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GENERAL SOLUTION OF A SPECIAL CLASS OF NONLINEAR BBM-B EQUATION BY USING THE (G'/G)-EXPANSION METHOD

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Abstract. In this work, the (G'/G)-expansion method is proposed for constructing more general exact solutions of a special class of the nonlinear Benjamin-Bona-Mahoney–Burgers (BBM-B) equations including exponential terms appearing more in biological science which may contain high nonlinear terms. Our work is motivated by the fact that the (G'/G)–expansion method provides not only more general forms of solutions but also periodic and solitary waves. If we set the parameters in the obtained wider set of solutions as special values, then some previously known solutions can be recovered. The method appears to be easier and faster by means of a symbolic computation system.

Key words: (G'/G)-expansion method, Generalized Benjamin–Bona–Mahony (gBBM) equation, Hyperbolic function solutions, Trigonometric function solutions, Solitary wave solutions.

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1. INTRODUCTION

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical–physical sciences such as physics, biology, chemistry, etc. The analytical solutions of such equations are of fundamental importance since a lot of mathematical–physical models are described by NLEEs.

Among the possible solutions to NLEEs, certain special form solutions may depend only on a single combination of variables such as traveling wave variables. In the literature, there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the Jacobi elliptic function method [1], inverse scattering method [2], Hirota's bilinear method [3], homogeneous balance method [4], homotopy perturbation method [5], Weierstrass function method [6], symmetry method [7], Adomian decomposition method [8], sine/cosine method [9], tanh/coth method [10], the Exp-function method [11], solitary wave ansatz method [12–17] and so on.

But, most of the methods may sometimes fail or can only lead to a kind of special solution and the solution procedures become very complex as the degree of nonlinearity increases.

Recently, the (G'/G)-expansion method, firstly introduced by Wang *et al.* [18], has become widely used to search for various exact solutions of NLEEs [18–26]. The value of the (G'/G)-expansion method is that one treats nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for travelling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation.

The generalized Benjamin-Bona-Mahony-Burgers equations has the following form

$$u_t - u_{xxt} - au_{xx} + bu_x + g(u)_x = 0, \quad x \in \mathbb{R}, t \geq 0. \tag{1}$$

where $u(x,t)$ represents the fluid velocity in the horizontal direction x, $a,b \in \mathbb{R}$, , b is a non-zero constant and finally, $g(u)$ is a C_2- smooth nonlinear function [27–30]. In the physical sense, equation (1) with the dissipative term $-au_{xx}$ is proposed if the good predictive power is desired, such problem arises in the phenomena for both the bore propagation and the water waves.

In [28], Ganji *et al.* used Exp-function method to obtained the travelling wave soliton of Eq. (1), in sample case $g(u) = \frac{1}{2}u^2$, and especially $a = 0, b = 1$. In this paper, we are going to consider the generalized Benjamin-Bona-Mahony-Burgers equation (1) when $g(u) = e^{\pm u}$. Generally, these kind of nonlinear PDEs including exponential terms appear more in biological science. (See [11, 31]).

The objectives of this work are twofold. First, we describe the (G'/G)-expansion method. Second, we aim to implement the present method to obtain general exact travelling wave solutions of nonlinear BBM-B equation when $g(u) = e^{\pm u}$.

2. DESCRIPTION OF THE (G'/G)-EXPANSION METHOD

The objective of this section is to outline the use of the (G'/G)-expansion method for solving certain nonlinear partial differential equations (PDEs). Suppose we have a nonlinear PDE for $u(x,t)$, in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, ...) = 0, \tag{2}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives. The transformation $u(x,t) = U(\xi), \xi = kx + \omega t$, reduces Eq. (2) to the ordinary differential equation (ODE)

$$P(U, kU', \omega U', k^2U'', k\omega U'', ...) = 0, \tag{3}$$

where $U = U(\xi)$, and prime denotes derivative with respect to ξ. We assume that the solution of Eq. (3) can be expressed by a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{i=1}^{n} \alpha_i \Big(\frac{G'}{G}\Big)^i + \alpha_0, \quad \alpha_n \neq 0. \tag{4}$$

where α_0, and α_i, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$\frac{d^2G(\xi)}{d\xi^2} + \lambda \frac{dG(\xi)}{d\xi} + \mu G(\xi) = 0. \tag{5}$$

where λ and μ are arbitrary constants. Using the general solutions of Eq. (5), we have

