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SHELL MODEL CALCULATIONS FOR NEUTRINOLESS DOUBLE BETA DECAY THROUGH THE EXCHANGE OF HEAVY NEUTRINOS

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Abstract. Nuclear matrix elements (NMEs) for neutrinoless double beta decay ($0\nu\beta\beta$) are computed for the mechanism of exchange of heavy neutrinos. The calculations are performed with a shell model (ShM) code recently developed, for three experimentally interested nuclei, ^{48}Ca, ^{76}Ge, and ^{82}Se. We study the different nuclear effects, such as short range correlations (SRCs), finite nuclear size (FNS) and higher order terms in the nucleon currents (HOC) on the final values of the NMEs, and find that their influence is stronger than in the case of the light neutrino exchange mechanism. We compare our results with similar results from literature and discuss the differences.

1. INTRODUCTION

The study of the $0\nu\beta\beta$ decay is very important because it could clarify the question on the lepton number conservation, decide on the neutrinos character (are they distinguished or not from their antiparticles?) and give a hint on the scale of their absolute masses. The importance of these fundamental issues has led to ample theoretical and experimental investigations of this process [1]- [6]. The largest uncertainties in the theoretical calculations for double beta decay (DBD) are related to the values of the NMEs. Their computation is currently performed by several methods, the proton-neutron Quasi Random Phase Approximation (pnQRPA) [7]- [11], Interacting Shell Model(ISM) [12]- [15], Interacting Boson Approximation (IBA) [16]- [18], Projected Hartree Fock Bogoliubov (PHFB) [19] and Energy Density Functional (EDS) method [20] being at present the most employed ones. DBD can occur through several possible mechanisms, the most common being the exchange of a left-handed light neutrino between two nucleons inside a nucleus. However, theories beyond the SM predict that other mechanisms as, for example, exchange of heavy left and/or right neutrinos in the presence of right-handed currents [21]- [23], exchange of SUSY particles [3], [24], DBD with Majoron emission [25], Kaluza-Klein neutrino exchange within an extra-dimensional model [26], etc, may contribute to $0\nu\beta\beta$ decay, as well. Specific NMEs are associated to each of these mechanisms. For the light-neutrino exchange mechanism there are still large differences in literature between the NMEs values computed with different methods and by different groups, and these discrepancies have been largely discussed in the literature (see for example [5]- [6]). For other mechanisms, until now, extended calculations of the corresponding NMEs have been performed only with pnQRPA [22], and IBA-2 [18] methods. ShM calculations do exist, as well, but for fewer cases and not discussed in detail [28], [29]. Such ShM calculations are now possible and needed in order to extract (using experimental limits for the $0\nu\beta\beta$ lifetimes) upper limits for heavy neutrino and SUSY masses and their couplings, in various scenarios. It is worth to mention that studies on the heavy neutrino and SUSY mechanisms are also under investigation at high energies in experiments at hadron colliders, particularly at LHCb, by analysing decay channels with same-sign dilepton emission, processes which violate by two units the lepton number conservation [31]. In this paper we report new values for the NMEs associated with the heavy neutrino mechanism for the $0\nu\beta\beta$ decay. The calculations are performed with a ShM code recently developed [32], [33] for three experimentally interested nuclei, ^{48}Ca, ^{76}Ge and ^{82}Se. We highlight the effect of inclusion of different nuclear effects on the NMEs values and compare our results with other similar ones reported in literature.

2. FORMALISM

The theoretical formalism leading to the NMEs formulae associated with the heavy neutrino exchange mechanism is similar to that for the case of the light neutrino exchange formalism [3], [18], [29]. The lifetimes for $0\nu\beta\beta$ decay read as:

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} \mid M_{H\nu}^{0\nu} \mid^2 \left(\langle\eta_k\rangle\right)^2 \tag{1}$$

