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THE QCD UNIVERSE WITH VISCOUS AND PARTICLE CREATION

S.M. MOTEVALLI, J. SADEGHI, A. NOUROUZI

Department of Physics, Faculty of Science, University of Mazandaran,
P. O. Box 47416-95447, Babolsar, Iran
E-mail: motavali@umz.ac.ir

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Abstract. In this paper, we study FLRW bulk viscous cosmology in presence of particle creation and obtain the Hubble expansion parameter. In order to find effect of particle creation and bulk viscosity on the cosmological quantities we use perturbation theory and recent lattice QCD simulations. Finally, we obtain the time evolution of energy density for viscous cosmological matter in the presence of particle creation.

Key words: Particle creation, Cosmological quantity, Perturbation theory, Bulk viscosity.

1. INTRODUCTION

The observed Universe has a high entropy, with high number of photons per baryon, about 10^8. This gives a total entropy in the observable Universe of about 10^{88} [1]. Inflationary cosmology predicts that nearly all of this entropy is generated by the reheating process at the end of inflation, *i.e.*, that all other dissipative processes in the evolution of the Universe make a negligible contribution to entropy production by comparison. In this model, the entropy would have to be modified to include the dissipation not only from the fluid effects, but also from particle creation. Particle creation leads to non-conservation of particle number. The contribution from particle production may be modelled as an effective bulk viscosity [1]. The first theoretical approach of the particle creation problem was investigated by Prigogine *et al.* [2]. They showed that the second law of thermodynamics may be modified to accommodate flow of energy from the gravitational field to the matter field, resulting in the creation of material particles [3]. The particle creation pressure, which is negative, might play the role of dark energy component [4]. Padmanbhan and Chitre discussed the role of the bulk viscosity in the entropy production in an expanding Universe [5]. Another peculiar characteristic of bulk viscosity is that it acts like a negative energy field in an expanding Universe (Johari and Sudharsan) [6]. The basic idea was that the bulk viscosity (particle creation) contributes at the level of the Einstein field equations as a negative pressure term. Barrow [7, 8] introduced this idea in the framework of new inflationary scenario.

We consider the cosmological model to study the evolution of the Universe as it goes from an inflationary phase to a radiation-dominated era in the presence of bulk viscosity and particle creation. We have discussed the role of bulk viscosity in the framework of the particle creation mechanism in homogeneous and isotropic flat FLRW model. Our results are strongly limited within a very narrow temperature - or time - interval in the QCD era, where the quark-gluon plasma is likely dominant. The background matter is assumed to be characterized by barotropic equations of state, obtained from recent lattice QCD simulations. Here, it is important to note that the QCD plasma be formed at temperatures $0.2 \leq T \leq 10$ GeV. In the recent work [9], Hubble expansion parameter in QCD Universe for finite bulk viscosity has been studied. We indeed used method of the Refs. [9, 10] and extended this work to the case of particle creation. Therefore, in this paper we see the effect of creation particle on the early Universe.

2. GEOMETRY AND BASIC EQUATIONS

In this section, we consider Friedmann-Lemaitre-Robertson-Walker (FLRW) bulk viscous cosmology with particle creation which described by the following metric,

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \tag{1}$$

where r, θ and φ are dimensionless co-moving coordinates and $a(t)$ is the scale factor. Throughout we use units such that $c = 1$. The Hubble parameter is defined as $H = \dot{a}/a$. The Einstein's field equations are given by,

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi G T_{ij}, \tag{2}$$

where G is gravity constant that $8\pi G = 1$ and the energy momentum tensor (T_{ij}) of the cosmic fluid in the presence of creation of particle and bulk viscosity includes the creation pressure term p_c and the bulk viscous stress. Thus it may be defined as

$$T_{ij} = (\rho + p + p_c + \Pi)\,u_i\,u_j - (p + p_c + \Pi)\,g_{ij}, \tag{3}$$

with $i, j = 0, 1, 2, 3$. ρ is the energy density, p is the thermodynamic pressure, Π is the bulk viscous pressure and u_i is the four velocity vector where $u^i\,u_i = -1$. In that background, the nontrivial EFE for a fluid endowed with matter creation, viscosity and the balance equation for the particle number density can be written as

