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PLASMA PHYSICS

ANALYSIS OF GRAVITO-MHD WAVES IN A STRATIFIED ISOTHERMAL PLASMA WITH CONSTANT ALFVÉN SPEED

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Abstract. Standard set of magneto-hydrodynamic equations (MHD) was used to derive dispersion equation for gravito-MHD waves. Properties of these waves were analysed in the limits of low-β (like solar photosphere) and high-β (like solar corona) plasmas. The results are represented analytically and graphically.

Key words: magnetohydrodynamics, gravito-MHD waves.

1. INTRODUCTION

There are three restoring forces that act on gas in the solar atmosphere: gas pressure, gravitation and magnetism. If a gas in stable equilibrium is disturbed it will oscillate. When only one of the above forces acts, the resulting characteristic oscillations, or modes, are known as sound, internal gravity (in this paper only gravity waves for brevity), or Alfvén waves, respectively. Although the physics of these ”pure” modes is quite simple, the properties of the oscillations that occur when two or three forces act simultaneously upon the gas have a very complicated dependence on frequency and wavelength. Analogy with the simple, single-force modes [1], when it is possible, is the easiest way to understand the complex multiforce wave modes.
Magnetic field embedded in astrophysical plasma environment allows various types of wave modes to propagate and eventually contribute to a complex dynamics of media such as stellar atmospheres, planetary magnetosphere etc. The large values of the viscous and magnetic Reynold's numbers in the solar atmosphere imply that the dissipative terms in the MHD equations are unimportant and MHD waves are accurately described by the equations of ideal MHD. These waves are: Alfvén wave, fast and slow magneto-acoustic wave. The presence of gravitational field introduces the additional non-magnetic gravity force $\vec{f} = \rho\vec{g}$ into MHD equations. In this paper we will assume a uniform, initially prescribed, gravitational field with constant acceleration $\vec{g}$. This means that the considered plasma does not affect the gravitational field in the same way as stellar atmospheres and planetary plasma spheres for example. Some

other media, like stellar interiors and large interstellar plasma clouds would require a self-consistent approach with the gravitational field resulting from the density distribution $\rho(\vec{r},t)$ and gravitational potential $\phi_g(\vec{r},t)$ computed by the Poisson equation $\nabla^2\phi_g = 4\pi G\rho$. Such self-gravitating plasmas with $\vec{g} = -\nabla\phi_g$ fall off the scope of this paper.
We will consider the gravito-MHD waves propagation in the model with constant sound and Alfvén speed, *i.e.* $v_s, v_A = const$ in the basic state, which provide that the density ρ_0 and horizontal magnetic field intensity B_0 are functions of the variable z taken in the vertical direction: and $\rho_0 = \rho_0(z)$ and $\vec{B_0} = B_0(z)\vec{e_x}$. Beneath the solar surface magnetic field can be described by confined, vertical thin flux tubes. When these flux tubes break through the photosphere, it is observed that the magnetic field lines incline in most cases from the vertical direction. They fan out and create a local magnetic canopy, *i.e.* structures with horizontal magnetic field, through the chromosphere. Lites *et al.* [2], presented observations of quiet regions near the centre of the solar disk using the Advanced Stokes Polarimeter. Horizontal (canopy) magnetic field with uniform Alfvén speed was introduced into the chromosphere of a simple planar solar-like model by Campbell and Roberts, [3]. A simple planar three-layers model, including non-magnetic interior and constant β chromosphere and corona was analysed by Pinter and Goossens, [4]. This type of the basic state also allows full analytical treatment and has been used by many authors for different purposes: Yu [5], Gilman [6], Pinter *et al.* [7], Pinter, Erdelyi and New [8], Pinter, Erdelyi and Goossens [9], Newington and Cally [10], among others.

2. MHD EQUATIONS

We start from the standard set of MHD equations describing the dynamics of a fully ionized plasma with properties of the ideal gas given by
- the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0,$$

- momentum equation

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho\vec{v}\cdot\nabla\vec{v} = -\nabla p + \rho\vec{g} + \frac{1}{\mu_0}\left(\nabla\times\vec{B}\right)\times\vec{B},$$

- magnetic induction equation

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}), \tag{1}$$

- Gauss law

$$\nabla\cdot\vec{B} = 0,$$

and adiabatic law for the ideal gas

$$\frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p = \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho\right).$$

3. THE BASIC STATE

The initial basic state is defined by the model itself which implies a stationary, static (*i.e.* the hydrostatic equilibrium), gravitationally stratified isothermal plasma with embedded horizontal magnetic field. The equilibrium is assumed to depend only on the z direction. In Cartesian coordinates, we have:

$$\vec{g} = -g\vec{e_z}, \quad g = \text{const},$$
$$\vec{B_0} = B_0(z)\vec{e_x} \tag{2}$$

and

$$\rho_0 = \rho_0(z), \quad p_0 = p_0(z).$$

A fully ionized plasma in magnetohydrostatic equilibrium satisfies equation:

$$\frac{d}{dz}\left(p_0(z) + \frac{B_0^2(z)}{2\mu_0}\right) + \rho_0(z)g = 0 \tag{3}$$

or

$$\frac{d}{dz}\left(\frac{v_s^2}{\gamma} + \frac{v_A^2}{2}\right) + \left(\frac{v_s^2}{\gamma} + \frac{v_A^2}{2}\right)\frac{d}{dz}\log\rho_0(z) + g = 0. \tag{4}$$