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1 \sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+C_2 \cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{C_1 \cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+C_2 \sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right) - \frac{\lambda}{2},\ \lambda^2 - 4\mu > 0, \\ \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1 \sin(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+C_2 \cos(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{C_1 \cos(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+C_2 \sin(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right) - \frac{\lambda}{2},\ \lambda^2 - 4\mu < 0, \end{cases} \tag{6}$$

and it follows, from (4) and (6), that

$$U' = -\sum_{i=1}^{n} i\ \alpha_i \Big((\frac{G'}{G})^{i+1} + \lambda (\frac{G'}{G})^{i} + \mu (\frac{G'}{G})^{i-1} \Big), \tag{7}$$

$$U'' = \sum_{i=1}^{n} i\ \alpha_i \Big((i+1)(\frac{G'}{G})^{i+2} + (2i+1)\lambda (\frac{G'}{G})^{i+1} + i(\lambda^2+2\mu)(\frac{G'}{G})^{i} + (2i-1)\lambda\mu (\frac{G'}{G})^{i-1} + (i-1)\mu^2 (\frac{G'}{G})^{i-2} \Big), \tag{8}$$

and so on, here the prime denotes the derivative with respective to ξ.

To determine u explicitly, we take the following four steps:

Step 1. Determine the integer n by substituting Eq. (7) along with Eq. (6) into Eq. (3), and balancing the highest order nonlinear term(s) and the highest order partial derivative.

Step 2. Substitute Eq. (7) give the value of n determined in *Step 1*, along with Eq. (6) into Eq. (3) and collect all terms with the same order of (G'/G) together, the left-hand side of Eq. (3) is converted into a polynomial in (G'/G). Then set each coefficient of this polynomial to zero to derive a set of algebraic equations for k, ω, α_0 and α_i.

Step 3. Solve the system of algebraic equations obtained in *Step 2*, for $k, \omega, \lambda, \mu, \alpha_0$ and α_i by use of Maple.

Step 4. Use the results obtained in above steps to derive a series of fundamental solutions $u(\xi)$ of Eq. (3) depending on (G'/G), since the solutions of Eq. (5) have been well known for us, then we can obtain exact solutions of Eq. (2).

3. APPLICATION

In this section, we will demonstrate the (G'/G)-expansion method on the one special form of Benjamin–Bona–Mahoney–Burgers (BBM-B) equation (1), when $g(u) = e^{\pm u}$, then Eq. (1) reduce to

$$u_t - u_{xxt} - au_{xx} + bu_x \pm u_x e^{\pm u} = 0, \tag{9}$$

where $a, b \in \mathbb{R}$ and b is a non-zero constant.

Using the wave variable $\xi = kx + \omega t$, in Eq. (9), we find

$$(\omega + bk)U' - ak^2U'' - k^2\omega U''' \pm kU'e^{\pm U} = 0, \tag{10}$$

By the transformation $e^{\pm U} = V$, Eq. (10) becomes

$$\begin{aligned}&(\omega + bk)V^2V' - k^2\omega V^2V''' + 3k^2\omega VV'V'' - 2k^2\omega(V')^3\\&- ak^2V^2V'' + ak^2V(V')^2 \pm kV'V^3 = 0,\end{aligned} \tag{11}$$

According to *Step 1*, we get $3m + 3 = 4m + 1$, hence $m = 2$. We then suppose that Eq. (11) has the following formal solutions:

$$V = \alpha_2(\frac{G'}{G})^2 + \alpha_1(\frac{G'}{G}) + \alpha_0, \qquad \alpha_2 \neq 0, \tag{12}$$

where α_2, α_1, and α_0, are constants which are unknown to be determined later.

Substituting Eq. (12) along with Eq. (7) into Eq. (11) and collecting all terms with the same order of (G'/G) together, the left-hand sides of Eq. (11) are converted into a polynomial in (G'/G).

Setting each coefficient of each polynomial to zero, we can derive a set of algebraic equations for $k, \omega, \lambda, \mu, \alpha_0, \alpha_1$ and α_2, and solving by use of Maple, we get the following results:

$$\begin{aligned}&\alpha_0 = \mp\frac{a^2 + 2abk\lambda + b^2k^2\lambda^2}{2b},\\&\alpha_1 = \mp\,(2ak + 2bk^2\lambda),\\&\alpha_2 = \mp\,2k^2b,\\&\mu = \frac{b^2k^2\lambda^2 - a^2}{4k^2b^2},\\&\omega = -bk,\end{aligned} \tag{13}$$

where k, λ are nonzero free parameters. Therefore, substitute the set result (13) in

(12), we get

$$V = \mp 2k^2b(\frac{G'}{G})^2 \mp (2ak + 2bk^2\lambda)(\frac{G'}{G}) \mp \frac{a^2 + 2abk\lambda + b^2k^2\lambda^2}{2b}, \qquad (14)$$