$$3\,\frac{\dot{a}^2}{a^2} = \rho, \tag{4}$$

$$2\,\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -(p + p_c + \Pi), \tag{5}$$

$$\frac{\dot{n}}{n} + 3\,\frac{\dot{a}}{a} = \frac{\psi(t)}{n} = \frac{\dot{N}}{N}, \tag{6}$$

where an overdot means time derivative and $n = N/V$ is the particle number density, N is the particle number and $\psi(t)$ is matter creation rate ($\psi(t) > 0$ corresponds to particle creation while $\psi(t) < 0$ to particle decay). We consider the simple phenomenological expression of the particle creation rate [11],

$$\psi(t) = 3\beta n H, \tag{7}$$

where the parameter β is defined on the interval $\beta \in [0, 1]$, which is assumed to be constant. In the case of adiabatic particle creation, the pressure p_c is given by [9, 12],

$$p_c = -\frac{(\rho + p)V}{N}\frac{dN}{dV}, \tag{8}$$

Substituting Eqs. (6) and (7) into Eq. (8), the particle creation pressure is given by,

$$p_c = -\beta(\rho + p). \tag{9}$$

The continuity equation in presence of particle creation and bulk viscosity obtained as the following,

$$\dot{\rho} + 3(\rho + p)H = -3(p_c + \Pi)H. \tag{10}$$

The evolution of Π is governed by a transport equation. This equation is known as the full Israel-Stewart equation [13, 14],

$$\Pi + \tau\dot{\Pi} = -3\xi H - \frac{1}{2}\tau\Pi\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{T}}{T} - \frac{\dot{\xi}}{\xi}\right], \tag{11}$$

where ξ denotes the coefficient of bulk viscosity, T is temperature of the fluid, and τ is the relaxation time associated with the dissipative effects.

We assume that the Universe is filled with a relativistic viscous QCD plasma, whose bulk viscosity is supposed to be finite. By comparing the Universe in phase transition we see some quark - gluon plasma and string gas phase, so we take some information from QCD which are obtained by lattice QCD simulation. Then, the equation of state, the temperature and the bulk viscosity of the quark-gluon plasma (QGP) will be as [15, 16],

$$p = (\gamma - 1)\rho, \qquad T = \beta\rho^r, \qquad \xi = \alpha\rho + \frac{9}{\omega_0}T_c^4, \tag{12}$$

$$\alpha = \frac{1}{9\omega_0}\frac{9\gamma^2 - 24\gamma + 16}{\gamma - 1}, \tag{13}$$

where $\gamma \simeq 1.183$, $r \simeq 0.213$, $\beta \simeq 0.718$, and $\omega_0 \simeq 0.5 - 1.5$ *GeV*, respectively. We assume that $\alpha\rho \gg 9/\omega_0 T_c^4$ and we take $\xi \simeq \alpha\rho$. In order to have a close system of the cosmological equations, we need to give the expression of the relaxation time τ as [1],

$$\tau = \frac{\xi}{\rho} \simeq \alpha. \tag{14}$$

Equations (12) and (13) are standard in the study of the viscous cosmological models, whereas the equation (14) for τ is a simple procedure to ensure that the speed of viscous pulses does not exceed the speed of light.

2.1. PERTURBATIVE SOLUTION

Here, we consider the viscous Universe filled with a viscous QCD plasma in presence of particle creation and bulk viscosity. In that case the Hubble parameter equation will be obtained by equations (4) and (9-14),

$$\alpha H\ddot{H}+\frac{3}{2}[1+\gamma(1-\beta)(1-r)]\alpha H^2\dot{H}+H\dot{H} \\ -(1+r)\alpha\dot{H}^2+\frac{9}{4}\alpha H^4[\gamma(1-\beta)-2]+\frac{3}{2}\gamma(1-\beta)H^3=0. \quad (15)$$