Here, $\gamma = \frac{c_p}{c_v} = \frac{5}{3}$ is the ratio of specific heats, $v_s^2 = \gamma\frac{p_0(z)}{\rho_0(z)} = \gamma RT_0$ and $v_A^2 = \frac{B_0^2(z)}{\mu_0\rho_0(z)}$ are squares of the sound and Alfvén speeds respectively. In this model both v_s and v_A are assumed constant. Therefore Eq.(4) for magnetohydrostatic equilibrium could be rewritten in the form:

$$\frac{d}{dz}\log\rho_0(z) + \frac{1}{H} = 0, \tag{5}$$

with density scale length

$$H = \frac{v_s^2}{\gamma g} + \frac{v_A^2}{2g} = \frac{1+\beta}{\beta}H_0,$$

where $H_0 = \frac{v_s^2}{\gamma g} = \text{const}$ is the isothermal density scale length in a non-magnetized atmosphere and β is the ratio of thermal to magnetic plasma pressure: $\beta = \frac{p_0(z)}{p_{0m}(z)} = \frac{2v_s^2}{\gamma v_A^2} = \text{const}$.

The solutions for density and magnetic field profiles that follow from Eq.(5) and Alfvén speed definition are obtained as:

$$\rho_0(z) = \rho_0(0)e^{-z/H}, \quad B_0(z) = B_0(0)e^{-z/2H}. \tag{6}$$

Plasma density, thermal plasma pressure $p_0(z)$ and magnetic plasma pressure $p_{0m}(z) = B_0^2/(2\mu_0)$ decrease exponentially in the vertical direction with the same constant H, while magnetic field strength $B_0(z)$ decreases with $2H$:

$$B_0(z) = B_0(0)e^{-z/2H}. \tag{7}$$

Here, $B_0(0) = v_A\sqrt{\mu_0\rho_0(0)}$.

4. LINEARISED MHD EQUATIONS

Plasma dynamics of small amplitude waves is described by standard set of non-linear MHD equations for ideal plasma (1), which are perturbed by taking any unknown physical quantity $f(x,y,z,t)$ as a sum:

$$f(x,y,z,t) = f_0(z) + \nabla f(x,y,z,t). \tag{8}$$

Perturbations $\nabla f(x,y,z,t)$ have the form:

$$\nabla f(x,y,z,t) = f_1(z)e^{(-i\omega t + ik_x x + ik_y y)},$$

while unperturbed quantities $f_0(z)$ satisfy the magnetohydrostatic balance equation Eq.(5) of the basic state.

Eqs.(1) linearised according to Eq.(8) can be reduced to a system of two coupled ordinary differential equations:

$$\frac{d\xi_{1z}}{dz} = \frac{C_1}{D}\xi_{1z} - \frac{C_2}{D}P_1, \quad \frac{dP_1}{dz} - g\frac{d\rho_0(z)}{dz}\xi_{1z} = C_3\xi_{1z} - C_4 P_1, \tag{9}$$

where $\xi_{1z} = iv_{1z}/\omega$ is the z-component (*i.e.* vertical component) of the fluid displacement and $P_1 = p_1 + p_{1m}$ is the total pressure perturbation made of the perturbed plasma pressure p_1 and the perturbed magnetic pressure $p_{1m} = B_0 B_1/\mu_0$, where B_1 is perturbed magnetic field. The coefficients in Eqs.(9) are:

$$\begin{gathered} D(z) = \rho_0(z)(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2), \\ C_1(z) = \rho_0(z)g\omega^2(\omega^2 - \omega_A^2), \\ C_2(z) = (\omega^2 - \omega_A^2)(\omega^2 - \omega_s^2) - k_y^2(v_s^2 + v_A^2)(\omega^2 - \omega_c^2), \\ C_3(z) = \rho_0(z)(\omega^2 - \omega_A^2) + \frac{\rho_0(z)g^2(\omega^2 - \omega_A^2)}{(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)}, \\ C_4(z) = \frac{g\omega^2}{(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)}. \end{gathered} \tag{10}$$

The density distribution is given by Eq.(6) and characteristic frequencies are:

$$\omega_A^2 = k_x^2 v_A^2, \ \omega_s^2 = k_x^2 v_s^2, \ \omega_c^2 = k_x^2 v_c^2,$$

where $v_c^2 = v_s^2 v_A^2/(v_s^2 + v_A^2)$ is cusp speed. Taking into account Eq.(6), the equations Eqs.(9)-(10) allow the following solutions for the fluid displacement ξ_{1z} and the total pressure perturbation P_1:

$$\xi_{1z}(z) = \xi_{1z}(0)e^{z/2H}e^{ik_z z},$$
$$P_1(z) = P_1(0)e^{-z/2H}e^{ik_z z}. \tag{11}$$