Substituting the general solution of Eq. (4) into Eq. (14), and use the transformation $U(\xi) = \pm \ln V(\xi)$, the hyperbolic and trigonometric function solutions of Eq. (9), will be respectively:

$$\begin{aligned} u_1 = \pm \ln \Bigg(&\mp 2k^2b \Bigg(\frac{\sqrt{\frac{a^2}{k^2b^2}}\Big[C_1 \sinh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi) + C_2 \cosh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi)\Big]}{2\Big[C_2 \sinh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi) + C_1 \cosh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi)\Big]} - \frac{\lambda}{2}\Bigg)^2 \\ &\mp (2ak + 2bk^2\lambda)\Bigg(\frac{\sqrt{\frac{a^2}{k^2b^2}}\Big[C_1 \sinh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi) + C_2 \cosh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi)\Big]}{2\Big[C_2 \sinh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi) + C_1 \cosh(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ \xi)\Big]} - \frac{\lambda}{2}\Bigg) \\ &\mp \frac{a^2 + 2abk\lambda + b^2k^2\lambda^2}{2b}\Bigg), \end{aligned} \qquad (15)$$

and

$$\begin{aligned} u_2 = \pm \ln \Bigg(&\mp 2k^2b \Bigg(\frac{\sqrt{\frac{-a^2}{k^2b^2}}\Big[-C_1 \sin(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi) + C_2 \cos(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi)\Big]}{2\Big[C_2 \sin(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi) + C_1 \cos(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi)\Big]} - \frac{\lambda}{2}\Bigg)^2 \\ &\mp (2ak + 2bk^2\lambda)\Bigg(\frac{\sqrt{\frac{-a^2}{k^2b^2}}\Big[-C_1 \sin(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi) + C_2 \cos(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi)\Big]}{2\Big[C_2 \sin(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi) + C_1 \cos(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\ \xi)\Big]} - \frac{\lambda}{2}\Bigg) \\ &\mp \frac{a^2 + 2abk\lambda + b^2k^2\lambda^2}{2b}\Bigg), \end{aligned} \qquad (16)$$

where $\xi = kx - bkt$, $a, b \in \mathbb{R}$, , b is a non-zero constant, C_1, C_2, λ, and $k \neq 0$ are arbitrary constants.

Now, to obtain some special cases of the solutions obtained above, we set $C_2 = 0$, then hyperbolic and trigonometric function solutions (15)–(16) reduces

$$\begin{aligned} u_3(x,t) = \pm \ln \Bigg(&\mp \frac{a^2}{2b} \pm \frac{a^2}{2b}\mathrm{sech}^2\Big(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ (kx - bkt)\Big) \\ &\mp ak\sqrt{\frac{a^2}{k^2b^2}}\tanh\Big(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\ (kx - bkt)\Big)\Bigg), \end{aligned} \qquad (17)$$

$$u_4(x,t) = \pm\ln\Bigg(\mp\frac{a^2}{2b} \pm \frac{a^2}{2b}\sec^2\Big(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\,(kx-bkt)\Big) \pm ak\sqrt{\frac{-a^2}{k^2b^2}}\tan\Big(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\,(kx-bkt)\Big)\Bigg), \tag{18}$$

and when $C_1 = 0$, the general solutions (15)–(16) reduces

$$u_5(x,t) = \pm\ln\Bigg(\mp\frac{a^2}{2b} \mp \frac{a^2}{2b}\operatorname{csch}^2\Big(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\,(kx-bkt)\Big) \mp ak\sqrt{\frac{a^2}{k^2b^2}}\coth\Big(\frac{1}{2}\sqrt{\frac{a^2}{k^2b^2}}\,(kx-bkt)\Big)\Bigg), \tag{19}$$

$$u_6(x,t) = \pm\ln\Bigg(\frac{a^2}{2b} \pm \frac{a^2}{2b}\csc^2\Big(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\,(kx-bkt)\Big) + ak\sqrt{\frac{-a^2}{k^2b^2}}\cot\Big(\frac{1}{2}\sqrt{\frac{-a^2}{k^2b^2}}\,(kx-bkt)\Big)\Bigg), \tag{20}$$

where $a, b \in \mathbb{R}$, and b, k, are a arbitrary non-zero constant.

The author would like to note that the obtained general solutions (15)–(16) with an explicit linear function in ξ, and the special solutions (17)–(20), have been checked with Maple by putting them back into the original Eqs. (9), in both cases.

4. RESULTS AND DISCUSSION

Remark 1. Our first interest in the present work is introducing the (G'/G)-expansion method to stress its power in handling nonlinear equations.

Remark 2. The travelling wave solutions of a new integrable class of Benjamin–Bona–Mahony–Burgers equation(BBM-B) obtained using the (G'/G)–expansion method for the hyperbolic and trigonometric function types solutions are presented in (15)–(16), with an explicit linear function in ξ, and in (17)–(20), as special solutions.