In the $\alpha\longrightarrow 0$ limit, the evolution equation reduced to the following relation,

$$\dot{H}_0(t)+\frac{3}{2}\gamma(1-\beta)H_0^2(t)=0, \quad (16)$$

where $H_0(t)$ is the Hubble parameter corresponding to the non-viscous background matter. Solving Eq. (16), we get,

$$H_0(t)=\frac{2}{3\gamma(1-\beta)}t^{-1}. \quad (17)$$

It is clear that increasing β in the range $0\leq\beta<1$ increased the Hubble expansion parameter. Therefore effect of particle creation is increasing the Hubble expansion parameter. For the flat Universe, by using the relation (4) one can obtain energy density as the following,

$$\rho_0(t)=\frac{4}{3\gamma^2(1-\beta)^2}t^{-2}, \quad (18)$$

which is expectable so energy density usually is proportional to t^{-2}. We try to obtain Hubble expansion parameter and energy density with bulk viscosity and particle creation. We consider also the effects of the bulk viscosity by using a perturbative method at the first-order. Solution of the equation (15) may be written as,

$$H(t)=H_0(t)+\alpha F(t), \quad (19)$$

where $F(t)$ is the first-order perturbative correction. Substituting the solution (19) into the equation (15) and ignore the second power of α leads to the following equation,

$$\alpha H_0(t)\ddot{H}_0(t)+\frac{3}{2}[1+\gamma(1-\beta)(1-r)]\alpha H_0^2(t)\dot{H}_0(t)+H_0(t)\dot{H}_0(t)$$
$$+\alpha F(t)\dot{H}_0(t)+\alpha H_0(t)\dot{F}(t)-(1+r)\alpha\dot{H}_0^2(t)+\frac{9}{4}\alpha[\gamma(1-\beta)-2]H_0^4(t)$$
$$+\frac{3}{2}\gamma(1-\beta)H_0^3(t)+\frac{9}{2}\alpha\gamma(1-\beta)F(t)H_0^2(t)=0. \tag{20}$$

We can rewrite the equation (20) in the form of,

$$\dot{F}(t)+K_1(t)F(t)+K_0(t)=0, \tag{21}$$

where

$$K_0(t) = -\ddot{H}_0(t)-\frac{3}{2}[1+\gamma(1-\beta)(1-r)]H_0(t)\dot{H}_0(t)+(1+r)\dot{H}_0^2(t)H_0^{-1}(t)$$
$$-\frac{\dot{H}_0(t)}{\alpha}-\frac{9}{4}[\gamma(1-\beta)-2]H_0^3(t)-\frac{3}{2}\frac{\gamma}{\alpha}(1-\beta)H_0^2(t), \tag{22}$$

and

$$K_1(t)=\dot{H}_0(t)H_0^{-1}(t)+\frac{9}{2}\gamma(1-\beta)H_0(t). \tag{23}$$

We can use the equation (17) for eliminate $H_0(t)$ and it's derivatives. Then $K_0(t)$ and $K_1(t)$ can be written as,

$$K_0(t)=\frac{4}{3}\frac{1}{\gamma^3(1-\beta)^3t^3}, \tag{24}$$

$$K_1(t)=\frac{2}{t}. \tag{25}$$

The equation (21) is a first-order differential equation with the following solution,

$$F(t)=\frac{\int e^U K_0(t)dt+A}{e^U}, \tag{26}$$

where $U=\int K_1(t)dt$, and A is an integration constant. Substituting equations (24) and (25) into the equation (26) and then integrating yields,

$$F(t)=\frac{\frac{4\ln(t)}{3\gamma^3(1-\beta)^3}-A}{t^2}. \tag{27}$$

Therefore, substituting equations (17) and (27) into (19) gives us the Hubble expansion parameter as the following,

$$H(t)=\frac{2}{3\gamma(1-\beta)}t^{-1}+\alpha\left[\frac{4}{3\gamma^3(1-\beta)^3}\ln(t)-A\right]t^{-2} \tag{28}$$

with assuming $A = \frac{4}{3\gamma^3(1-\beta)^3} \ln t_0$, the Hubble expansion parameter in the viscous QGP model in presence of particle creation will be as,

$$H(t) = \frac{2}{3\gamma(1-\beta)} t^{-1} + \frac{4\alpha t^{-2}}{3\gamma^3(1-\beta)^3} \ln\left(\frac{t}{t_0}\right). \tag{29}$$