Finally, Eqs.(9) with solutions in Eqs.(11) yield the dispersion equation for gravito-MHD waves:

$$k_z^2 = \frac{\omega^2(\omega^2 - \omega_A^2) - \frac{g}{H}\omega_A^2}{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2)}\omega^2 - \frac{1}{4H^2}$$
$$-\frac{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2) + g^2(\omega^2 - \omega_A^2) - \frac{g}{H}(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)}{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2)}k_p^2, \tag{12}$$

where $k_p^2 = k_x^2 + k_y^2$ is the horizontal component of the wave vector $\vec{k}$. We can rewrite dispersion equation Eq.(12) in the dimensionless form by using dimensionless physical quantities: $K_p = k_p H$, $K_z = k_z H$, where K_p and K_z are dimensionless horizontal and vertical wave numbers scaled to $1/H$. Dimensionless frequencies scaled to v_s/H are: $\Omega = \omega H/v_s$, $\Omega_A = \omega_A H/v_s$, $\Omega_s = \omega_s H/v_s$ and $\Omega_c = \omega_c H/v_s$. Dimensionless dispersion equation for gravito-MHD waves is:

$$K_z^2 = \frac{\Omega^2(\Omega^2 - \Omega_A^2) - \frac{(1+\beta)}{\gamma\beta}\Omega_A^2}{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2)}\Omega^2 - \frac{1}{4}$$
$$-\frac{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2) + \frac{(1+\beta)^2}{\gamma^2\beta^2}(\Omega^2 - \Omega_A^2) - \frac{(1+\beta)}{\gamma\beta}\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_c^2)}{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2)}K_p^2. \tag{13}$$

Inspection of this equation shows that it is cubic in Ω^2, implying the existence of three distinct MHD waves in the considered model of a stratified atmosphere in magnetic field. Gravitational effects promote no new modes but they modify the dispersion properties of the modes existing in homogeneous magnetized plasmas. The restoring force acting upon perturbations is a combination of gravitational and magnetic forces. Proportional efficiency of these forces depends on the wave parameters and on the magnetic field strength. The dispersion properties depend qualitatively on the parameter β, measuring the relative contributions of hydrodynamic and magnetic effects. In the case without magnetic field, when $1/\beta = 0$ (or $H = H_0$) and $\Omega_A = \Omega_c = 0$, dispersion equation (13) becomes:

$$K_z^2 = \Omega^2 - \frac{1}{4} - \frac{K_p^2}{\Omega^2}\left(\Omega^2 - \frac{(\gamma - 1)}{\gamma^2}\right),$$

or in the terms of dimensionless characteristic frequencies- acoustic cut-off frequency $\Omega_{co} = \frac{\omega_{co} H_0}{v_s} = \frac{1}{2}$ and Brunt-Väisäläa frequency $\Omega_{BV} = \frac{\omega_{BV} H_0}{v_s} = \sqrt{\frac{(\gamma-1)}{\gamma^2}}$:

$$K_z^2 = \Omega^2 - \Omega_{co}^2 - \frac{K_p^2}{\Omega^2}(\Omega^2 - \Omega_{BV}^2). \tag{14}$$

This equation describes gravito-acoustic waves in an isothermal stratified atmosphere as in the case of the quiet Sun. For a high-β plasmas, $\beta \gg 1$, when the magnetic pressure terms can be neglected and the plasma pressure is dominant as in the solar interior, dispersion equation Eq.(13) could be developed in order by a small parameter $1/\beta$ to get the first order correction of the Eq.(14):

$$K_z^2 = \Omega^2 - \Omega_{co}^2 - \frac{K_p^2}{\Omega^2}(\Omega^2 - \Omega_{BV}^2) - \frac{2}{\gamma\beta}\left[\Omega^2 - \frac{K_p^2}{\Omega^2}\left(\frac{1}{2} - \Omega_{BV}^2\right)\right]. \tag{15}$$

In Fig.1 and Fig.2 are presented corrections for the gravity waves which propagate in the frequency range $0 < \Omega^2 < \Omega_{BV}^2 = \frac{(\gamma-1)}{\gamma^2} = 0.24$ and for the acoustic waves (modified by gravity) with propagation in the frequency range $\Omega^2 > \Omega_{co}^2 = 0.25$. We were separated this two frequency regions because we were interested in propagating waves only. Waves in the frequency range $\Omega_{BV}^2 < \Omega^2 < \Omega_{co}^2$ are evanescent.

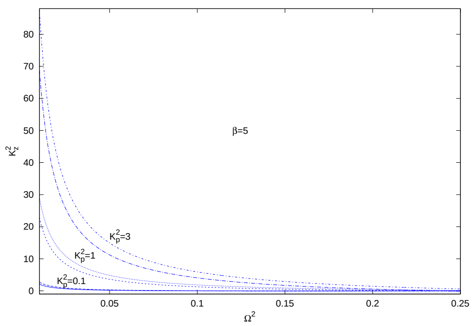

Fig. 1 – Dimensionless vertical wave number K_z^2 of gravity waves as a function of dimensionless frequency $\Omega^2 < \Omega_{BV}^2 = 0.24$ for a given K_p^2 and parameter $\beta = 5$. We compared three pairs of curves for a given K_p^2, where the first one curves are for gravity waves without magnetic field influence (Eq.14) and the second one curves are for gravity waves with small magnetic field influence (Eq.15).

