

## PHASE SPACE CALCULATIONS FOR $\beta^-\beta^-$ DECAYS TO FINAL EXCITED $2_1^+$ STATES

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*Abstract.* We report new values of the phase spaces factors (PSFs) for  $2\nu$ - and  $0\nu$ - $\beta^-\beta^-$  decays calculated for transitions to final excited  $2_1^+$  states, using exact Dirac electron wave functions (w.f) with inclusion of finite nuclear size (FNS) and screening effects, and using a Coulomb potential derived from a realistic proton density distribution in the daughter nucleus. We found important differences when comparing our results with other previous ones from literature, which are obtained with approximative electron wave functions.

*Key words:* double beta decay, phase space factors, electron wave functions.

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### 1. INTRODUCTION

To predict lifetimes for the double beta decay (DBD) process and extract upper limits for the neutrino parameters, the precise knowledge of the nuclear matrix elements (NMEs) and phase space factors is required [1]- [5]. Until recently, the NMEs were considered the main factor of uncertainty, and this issue has been, and still is, widely debated in literature [6]- [19]. By comparison, the PSFs have been less discussed, since they were considered to be computed with good precision [20]- [26]. However, recently, they were recalculated with an improved approach, by using exact electron Dirac wave functions (w.f.) and taking into account the finite nuclear size (FNS) and screening effects [27], and some differences are found as compared to previously calculations performed with approximative electron w.f., especially for heavier nuclei. Independently, PSFs were also calculated in Ref. [28], within a similar approximation, but using a Coulomb potential derived from a realistic proton distribution in the daughter nuclei. In Refs. [27] and [28] the computation of the PSFs refer to the  $2\nu$ - and  $0\nu$ - $\beta^-\beta^-$  decays for transitions to final g.s. and excited  $0_1^+$

states, and was performed for twelve nuclei of experimental interest. The results obtained in these references are in good agreement, with a few exceptions (for example  $^{238}\text{U}$ ), where recalculations are required. However, the calculations do not include  $\beta^- \beta^-$  transitions to excited  $2_1^+$  states which are of interest, as well, since they are relevant for the study of the DBD mechanisms and the existence of right handed (RH) currents in the weak interaction. Particularly, these transitions can be investigated in experiments where the spectra of the individual electrons can be recorded, such as NEMO and SNEMO [29]. In this work, we compute PSFs associated with  $\beta^- \beta^-$  transitions to final excited  $2_1^+$  states. We use exact Dirac electron w. f. with inclusion of FNS and screening effects, and a Coulomb potential derived from a realistic proton density distribution in the daughter nucleus. We compare our results with other previous ones from literature, which are obtained within a less rigorous formalism, and discuss the differences.

## 2. FORMALISM

We shortly present the main ingredients of our formalism described in more detail in [28]. The electron w.f. are expressed as a superposition of  $s$  and  $p$  Coulomb distorted spherical waves, whose radial parts are obtained as solutions of the Dirac equations with a central field [22]- [25], [27]:

$$\frac{dg_\kappa(\epsilon, r)}{dr} = -\frac{\kappa}{r}g_\kappa(\epsilon, r) + \frac{\epsilon - V + m_e c^2}{c\hbar}f_\kappa(\epsilon, r) \quad (1)$$

$$\frac{df_\kappa(\epsilon, r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar}g_\kappa(\epsilon, r) + \frac{\kappa}{r}f_\kappa(\epsilon, r) \quad (2)$$

that depends on the relativistic quantum number  $\kappa = (l - j)(2j + 1)$ . The quantities  $g_\kappa(\epsilon, r)$  and  $f_\kappa(\epsilon, r)$  are the small and large components of the solutions, which have a specific asymptotic behavior [25], [27]: Here,  $c$  is the velocity of the light,  $m_e/\epsilon$  are the electron mass/energy and  $V$  is the Coulomb potential between the electron and the daughter nucleus. The nuclear size corrections are usually taken into account by considering an unscreened potential  $V(r)$  obtained for a uniform charge distribution in a sphere of radius equal to the nuclear radius [22], [27]. The screening effect can be introduced by multiplying  $V(r)$  with a function  $\phi(r)$ , which is the solution of the Thomas Fermi equation:  $d^2\phi/dx^2 = \phi^{3/2}/\sqrt{x}$ , with  $x = r/b$ ,  $b \approx 0.8853a_0Z^{-1/3}$  and  $a_0 = \text{Bohr radius}$ . It is calculated within the Majorana method [30]. This approach described above was used by the authors from Ref. [27] to calculate the PSFs.