Finally, we can use relation (4) to obtain time-dependent energy density as the following form,

$$\rho(t) = \frac{4}{3}\left[\frac{1}{\gamma(1-\beta)} t^{-1} + \frac{2\alpha t^{-2}}{\gamma^3(1-\beta)^3} \ln\left(\frac{t}{t_0}\right)\right]^2. \tag{30}$$

We draw the corresponding Hubble parameter $H(t)$ in terms of time which show the variation of this parameter with respect to time for various particle creation coefficient (β), in viscous and non-viscous early Universe (Fig. 1). In the Fig. 2 we use QCD data and draw energy density in terms of time. In these figures, we see the time evolution of Hubble parameter and energy density are depicted for viscous cosmological matter in presence of particle creation and a non-viscous cosmological matter in the presence of particle creation. Also, considering QCD data, we see that the time evolution of the Hubble expansion parameter in the presence of particle creation for viscous cosmological matter is faster than its evolution for non-viscous cosmological matter.

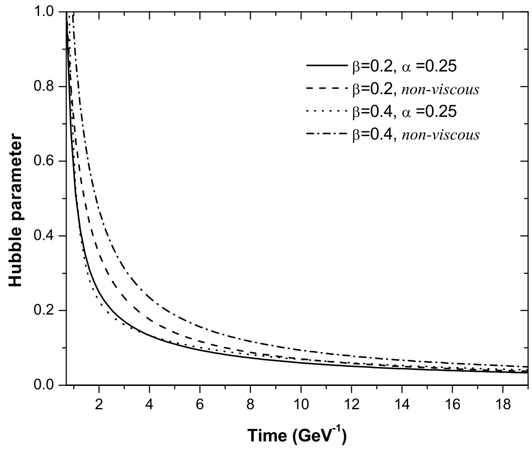

Fig. 1 – The dependence of Hubble parameter on co-moving time for bulk viscosity coefficient, $\alpha = 0.25$ and various particle creation coefficient, $\beta = 0.2$ (solid line); $\beta = 0.2$, *non-viscous* (dashed line); $\beta = 0.4$ (dotted line) and $\beta = 0.4$, *non-viscous* (dash-dotted line).

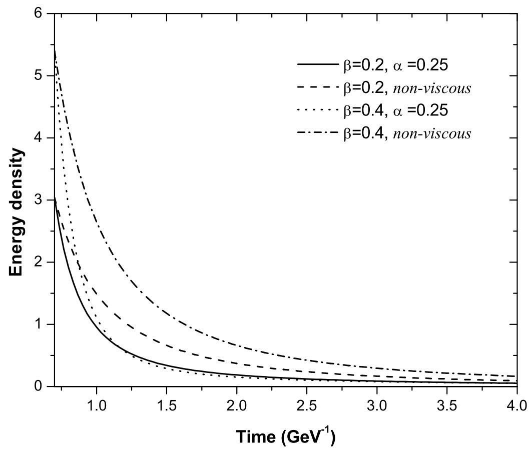

Fig. 2 – The dependence of energy density on co-moving time for bulk viscosity coefficient, $\alpha = 0.25$ and various particle creation coefficient, $\beta = 0.2$ (solid line); $\beta = 0.2$, *non-viscous* (dashed line); $\beta = 0.4$ (dotted line) and $\beta = 0.4$, *non-viscous* (dash-dotted line).

3. CONCLUSION

In this paper, we investigated the time evolution of Hubble parameter and energy density in QCD Universe with viscous FLRW model and particle creation, and compared these with their evolution for non-viscous cosmological matter. In order to obtain the corresponding Hubble expansion parameter and energy density we took data from lattice QCD simulations for flat Universe. Also, we used the truncated Israel-Stewart equation for the Universe that filled with viscous QGP in presence of particle creation and bulk viscosity. In the present work, we applied the first order perturbative correction and obtained the Hubble expansion parameter and energy density. The obtained results show that Hubble parameter and energy density increase with increasing particle creation coefficient for non-viscous cosmological matter. Finally, our calculations show that the time evolution of Hubble parameter and energy density in the presence of particle creation for viscous cosmological matter are faster than their evolution for non- viscous cosmological matter.

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