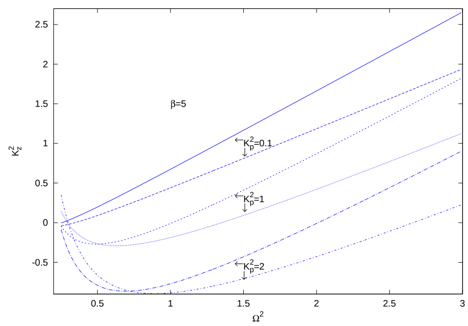

Fig. 2 – Dimensionless vertical wave number K_z^2 of acoustic waves as a function of dimensionless frequency $\Omega^2 > \Omega_{co}^2 = 0.25$ for a given K_p^2 and parameter $\beta = 5$. For a chosen K_p^2 values, there are three pairs of curves-first ones are for acoustic waves without magnetic field influence (Eq.14) and the second ones are for acoustic waves with small magnetic field influence (Eq.15).

Fig. 1 shows dimensionless vertical wave number K_z^2 of a gravity waves as a function of dimensionless frequency Ω^2 for a given K_p^2 and parameter $\beta = 5$. For

$K_p^2 < 0.22^*$, there are characteristic frequencies $\Omega^2 = K_p\sqrt{1/2-\Omega_{BV}^2}$ for which the correction term in Eq.(15) is equal to zero. In these points magnetic influence on the gravity waves propagation vanishes. For the frequencies smaller than this characteristic frequency, corrected K_z^2 in Eq.(15) is larger than K_z^2 of the gravity waves in Eq.(14). For the frequencies higher than characteristic frequency for a given K_p^2, there is an opposite situation.

If $K_p^2 > 0.22$, K_z^2 in Eq.(15) is always higher than K_z^2 in Eq.(14) for gravity waves. Magnetic influence increases vertical wave number of gravity waves without any restrictions. Note that $K_p^2 > 0.22$ in dimensional form means:

$$k_p^2 > \frac{0.22}{H^2} \approx \frac{0.22}{H_0^2}.$$

For the solar photosphere, where $v_s^2 = 1.6\times10^8\mathrm{m}^2/\mathrm{s}^2$ and $g = 273$ m/s^2, we have: $k_p^2 = 177.9\times10^{-14}\mathrm{m}^{-2}$ or $k_p = 13.34\times10^{-7}\mathrm{m}^{-1}$.

Fig.2 for acoustic branch of gravito-acoustic waves in a high-β plasma for the frequency range $\Omega^2 > \Omega_{co}^2$, also presents dimensionless vertical wave number K_z^2 as a function of dimensionless frequency Ω^2 for a given K_p^2 and parameter $\beta = 5$. Here, for $K_p^2 > 0.24$,[†] the curves for K_z^2 given by Eqs.(14)-(15) intersect in the characteristic frequencies $\Omega^2 = K_p\sqrt{1/2-\Omega_{BV}^2}$. In these points there are no magnetic field influence at all. Note that K_z^2 values for this characteristic frequencies become negative and acoustic waves are evanescent. Furthermore, for a given dimensionless horizontal wave number K_p^2 and for frequencies $\Omega_{co}^2 < \ Omega^2 < K_p^2$, the acoustic waves are evanescent. There are propagating acoustic waves only if $K_z^2 > 0$, *i.e.* for $\Omega^2 > K_p^2$ (and $\Omega^2 > \Omega_{co}^2$). Influence of the small magnetic field could be analysed for the values $K_p^2 > 0.24$ and $K_p^2 < 0.24$.

In the case when $K_p^2 > 0.24$, there is a characteristic frequency which separates the frequency region with increasing influence of a small magnetic field on acoustic waves propagation (frequencies smaller than $\Omega^2 = K_p\sqrt{1/2-\Omega_{BV}^2}$) and the frequency region with decreasing a small magnetic field influence on acoustic waves propagation (frequencies higher than $\Omega^2 = K_p\sqrt{1/2-\Omega_{BV}^2}$).

Increasing influence of a small magnetic field is, although low, the most pronounced for the frequencies near acoustic cut-off frequency $\Omega_{co}^2 = 0.25$, or in photospheric case for example, $\omega_{co}^2 \approx 3.2\times10^{-4}\mathrm{s}^{-2}$. This influence can make the acoustic waves, evanescent in these frequencies without magnetic field, to propagate.

Decreasing magnetic field influence on the acoustic waves with $K_p^2 > 0.24$ increases with increasing frequencies. Their dimensionless vertical wave number

*Maximal frequency of gravity waves is Ω_{BV}, so: $K_p^2 = \Omega_{BV}^4/(1/2-\Omega_{BV}^2) = 0.22$

†$K_p^2 = \Omega_{co}^4/(1/2-\Omega_{BV}^2) = 0.24$

K_z^2 in Eq.(14) is significantly reduced by the small magnetic field.

