner [19].

The semi-empirical results concerning the possibility of a unitary (in time) description of the growth processes were scientifically confirmed by the works of

A. M. Turing “The chemical basis of morpho- genesis”, 1952; J. Ross, C. Müller, C. Vidal “Chemical waves”, 1988 [7] in the frame of the study of some processes considerably more intricate – those of morpho-genesis.

Starting from the findings of the work [7], which pointed out that the living beings morphology is achieved by a continuous chemical process starting from the embryo phase, this work used the method of phenomenological universality (PUN) [13] to express the 3 functions of our unitary description of the “reduced” weight growth (m_0 – the birth mass):

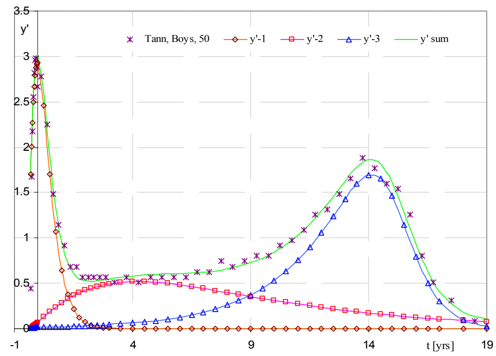

Fig. 5 – The three components of the growth rate, represented separately, their resultant and the rate of observed data (the used data – [19,20]).

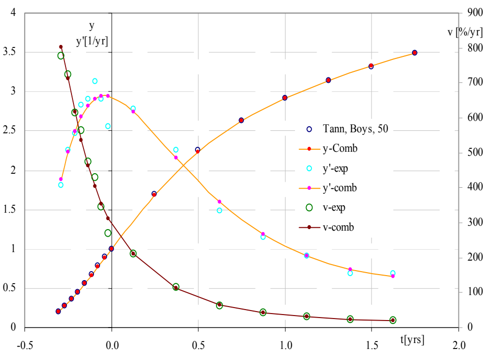

Fig. 6 – Some details about the evolution of the reduced mass (y) and the growth rates: the absolute – y' and the relative one – v for the interval 24 weeks before the birth ÷ 1.5 years (exp = experimental data, comb = analytical result).

$$\frac{m}{m_o} = y(t) = y_{01} F_{1U}(t) + y_{02} F_{2U}(t) + y_{03} F_{3U}(t). \tag{4.1}$$

The symbols $F_{1U}(t), F_{2U}(t), F_{3U}(t)$ from expression (4.1) stand for the PUN functions used for description, and y_{01}, y_{02}, y_{03}, $(y_{01} + y_{02} + y_{03} = 1)$ are the corresponding weights. Of course, the chosen functions must have an accommodation-stagnation character (ACS), their number (3) being the least possible taking into account the global shape of the growth plot, involving exactly the three accelerated growth stages found before [41].

We have studied a rather large number (10) of different basic semi-empirical relations (linear-logarithmic, exponential, mono-, double-, triple-, particular logistic, Gompertzian, sum of powers or exponentials, etc) used to describe the human body growth, proposed in the frame of the works [45–55].

4.1. PHENOMENOLOGICAL APPROXIMATION

Given being we have found the possibility of coincidence – in certain conditions – of the model of auto-catalytic growth followed by stagnation (ACS) with the West's model (U2), the 3 functions of the unitary description of the "reduced" mass growth were represented by the model U2 of the phenomenological universality:

$$y_i(t) = y_{oi}\left[1 + \frac{\nu_{oi}\gamma_i}{\beta}\left(1 - \exp(\beta_i t)\right)\right]^{-1/\gamma_i} \quad \text{(for } i = 1, 2, 3\text{)}. \tag{4.2}$$

It was met so a problem of adjustment (fitting) of a rather large number (12) of parameters of a non-linear relation (1), by a *minimization requirement* of a square mean deviation of the analytical solution values relative to the "experimental" ones:

a) absolute, of a: (i) growth parameter $y_i(t)$ (the "reduced" mass), (ii) the absolute growth rate $\dot{y}_i(t)$, (iii) and the relative growth rate $\nu_i = \dot{y}_i / y_i$, respectively, relative of the 3 indicated parameters,

b) by means of a *maximization requirement* of one of the correlation coefficients $R(\tilde{y}, \bar{y})$, $R(\tilde{\dot{y}}, \bar{\dot{y}})$ and $R(\tilde{\nu}, \bar{\nu})$ between the calculated values $\tilde{y}_i$, $\tilde{\dot{y}}_i$, $\tilde{\nu}_i$ of the 3 indicated parameters and the corresponding "experimental" values $\bar{y}_i$, $\bar{\dot{y}}_i$, $\bar{\nu}_i$, respectively.

The adjustment (fitting) method of the studied parameters used the computing code Solver from Excel, itself this program representing an application of the method of the conjugated gradient. Given being the: a) huge diversity of conjugate gradient method versions [33], b) the rather numerous various criteria of minimization/maximization (not less than 9 in the studied problem), leading to sensibly different results concerning the parameters of the examined non-linear expression, c) the fact that the usual conjugate gradient method versions and their computing code Solver application do not take into consideration the existing experimental errors and do not evaluate the compatibility criterion (3.4), it results that there remain considerable improvement possibilities in the approach of such problems.