In this work, we take into account the influence of the nuclear structure by deriving the potential  $V(r)$  from a realistic proton density distribution in the daughter

nucleus.

$$V(r) = \alpha \hbar c \int \frac{\rho_e(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}', \quad \text{with} \quad \rho_e(\vec{r}) = 2 \sum_i v_i^2 |\Psi_i(\vec{r})|^2 \quad (3)$$

$\Psi_i$  is the proton (WS) w. f. of the single particle state  $i$  and  $v_i$  is its occupation amplitude. The factor 2 reflects the time reversal degeneracy. The screening effect is taken into account in the same manner as in Ref. [27].

To compute the PSF, the electron phase factors  $f_{jk}^{(0)}$  must be obtained from the solutions of the Dirac equation by neglecting the neutrino mass:

$$f_{11}^{(0)} = |f^{-1-1}|^2 + |f_{11}|^2 + |f_{-1}^{-1}|^2 + |f_1^{-1}|^2 \quad (4)$$

with

$$f^{-1-1} = g_{-1}(\epsilon_1)g_{-1}(\epsilon_2); \quad f_{11} = f_1(\epsilon_1)f_1(\epsilon_2), \quad (5)$$

$$f_{-1}^{-1} = g_{-1}(\epsilon_1)f_1(\epsilon_2); \quad f_1^{-1} = f_1(\epsilon_1)g_1(\epsilon_2) \quad (6)$$

The values of the  $f$  and  $g$  functions are approximated with the solutions on the surface (the method I from [27]).

$$g_{-1}(\epsilon) = g_{-1}(\epsilon, R); \quad f_1(\epsilon) = f_1(\epsilon, R) \quad (7)$$

For the two neutrino  $\beta^- \beta^-$  decay, the PSF are:

$$G_{2\nu} = \frac{2\tilde{A}^2}{3 \ln 2 g_A^4 (m_e c^2)^2} \int_{m_e c^2}^{T_0 - m_e c^2} \int_{m_e c^2}^{T_0 - \epsilon_1} \int_0^{T_0 - \epsilon_1 - \epsilon_2} \times d\epsilon_1 d\epsilon_2 d\omega_1 f_{11}^{(0)} w_{2\nu} (\langle K_N \rangle^2 + \langle L_N \rangle^2 + \langle K_N \rangle \langle L_N \rangle) \quad (8)$$

where  $T_0 = Q_{\beta\beta}^2 + 2m_e c^2$  is the total energy released in the decay and  $\langle K_N \rangle$ ,  $\langle L_N \rangle$  are expressions (known in the theory of DBD) that depend on the electron and neutrino ( $\omega_{1,2}$ ) energies, and on the g.s. energies of the initial nucleus and excited states of the intermediate nucleus [21]- [27].  $\tilde{A} = 1.12A^{1/2}$  (in MeV) gives the energy of the giant Gamow-Teller resonance in the intermediate nucleus. The PSF are renormalized to the electron rest energy and are reported in [ $yr^{-1}$ ].

For the  $0\nu\beta\beta$  decay, the PSFs are

$$G_{0\nu} = \frac{2}{4g_A^4 R^2 \ln 2} \int_{m_e c^2}^{T_0 - m_e c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1 \quad (9)$$

### 3. RESULTS

The single particle densities inside the daughter nucleus, needed to derive the potential  $V(r)$ , are obtained by solving the Schrödinger equation for a spherical WS potential, including spin-orbit and Coulomb terms. The model was widely used in

the investigations of fission [31], cluster decay [32] and heavy elements synthesis [33]. The occupation amplitudes are obtained within the BCS approach. Further, the Dirac equation is solved for the electron moving in the potential  $V(r)$ , created by the proton distribution, by using the power series method from Ref. [34]. We built up a numerical code that uses an algorithm similar to that described in Ref. [35]. The asymptotic normalization to unity is done as in Ref. [36]. The calculations are performed in a similar manner as that described in [28]. Because the w. f. values at the nuclear surface vary rapidly for energies close to  $m_e c^2$ , we took additional mesh points in the vicinity of this region for improving the numerical accuracy. Our results are presented in Tables I for twelve nuclei of experimental interest.

In the first (upper) part of the table we display the PSF values,  $G_{2\nu}$ , for the  $2\nu\beta\beta$  decay mode, for the transitions to excited  $2_1^+$  states. For comparison, similar results are also shown, indicating the references where they are taken from. The maximum available kinetic energies  $Q^2\beta\beta$  are given as well. In the last rows the relative differences in percentages between other results and ours are showed for comparison. The similar results,  $G_{0\nu}$ , for the  $0\nu\beta\beta$  decay mode are presented in the second (lower) part of the table.

We remark important differences between our results and the results from [22] and [26], especially for the  $G_{2\nu}$  values. The differences may come from two sources: i) the use of a more rigorous approach, as we did, in comparison to the use of approximative electron w.f., and ii) the accuracy of the method of numerical integration employed. While, errors up to, say, 70% may be accepted to come mainly from the use of an improved approach, the larger differences may be associated as well with the numerical treatment of the integrals. This is supported by noting that  $G_{2\nu}$  are obtained by performing three integrals, while for  $G_{0\nu}$  one needs to manage only one integral. These integrals, as we already mentioned are sensitive for the energy values close to  $m_e c^2$ , and the largest differences appear for the nuclei where the upper limits of the integrals are closer to this value.