Acoustic waves with $K_p^2 < 0.24$ have no any characteristic frequency and a small magnetic field reduces their K_z^2 values. Note that for the case of the solar photosphere there is dimensional form of horizontal wave number (equivalent with K_p^2): $k_p^2 < 194.1 \times 10^{-14}\mathrm{m}^{-2}$ or $k_p < 13.93 \times 10^{-7}\mathrm{m}^{-1}$.

In the limit of large wave numbers $k \gg 1/H$, *i.e.* short wavelengths ($\lambda \ll 2\pi H$), gravitational effects on gravito-MHD waves are negligible and the waves propagate as in a locally uniform magnetized plasma with no density stratification. Dispersion equation (12) in this approximation, with negligible terms proportional with $1/H$ and g, could be written as dimensionless dispersion equation for the three MHD modes in uniform magnetized plasmas:

$$(\Omega^2 - \Omega_A^2)\left[\Omega^4 - K^2\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_c^2)\right] = 0, \tag{16}$$

with solutions:

$$\Omega^2 = \Omega_A^2, \tag{17}$$

for Alfvén waves and

$$\Omega_{1,2}^2 = \frac{K^2\left(1 + \frac{2}{\gamma\beta}\right)}{2}\left[1 \pm \sqrt{1 - \frac{4\Omega_c^2}{K^2\left(1 + \frac{2}{\gamma\beta}\right)}}\right], \tag{18}$$

for fast (with plus sign) and slow (with minus sign) magneto-acoustic waves. Here, $K = \sqrt{K_p^2 + K_z^2}$ is dimensionless wave number. In a very low-β plasmas, when $\beta \ll 1$, ‡ the effect of the plasma pressure is very small compared to the magnetic pressure. Using this approximation, from dimensionless dispersion equation (18) we can get:

$$\Omega^2 \approx K_x^2, \tag{19}$$

for the slow magneto-acoustic wave and

$$\Omega^2 \approx \frac{2K^2}{\gamma\beta}\left(1 - \frac{\gamma\beta K_x^2}{2K^2}\right), \tag{20}$$

for the fast magneto-acoustic wave. This means that in a very low-β plasmas slow magneto-acoustic waves in Eq.(19) have almost the same frequency as acoustic waves that propagate parallel with a magnetic field B_0§. Eq.(20) represents first order correction of degenerate fast magneto-acoustic wave whose dimensional frequency is

‡The assumption $\beta \ll 1$, or $v_s \ll v_A$, is valid for quite a number of relevant plasmas like those in tokamaks and in the solar corona.

§$\Omega_s^2 = \frac{\omega_s^2 H^2}{v_s^2} = \frac{k_x^2 v_s^2 H^2}{v_s^2} = K_x^2$. Note that in this approximation we have:
$\Omega_c^2 = \frac{k_x^2 v_s^2 v_A^2 H^2}{v_s^2(v_s^2 + v_A^2)} = \frac{K_x^2}{1 + \frac{\gamma\beta}{2}} \approx K_x^2\left(1 - \frac{\gamma\beta}{2}\right)$ if $\beta \ll 1$.

$\omega^2 = k^2 v_A^2 - \omega_s^2$. Note that K_x^2 component of the wave number could be represented as: $K_x^2 = K_p^2 \cos^2\varphi$, where φ is the angle between horizontal wave number K_p and magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$. The equation Eq.(20) is illustrated with Fig.3, Fig.4, Fig.5 and Fig.6. In these figures dimensionless frequency Ω^2 is a function of dimensionless wave number K^2 for a given: dimensionless horizontal wave number K_p^2, angles φ and for a given parameter β.

In Fig.3, where $\beta = 0.1$, and $K_p^2 = 0.5$, for a given angles φ, we can see that for $K^2 = K_p^2 + K_z^2 \gg 1$, there are almost vertical fast magneto-acoustic waves with respect to magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$. In this case $K_z^2 \gg K_p^2$. If $\varphi = 90^0$, then $K_x^2 = K_p^2 \cos^2\varphi = 0$, *i.e.* Eq.(20) shows that $\Omega^2 \approx \frac{2K^2}{\gamma\beta}$, or in dimensional form: $\omega^2 = k^2 v_A^2$ which is degenerate fast magneto-acoustic wave. This wave propagates perpendicular with respect to the magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$. For the angles $0^0 < \varphi < 90^0$, wave frequencies are lower for the same wave numbers. If $\varphi = 0^0$, then $K_x^2 = K_p^2$ and the waves have wave vector components in x direction (parallel with magnetic field) and in z direction (perpendicular with magnetic field). Note that for increasing K^2 (*i.e.* K_z^2 wave number component), wave frequencies increases linearly.

Fig.4 shows dimensionless frequency Ω^2 as a function of the same K_p^2 and angles φ as in Fig.3 but with a smaller parameter $\beta = 0.05$. We can noticed that almost vertical fast magneto-acoustic waves have a frequencies higher for $N = 0.1/0.05 = 2$ times than frequencies in Fig.3. The difference between the frequencies for the same K_p^2 value is smaller in Fig.4 than in Fig.3, which is reasonable because in Eq.(20), Ω^2 is proportional with $\frac{1}{\beta}$. This means that in the solar corona, where $\beta \approx 5 \times 10^{-4}$, all these frequencies tend to $\Omega^2 = \frac{2K^2}{\gamma\beta}$.