$G^{0\nu}$ is the phase space factor for this decay mode and $\langle\eta_k\rangle$ is the heavy neutrino mass parameter. $M_{H\nu}^{0\nu}$ are the NMEs depending on the nuclear structure of the nuclei involved in the decay. They can be expressed as a sum of products of two-body transition densities (TBTDs) and matrix elements of the two-body transition operators for two-particle states, shortly, two-body matrix elements (TBMEs):

$$M_{\alpha,H}^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTDs\left(j_p j_{p'}, j_n j_{n'}; J_\pi\right) \cdot TBMEs\left(j_p j_{p'}, j_n j_{n'}; J_\pi\right) \tag{2}$$

with:

$$TBMEs\left(j_p j_{p'}, j_n j_{n'}; J_\pi\right) = \left\langle j_p j_{p'}; J_\pi \| \tau_{-1}\tau_{-2} O_{12}^{\alpha} \| j_n j_{n'}; S_\alpha J_\pi \right\rangle, \tag{3}$$

where $|jj'; J^\pi >$ represent the antisymmetrized two-particle states and O_{12}^{α} are two-body transition operators whose expressions can be found in [3], [10]. The challenging issue is the computation of the radial part of the TBMEs which contains the neutrino potetials. In the case of heavy neutrinos the expressions of the neutrino potentials differ from those of light neutrino mechanism, and read as: [10]

$$H_\alpha(r) = \frac{2R}{\pi m_e m_p} \int_0^\infty j_i(qr) h_\alpha(q^2) q^2 dq, \tag{4}$$

where $R = 1.2A^{1/3}$ fm and $j_i(qr)$ is the spherical Bessel function (i = 0, 0 and 2 for Gamow-Teller (GT), Fermi (F), and tensor (T), parts, respectively). The expressions of h_α factors are also given in [3], [10] and include FNS effect and HOC. In the computation of the radial matrix elements $\langle nl|H_\alpha|n'l'\rangle$ we also included the SRCs effects. This is done by using harmonic oscillator wave functions $\psi_{nl}(lr)$ corrected by a factor $[1+f(r)]$, with $f(r) = -c \cdot e^{-ar^2}\left(1-br^2\right)$. We take the values of the a, b and c constants in different parameterizations: Jastrow with Miller-Spencer (J-MS) parametrisation [34], AV18 and CD-Bonn parametrisations [35]- [36]. The radial matrix elements of the neutrino potentials become:

$$\left\langle nl \mid H_\alpha(r) \mid n'l' \right\rangle = \int_0^\infty r^2 dr \psi_{nl}(r) \psi_{n'l'}(r) \left[1+f(r)\right]^2 \times \int_0^\infty q^2 dq V_\alpha(q) j_i(qr), \tag{5}$$

where $V_\alpha(q)$ is the Fourier transformation of the neutrino potential.

3. NUMERICAL RESULTS AND DISCUSSIONS

We perform the calculations with ShM code described in Refs. [32,33]. It allows a rapid end efficient procedure for computing the radial matrix elements of the neutrino potentials (5) by reducing the computation of the two-dimensional integral to the computation of a single-dimensional integral over momentum. The two-body transition densities needed to calculate the $M^{0\nu}$ nuclear matrix element, are computed with the code ANTOINE [37] and using the method described in Ref. [15]. We include in the calculations all the nuclear effects which are usually taken into account, i.e. FNS, HOC and SRCs introduced by the J-MS [34], AV18 and CD-Bonn [35,36] parameterizations. The SRC parameters entering the $f(r)$ expression are the same as in Ref. [15]. For the vector and axial coupling constants we use the values $g_V = 1$ and the unquenched value $g_A = 1.25$, while the values of the vector and axial vectors form factors are $\Lambda_V = 850 MeV$ and $\Lambda_A = 1086 MeV$ [1], respectively. For ^{48}Ca we use two different NN effective interactions GXPF1A [38] and KB3G [39] in the full pf model space, while for ^{76}Ge and ^{82}Se we use JUN-45 [40] effective interactions in the $1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2}$ valence space $(jj44)$. We perform the calculations within the closure approximation, with the value of the average energy $< E >$ given by the formula $< E >= 1.12A^{1/2}$. Our results are presented in Table 1, columns 1-5. Besides the total values of $M^{0\nu}_{H\nu}$ which include all the nuclear effects, we also display the NMEs values where the nuclear effects are introduced gradually: bare (b)means no nuclear effects, while (F), (S) and (H) means that FNS SRC and HOC effects are included. One observes that inclusion of SRCs and HOC have a strong influence on the bare value of $M^{0\nu}_{H\nu}$. It is worth to mention that their inclusion affects much stronger the bare values than in the case of light neutrino exchange mechanism. For example, for ^{48}Ca and in the case of KB3G NN interaction, their inclusion diminishes the bare NME value by factors of 10, 4 and 2.5 if J-MS, AV18 or CD-Bonn SRC parametrizations are used. Such decreasing is almost similar when GXPF1A NN interaction is used. Also, the results are sensitive significantly to