Remark 3. On comparing between the (G'/G)-expansion method and the modified tanh–coth method [10], we first summarize the last method as follows: Suppose the solution of Eq. (3) in a finite series of function of the form:

$$u(\xi) = \alpha_0 + \sum_{\ell=1}^{m}\big\{\alpha_\ell Y^{\ell}(\xi) + \beta_\ell Y^{-\ell}(\xi)\big\}, \tag{21}$$

where $Y(\xi)$ satisfies the Riccati equation

$$\frac{dY(\xi)}{d\xi^2} + Y^2(\xi) + \lambda Y(\xi) + \mu = 0, \tag{22}$$

The parameter "m" can be determined by the homogeneous balance. Inserting (21) into (3) and using (22) results a set of algebraic equations which can be solved to determine $k, \omega, \alpha_\ell, \beta_\ell, \lambda, \mu$ for $\ell = 1, 2, ..., m$ where the general solution of (22) is well known. Having these parameters we obtain an analytic solution $u(x,t)$ in a closed form. If we put $\beta_\ell = 0$, and $Y(\xi) = \frac{G'(\xi)}{G(\xi)}$ in (21) and (22), then we get immediately (4) and (5), respectively. This shows that the (G'/G)–expansion method is more effective and convenient than the modified tanh-coth method for the following reasons:

(1) In the case of the (G'/G)–expansion method we use the solutions of the second order linear ODE (5) which are easier than the solutions of the Riccati Eq. (22).

(2) The exact solutions of Eq. (2) using the (G'/G)–expansion method contain more arbitrary constants compared to the exact solutions presented by the modified tanh-coth method.

Remark 4. Now to comparing between the (G'/G)-expansion method and the Exp-function method, we use the Exp-function method to deal with Eq. (5) and can obtain the following two solutions:

$$G(\xi) = \frac{A_0}{B_0} e^{\frac{-\lambda + \sqrt{\lambda^2 - 4\mu}}{2}\xi}, \tag{23}$$

$$G(\xi) = \frac{A_0}{B_0} e^{\frac{-\lambda - \sqrt{\lambda^2 - 4\mu}}{2}\xi}, \tag{24}$$

where A_0 and B_0 are free parameters. Because Eq. (5) is a linear equation, the linear combination of solutions (23) and (24) is its general solution:

$$G(\xi) = \widetilde{C}_1 \frac{A_0}{B_0} e^{\frac{-\lambda + \sqrt{\lambda^2 - 4\mu}}{2}\xi} + \widetilde{C}_2 \frac{A_0}{B_0} e^{\frac{-\lambda - \sqrt{\lambda^2 - 4\mu}}{2}\xi}, \tag{25}$$

where $\widetilde{C}_1$ and $\widetilde{C}_2$ are arbitrary constants. We therefore have

$$\frac{G'}{G} = \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{\widetilde{C}_1 e^{\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi} - \widetilde{C}_2 e^{\frac{-\sqrt{\lambda^2 - 4\mu}}{2}\xi}}{\widetilde{C}_1 e^{\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi} + \widetilde{C}_2 e^{\frac{-\sqrt{\lambda^2 - 4\mu}}{2}\xi}} \right) - \frac{\lambda}{2}, \tag{26}$$

further setting $\widetilde{C}_1 = C_1 + C_2$ and $\widetilde{C}_2 = C_1 - C_2$, and letting $\lambda^2 - 4\mu > 0$, we obtain

$$\frac{G'}{G} = \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)} \right) - \frac{\lambda}{2}, \tag{27}$$

where C_1 and C_2 are arbitrary constants. If set $\widetilde{C}_1 = C_1 - iC_2$ and $\widetilde{C}_2 = C_1 + iC_2$, here $i \equiv \sqrt{-1}$ and let $\lambda^2 - 4\mu < 0$, we also obtain

$$\frac{G'}{G} = \frac{\sqrt{4\mu - \lambda^2}}{2}\left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,\xi\right) + C_2 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,\xi\right)}{C_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,\xi\right) + C_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,\xi\right)}\right) - \frac{\lambda}{2}, \tag{28}$$

Employing Eq. (5) and solutions (27) and (28), and using the Exp-function method, we can obtain solutions same as the solutions of (G'/G)–expansion method.

5. CONCLUSIONS

This study shows that the (G'/G)–expansion method is quite efficient and practically well suited for use in finding exact solutions for a new integrable class of Benjamin–Bona–Mahony–Burgers equation (BBM-B). The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. With the aid of Maple, we have assured the correctness of the obtained solutions by putting them back into the original equation. We hope that they will be useful for further studies in applied sciences and engineering.

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