In Fig.5 and Fig.6, the frequencies Ω^2 of the fast magneto-acoustic waves are presented as a function of angles φ, parameter β and much higher horizontal wave number K_p^2. These waves are not vertical waves any more. If $K_z^2 = 0$, *i.e.* $K^2 = K_p^2 = 5$, we are dealing with a pure horizontal waves which propagate in $x - y$ plane. Their frequencies decreasing with decreasing the angle φ values. With increasing K^2 (*i.e.* K_z^2), wave frequencies linearly increase for the each angle φ. As in Fig.3 and Fig.4, we can noticed that fast magneto-acoustic wave frequencies are higher for the lower $\beta = 0.05$ than for a higher $\beta = 0.1$, for $N = 2$ times.

For a high-β plasmas, in the limit of very large dimensionless wave numbers $K \gg 1$, dimensionless dispersion equations (17)-(18) show that Alfvén and slow magneto-acoustic waves disappear (their frequencies are very low-$\Omega^2 \approx \Omega_A^2 \ll 1$¶ and $\Omega^2 \approx \Omega_c^2 \ll 1$) and fast magneto-acoustic waves become a pure acoustic waves with a first

¶ $\Omega_A^2 = \frac{k_x^2 v_A^2 H^2}{v_s^2} = \frac{2K_x^2}{\gamma\beta}$ and $\Omega_A^2 = \Omega_c^2 \ll 1$ for finite K_x^2 and for $\beta \gg 1$.

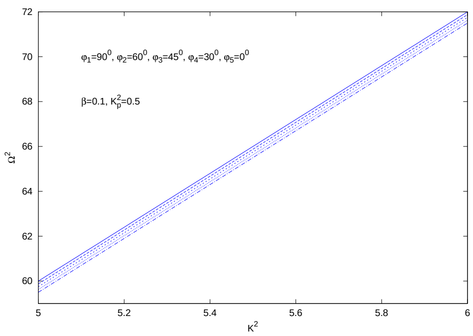

Fig. 3 – Dimensionless frequency Ω^2 as a function of dimensionless wave number K^2, for a given $K_p^2 = 0.5$, angles φ and parameter $\beta = 0.1$. This figure represents almost vertical fast magneto-acoustic waves derived from Eq.(12) for gravito-MHD waves in the limit of large wave numbers ($K \gg 1$) and in low-β plasmas.

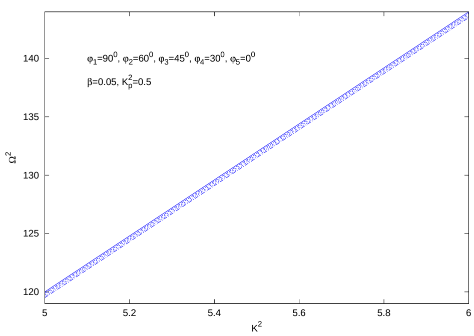

Fig. 4 – Dimensionless frequency Ω^2 as a function of dimensionless wave number K^2, for a given $K_p^2 = 0.5$, angles φ and parameter $\beta = 0.05$ which is $N = 2$ times smaller than parameter β in Fig.3. Because of that, frequencies of almost vertical fast magneto-acoustic waves are higher for $N = 2$ times.

order correction term:

$$\Omega^2 \approx K^2 \left[1 + \frac{2}{\gamma\beta} \left(1 - \frac{K_x^2}{K^2} \right) \right], \tag{21}$$

i.e. in the dimensional form $\omega^2 = k^2 v_s^2 + v_A^2(k^2 - k_x^2)$. Dimensionless frequencies Ω^2 for the fast magneto-acoustic waves in this case are presented in Fig.7 and Fig.8. Frequencies increase linearly with increasing wave number K^2 and decrease with decreasing angle φ. Eq.(21) shows that for $K_x^2 \ll K^2$ (*i.e.* $K_p^2 cos^2\varphi \ll K^2$), when the waves propagate almost perpendicular with respect to magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$, their frequencies are $\Omega^2 \approx K^2 \left(1 + \frac{2}{\gamma\beta} \right)$, Fig.7. For $K_x^2 = K_p^2 cos^2\varphi = K^2$, when the waves propagate horizontally with respect to magnetic field, their frequencies become $\Omega^2 \approx K^2$. This is the frequency of a pure acoustic wave as in the point $K^2 = 5$, for $\varphi = 0^0$, Fig.8. Furthermore, in the case when $\beta \gg 1$ as in the solar photosphere, where $v_A^2 \approx 0$, fast magneto- acoustic wave becomes a pure acoustic wave with frequency $\omega^2 \approx k^2 v_s^2$ for each k_x^2.