Table 1

The NMEs obtained with inclusion of different nuclear effects. ”b” denotes the value obtained without any effect included, while “F”, H” “S” and “total” indices denote the $M^{0\nu}$ values obtained when FNS, HOC, SRC and all effects, are, respectively, included. The set of the three values from the columns with SRC effects included refers to the particular prescriptions: (a)=Jastrow with MS parameterization, (b)=CCM-AV18 and (c)=CCM-CD-Bonn type.

	M_b	M_{b+F}	M_{b+H}	M_{b+F+H}	$M^{0\nu}_{total}$	ISM [29]	IBM2 [18]	QRPA [22]
					(a)-13.8		16.3	
					20.6			
^{48}Ca	-148.1	-104.1	-111.0	-81.1	(b)-37.4	52.9	46.3	
$^{48}Ca^{*}$	168.6	118.4	132.8	97.3	47.5			
					(c)-60.2	75.5	76.0	
					73.5			
					(a) -57.5		48.1	32.6
^{76}Ge	-396.1	-284.4	-321.3	-239.9	(b)-122.1		107	233
					(c)-183.5		163	351
					(a) -56.8		35.6	30.0
^{82}Se	-370.3	-267.4	-303.3	-227.8	(b)-117.5		84.4	226
					(c)-175.1		132	340

$*$KB3G NN-interaction.

the use of different NN effective interactions. These features are also similar in the case of the other three isotopes, ^{76}Ge and ^{82}Se. In columns 6-8 of the Table I we display other results from literature, obtained with other ShM code or with IBA-2 and pnQRPA methods, indicating the references where they are taken from. For ^{48}Ca we compare our results with those from the Refs. [29] and [18]. The difference between the results from [18] and ours is 15% or 20% depending on the NN interaction used, GXPF1A or KB3G, respectively. This can be considered a good agreement having in view that they were obtained with different methods (ShM and IBA-2). In Ref. [29] the calculations are performed with a ShM code and with AV18 and CD-Bonn SRC parametrisations. In this case we suspect the use of an incorrect sign of the tensor component (this contribution should have the opposite sign as the GT contribution). If this contribution would be correctly added, we would have a very good agreement with their results. For the other two isotopes the agreement is good when comparing with IBA-2 results. However, there are significant differences when comparing with QRPA results, which should be further investigated.

4. CONCLUSIONS

We report new values of the neutrinoles DBD matrix elements for the heavy neutrino exchange mechanism. The calculations are performed with a ShM code which allows for a rapid and efficient computation of the TBMEs, and refer to three isotopes: ^{48}Ca, ^{76}Ge and ^{82}Se. For the last two isotopes the NMEs values performed with a ShM-based code are reported for the first time. The results are much sensitive on the nuclear effects than in the case of the light neutrino exchange mechanism, especially when HOC and SRC effects are included. The agreement with other similar values from literature is good for ^{48}Ca, even when the calculations are performed with different methods, as for example IBA-2. For the other two isotopes there are important differences between our calculations and QRPA calculations, while the agreement is again quite good when comparing with IBA-2 calculations. However, the existent differences between the results obtained with different methods and the sensitivity of the NMEs values to the inclusion of different nuclear effects are features which need further investigations.

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