Due to the rather large errors intervening in the evaluation of the relative square mean deviations $\varepsilon(y) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\tilde{y}_i / \bar{y}_i - 1\right)^2}$ (of the analytical solution $\tilde{y}_i / \bar{y}_i$ relative to the experimental one), especially, and of the absolute square deviations $\Delta(y) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\tilde{y}_i - \bar{y}_i\right)^2}$ of the same quantities, these parameters

[$\varepsilon(y)$, $\Delta(y)$] were studied mainly by this work. Of course, the use of the conjugate gradient method requires also a suitable choice of the zero-order approximations of the parameters (12, for the approached problem) of the studied nonlinear relation. As a preliminary stage of the choice of the zero-order approximations, there were studied in detail the inflexion points and the asymptotes specific to the phenomenological universality functions (PUN).

Table 3 below presents (as a particular example, for the Tanner's data [19] corresponding to the British boys growth) the results obtained for each U2 (Fig. 4) component of the unitary growth curve (4.1) referring to the: a) weight y_{oi} (i = 1, 3), b) relative growth rate v_o, c) damping coefficient β(years^{-1}), d) generation coefficient γ, e) inflexion time $T_{infl.}$ of the corresponding $y = f(t)$ plot, f) the relative and the absolute mean square deviations $\varepsilon(y)$, $\Delta(y)$ for: (i) the non-dimensional growth parameter y, (ii), (iii) its absolute $\dot{y}$ and relative v growth rate, g) the correlation coefficients $R(\tilde{y},\ \overline{y})$, $R(\tilde{\dot{y}},\ \overline{\dot{y}})$, $R(\tilde{\text{v}},\ \overline{\text{v}})$ between the numerically evaluated growth quantities ($\tilde{y}$) and the experimental ones ($\overline{y}$) for the above indicated parameters.

Fig. 5 shows that each component act individually: the intensity of it's action is maximal in the intervals when the other two are working "in background".

Table 3

The evaluated parameters of the unitary (3U2) description of the growth process, for the Tanner's data [19] concerning the British boys

y_o	v_o [yr $^{-1}$]	β [yr $^{-1}$]	γ	T_{infl} [yr]	
0.8000826	3.5965152	–1.8551141	–0.6115777	–0.065	U2-1
0.0121928	6.0715281	–0.2014191	–0.4623122	3.486	U2-2
0.1877246	0.2630745	–1.3050361	4.9607092	14.312	U2-3
Δ(*y*)=	0.1273829	***Ry*=**	0.999995	***ε*(*y*)=**	0.42%
Δ(*y'*)=	0.4378222	***Ry'*=**	0.997143	***ε*(*y'*)=**	5.9%
Δ(*v*)=	41.436459	***Rv*=**	0.999591	***ε*(*v*)=**	5.9%

The accomplished study reported also the obtained results corresponding to the boys and girls growths, respectively, both for the National Center for Health Statistics (USA) [56] and the Tanner's results [19], as well as multiple growth plots presenting both the "experimental" data and the evaluated ones for the basic studied parameters [the non-dimensional ("reduced") mass and its absolute and relative growth rates], finding usually a very good agreement of the studied unitary description of growth processes with the observed data (Fig. 6).

5. STUDY OF THE GRADUAL DEGRADATION OF THE STRUCTURE OF FD SIMULATIONS OF SOME SOLITONS, AS EVOLVING COMPLEX SYSTEMS

After the achievement (by means of some organization/auto-organization processes) of the optimal structure of an evolving pulse, this pulse is subject to some local interactions of different intensities, which lead to the appearance and gradual development of certain distortions, determining finally the appearance of instabilities and the annihilation of the evolving pulse. An important parameter for the characterization of the gradual degradation of the evolving pulses structure is their life-duration t_{max} (up to the destruction of their optimal profile). To evaluate numerically this duration for one of the simplest complex systems (the KdV solitons), it was used the FD method, simulating the local interactions by means of the local deviations (errors) of certain FD schemes relative to the theoretical values corresponding to the solitons evolution. The used numerical method was that of LISA [39], in the versions: a) strong-LISA, corresponding to the usual numerical schemes (in 2 steps), without stabilization elements, b) medium-LISA, for the usual FD schemes in 2 steps, involving stabilization elements of the Zabusky-Kruskal [11a] and Landau-Pàez type [11b], respectively, c) weak-LISA, for the FD schemes with 4 steps introduced in the frame of the work [40].

To study the appearance and development of the numerical distortions which lead finally to instabilities (the destruction of the evolving pulses) there were evaluated the statistical parameters skewness and kurtosis, respectively, the obtained presents the upper/lower branches between whom produce the oscillations with the period $T = 2$ iterations of the parameters skewness and kurtosis. The life duration t_{max} of an evolving pulse is evaluated by means of the finite differences (FD) method, using the elementary relation $t_{\max} = N_{inst.} \cdot \Delta t_{\max}$, where $N_{inst.}$ is the number of successive approximations (iterations) up to the instability regime installation (destruction of the evolving pulse), and $\Delta t_{\max}$ is the maximum value of the FD time step corresponding to a given space step Δs. Given being the our study proved that the Vliegenthart's stability criterion of the FD simulations of the KdV pulses propagation is only a sufficient condition (but not a necessary one, also), this study has accomplished (see also [40]) a detailed study of the necessity condition, pointing out that usually are stable the FD simulations with considerably larger (1,6 … 1,7 times) values of $\Delta t_{\max}$ than the threshold indicated by the Vliegenthart's sufficiency condition [41].

6. CONCLUSIONS

The evolution of many complex physical, technical and biological systems, characterized by growth (autocatalytic – stagnation) processes can be described with satisfactory accuracy by the phenomenological universality model (or of some power laws), especially by means of the U2 class functions (or a linear combination thereof, where each component is more potent than others in a given period of time).

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