Waves with small wave numbers are modified by the stratification if their wavelengths are comparable with or larger than the density scale length H. When $k \ll 1/H$, dimensionless dispersion equation for modified by gravity fast magneto-acoustic waves, derived from Eq.(13) for gravito-MHD waves, is:

$$\Omega^2 = \frac{1}{4} \left(1 + \frac{2}{\gamma\beta} \right) = \Omega_{co}^2 \left(1 + \frac{2}{\gamma\beta} \right). \tag{22}$$

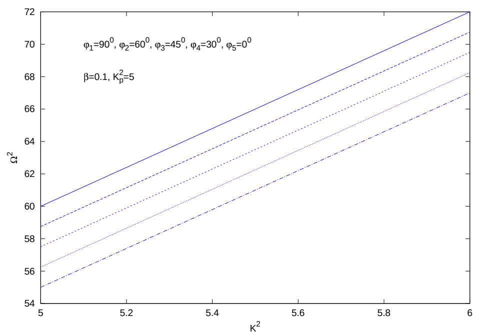

Fig. 5 – Dimensionless frequency Ω^2 of the fast magneto-acoustic waves as a function of dimensionless wave number K^2, for a given $K_p^2 = 5$, angles φ and parameter $\beta = 0.1$. In the point $K^2 = K_p^2$ there are a pure horizontal fast magneto-acoustic waves which propagate in $x - y$ plane. With increasing K^2, (*i.e.* with increasing vertical wave number K_z^2), fast magneto-acoustic waves frequencies linearly increase.

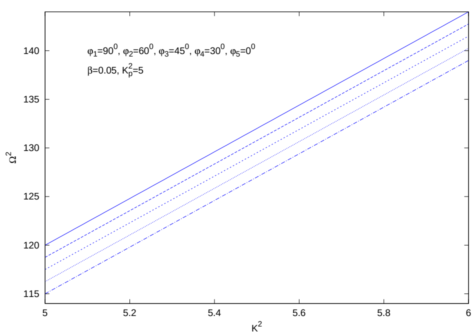

Fig. 6 – Dimensionless frequency Ω^2 of the fast magneto-acoustic waves as a function of dimensionless wave number K^2, for a given $K_p^2 = 5$, angles φ and parameter $\beta = 0.05$, *i.e.* β is $N = 2$ times smaller than in Fig.5. As a result, the fast magneto-acoustic waves frequencies are $N = 2$ times higher for a such β value. As in Fig.5, the smallest frequencies are for horizontal fast magneto-acoustic waves in the point $K^2 = K_p^2$.

Dimensional frequency is: $\omega^2 = (v_s^2 + v_A^2)/4H^2$. This equation describes oscillations of the system as a whole which is characteristic only for plasmas in a gravitational field. Alfvén and slow magneto-acoustic waves in this approximation disappear. In a low-β plasma, when $\beta \ll 1$, Eq.(22) becomes:

$$\Omega^2 \approx \frac{1}{2\gamma\beta}, \tag{23}$$

in dimensional form $\omega^2 = v_A^2/4H^2$. For the solar coronal low-β plasma where $v_A^2 = 7 \times 10^{13}\mathrm{m}^2/\mathrm{s}^2$ this frequency is: $\omega^2 = 1 \times 10^{-9}\mathrm{s}^{-2}$.

In a high-β plasma, when $\beta \gg 1$, Eq.(22) has a form:

$$\Omega^2 \approx \frac{1}{4} = \Omega_{co}^2, \tag{24}$$

i.e. $\omega^2 = v_s^2/4H^2 = 3.2 \times 10^{-4}\mathrm{s}^{-2}$ in the solar photosphere where: $v_s^2 = 1.6 \times 10^8\mathrm{m}^2\mathrm{s}^{-2}$ and $v_A^2 \approx 0$. This means that the frequency of fast magneto-acoustic waves, modified by gravity in a high-β plasma, becomes equal to the acoustic cut-off frequency below which this waves can not propagate. In Fig.9 dimensionless frequency Ω^2, given by Eq.(22) as a function of small β, is presented by solid line. Dimensionless frequency Ω^2, given by Eq.(23), is presented by dashed line. For a very low-β plasmas, like in the solar corona where $\beta = 5 \times 10^{-4}$, the frequency of fast magneto-acoustic wave modified by gravity is given by Eq.(23). For a low-β plasmas with $K \ll 1$ and with $\beta > 0.2$, Eq.(22) can be used for the wave frequency.

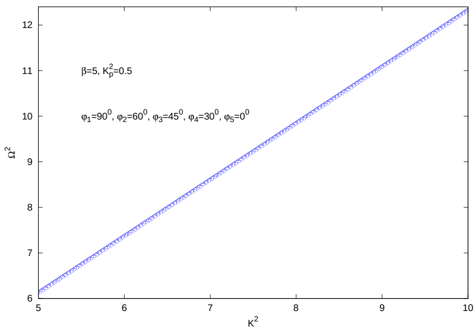

Fig. 7 – Dimensionless frequency Ω^2 of the fast magneto-acoustic waves as a function of dimensionless wave number K^2, for a given $K_p^2 = 0.5$, angles φ and parameter $\beta = 5$. If $K_p^2 \cos^2\varphi \ll K^2$, fast magneto-acoustic waves propagate almost perpendicular with respect to the magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$.