In conclusion, we computed the PSFs  $2\nu$ - and  $0\nu$ - $\beta^-\beta^-$  decays for transitions to final excited  $2_1^+$  states using an improved approach, as compared to the approaches employed until present in literature. The novelty of our approach is the use of a Coulomb potential derived from a realistic proton density distribution in the daughter nucleus. It is for the first time when these PSFs are re-calculated with a more rigorous approach, since 1998. We found important differences between our results and similar ones from literature, whose understanding needs further studies.

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Table 1

Values of the PSF  $G_{2\nu}[yr^{-1}]$  for transitions to excited  $2_1^+$  state. The relative differences in percentage  $\varepsilon[\%]$  between other previous calculations (indicated by references) and our results are displayed in the last rows. The  $Q_{\beta\beta}^{(2)}$  are the kinetic energies available in the corresponding decays.

| Nucleus                | $G_{2\nu}[yr^{-1}]$ |                   |                   |                   |                   |                   |
|------------------------|---------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                        | <sup>48</sup> Ca    | <sup>76</sup> Ge  | <sup>82</sup> Se  | <sup>96</sup> Zr  | <sup>100</sup> Mo | <sup>110</sup> Pd |
| $Q_{\beta\beta}^{(2)}$ | 3.289               | 1.480             | 2.218             | 2.572             | 2.494             | 1.360             |
| [22]                   | 4.41E-18            | 4.83E-22          | 9.06E-20          |                   |                   |                   |
| [26]                   | 4.4E-18             | 4.9E-22           | 8.5E-20           | 8.5E-19           | 6.9E-19           |                   |
| Present work           | 1.33E-18            | 2.98E-21          | 1.08E-19          | 5.74E-19          | 5.61E-19          | 4.72E-21          |
| $\varepsilon$ [22]     | 69                  | >100              | -19               |                   |                   |                   |
| $\varepsilon$ [26]     | 69                  | >100              | -27               | 32                | 19                |                   |
| Nucleus                | <sup>116</sup> Cd   | <sup>128</sup> Te | <sup>130</sup> Te | <sup>136</sup> Xe | <sup>150</sup> Nd | <sup>238</sup> U  |
| $Q_{\beta\beta}^{(2)}$ | 1.521               | 0.425             | 1.989             | 1.640             | 3.037             | 1.100             |
| [22]                   |                     | 1.36E-27          | 1.16E-19          |                   | 4.56E-17          |                   |
| [26]                   | 2.3E-21             | 1.3E-27           | 1.2E-19           | 1.5E-20           | 4.9E-17           |                   |
| Present work           | 1.33E-20            | 5.49E-25          | 1.78E-19          | 4.05E-20          | 1.25E-17          | 7.10E-20          |
| $\varepsilon$ [22]     |                     | >100              | -53               |                   | 72                |                   |
| $\varepsilon$ [26]     | >100                | >100              | -48               | >100              | 74                |                   |
| Nucleus                | $G_{0\nu}[yr^{-1}]$ |                   |                   |                   |                   |                   |
|                        | <sup>48</sup> Ca    | <sup>76</sup> Ge  | <sup>82</sup> Se  | <sup>96</sup> Zr  | <sup>100</sup> Mo | <sup>110</sup> Pd |
| [22]                   | 6.04E-14            | 1.84E-15          | 1.38E-14          |                   |                   |                   |
| Present work           | 8.76E-15            | 8.16E-16          | 3.44E-15          | 7.78E-15          | 7.91E-15          | 1.41E-15          |
| $\varepsilon$ [22]     | 85                  | 56                | 75                |                   |                   |                   |
| [22]                   |                     | 6.68E-17          | 2.28E-14          |                   | 3.01E-13          |                   |
| Present work           | 2.17E-15            | 9.87E-17          | 6.40E-15          | 3.90E-15          | 4.34E-14          | 2.94E-14          |
| $\varepsilon$ [22]     |                     | -48               | 72                |                   | 44                |                   |
| Nucleus                | <sup>116</sup> Cd   | <sup>128</sup> Te | <sup>130</sup> Te | <sup>136</sup> Xe | <sup>150</sup> Nd | <sup>238</sup> U  |
| [22]                   |                     | 6.68E-17          | 2.28E-14          |                   | 3.01E-13          |                   |
| Present work           | 2.17E-15            | 9.87E-17          | 6.40E-15          | 3.90E-15          | 4.34E-14          | 2.94E-14          |
| $\varepsilon$ [22]     |                     | -48               | 72                |                   | 44                |                   |

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