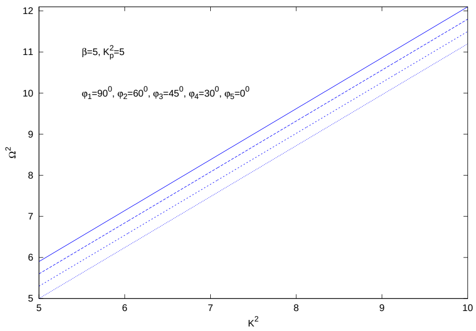

Fig. 8 – Dimensionless frequency Ω^2 of the fast magneto-acoustic waves as a function of dimensionless wave number K^2, for a given $K_p^2 = 5$, angles φ and parameter $\beta = 5$. If $K_x^2 = K_p^2 \cos^2\varphi = K^2$, fast magneto-acoustic waves propagate parallel with magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$ and they become a pure sound waves.

Fig.10 has been made for dimensionless frequency Ω^2 of fast magneto-acoustic waves modified by gravity, Eq.(22), as a function of high β. Frequency given by Eq.(22) slowly decreases with increasing β. For $\beta \gg 1$, this frequency tends to the acoustic cut-off frequency. In high-β plasmas with $K \ll 1$, for $\beta < 250$, Eq.(22) can be used for the wave frequency.

5. CONCLUSION

Dispersion equation Eq.(12) (or Eq.(13)) for gravito-MHD waves is derived using the standard set of MHD equations for the gravitationally stratified isothermal plasma with embedded horizontal magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$. This equation was analysed in several cases:

1. Without magnetic field a dispersion equation for gravito-acoustic wave was obtained, Eq.(14),
2. Small magnetic field influence, when $\beta \gg 1$, the first order correction of the gravito-acoustic waves dispersion equation was derived, Eq.(15),
3. In the limit of large wave numbers, $K \gg 1$, when gravitation effects on gravito-MHD waves are negligible, MHD modes propagate as in a locally uniform magnetized plasma. These waves in a low-β plasmas, ($\beta \ll 1$), propagate as acoustic waves, Eq.(19), and as a degenerate fast magneto-acoustic wave with small correction term, Eq.(20).
4. For a high-β plasmas in the limit of large wave numbers, $K \gg 1$, Alfvén and

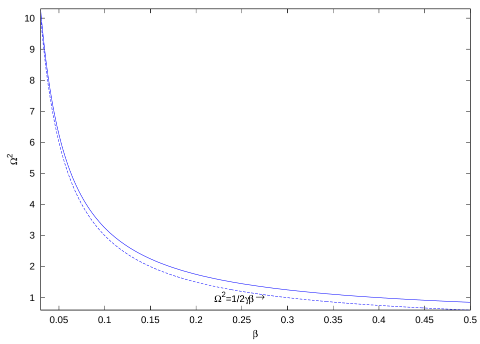

Fig. 9 – Dimensionless frequency Ω^2 of a fast magneto-acoustic waves derived from Eq.(13) for gravito-MHD waves in the limit of small wave numbers $K \ll 1$ as a function of low-β. Solid line represents frequency of fast magneto-acoustic wave, given by Eq.(22), and dashed line represents frequency of these waves given by Eq.(23) in a very low-β plasmas ($\beta \ll 1$).

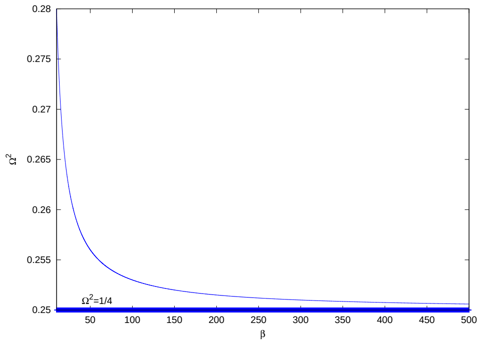

Fig. 10 – Dimensionless frequency Ω^2 of a fast magneto-acoustic waves derived from Eq.(13) for gravito-MHD waves in the limit of small wave numbers $K \ll 1$ as a function of high-β. Solid line represents frequency of fast magneto-acoustic wave, given by Eq.(22) and the line $\Omega^2 = \Omega_{co}^2 = 1/4$ shows that the fast magneto-acoustic wave frequency tends to acoustic cut-off frequency in the case when $\beta \gg 1$.

slow magneto-acoustic waves disappear, while fast magneto-acoustic waves become a pure acoustic waves with first order correction term, Eq.(21). In the large wave numbers limit, waves frequencies depend on the angle φ between horizontal wave number K_p and magnetic field $\vec{B_0} = B_0(z)\vec{e_x}$. This was shown in: Fig.3, Fig.4, Fig.5 and Fig.6.
5. In the limit of small wave numbers, $K \ll 1$, gravito-MHD waves are affected by the non-uniformity of the medium resulting from the action of gravitation. The frequencies of these waves are given by Eq.(22). These frequencies depend on the β values, Fig.9 for low-β plasmas and Fig.10 for high-β plasmas.

In this paper the method of deduction was used to derive the "pure" waves modes from the general gravito-MHD dispersion equation, Eq.(12) or Eq.(13). We derived a first order corrections for the waves frequencies in the cases of very low and very high-β plasmas, such as solar photosphere and solar corona respectively.

In the further work the conditions for instability (stability) of this plasma configuration will be examined.

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