

## STUDY OF THE EXTENSIVE AIR SHOWER SIZE AND THE ENERGY SPECTRA OF COSMIC RAY MUONS AT SEA LEVEL\*

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*Abstract.* In this paper I presented the results obtained for the differential energy spectra of muons at sea level determined as a function of three zenith angles. I calculated the shower size and the number of muons per square meter as a function of the lateral distance from the center of the shower. The data interpretation provide the most significant physical information for understanding the main problems in cosmic ray physics. The problem of the energy dependence of the muon intensity at sea level is in this respect a specific goal to which the present article is devoted.

*Key words:* shower size, muon densities, atmospheric depth, chemical composition, energy threshold.

### 1. INTRODUCTION

Cosmic ray particles at sea level include charged hadrons, neutrons, gamma rays, electrons, muons, nuclei and antinucleons. The probability of surviving from the point of production to sea level depends on the particle type, its charge state, energy, zenith angle of propagation and mean lifetime at rest. Ultra high energy cosmic rays are presumed to be of extragalactic origin [1]. Energetic protons and nucleons lose energy mostly by strong interactions, heavy nuclei are fragmented in collisions with nuclei of air molecules, and electrons and photons undergo electromagnetic energy loss processes. The primary cosmic radiation which consists predominantly of protons, alpha particles and heavier nuclei is influenced by the galactic, the interplanetary, the magnetospheric and the geomagnetic magnetic fields while approaching the Earth. The electrically charged secondary cosmic ray component produced in the atmosphere is also subject to geomagnetic effects. The cosmic ray flux is modulated by solar activity. The solar minimum and maximum cycle in 11 years and both cycles cause various effects on the galactic cosmic radiation. The modulation effects decrease with increasing energy and

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become insignificant for particles with rigidities in excess of  $\sim 10$  GV [2]. All particles suffer energy losses through hadronic and / or electromagnetic processes. Above an energy of a few GeV, local penetrating particle showers are produced, resulting from the creation of mesons and other secondary particles in the collisions. Energetic primary protons undergo on average 12 interactions along a vertical trajectory through the atmosphere to sea level, corresponding to an interaction mean free path, of about  $80 \text{ g}\cdot\text{cm}^{-2}$ . For a projectile nucleus with mass number  $A = 25$ , the interaction mean free path is approximately  $23 \text{ g}\cdot\text{cm}^{-2}$  in air, corresponding to about 50 interactions for a vertical trajectory through the atmosphere. Consequently, there is practically no chance for a heavy nucleus to penetrate down to sea level. The relation between cross section and interaction mean free path is described the expression (1) by [2]:

$$\lambda_i = \left[ \frac{N_A}{A} \cdot \sigma_i \right]^{-1}, \quad (1)$$

where  $N_A$  is Avogadro's number,  $A$  is the rest mass number of the target nucleus and  $\sigma_i$  is the cross section for the particular interaction [2].

The problem of the altitude dependence of the muon intensity in the  $1033 \text{ g}\cdot\text{cm}^{-2}$  atmospheric depth is in this respect a specific case to which the present paper is devoted. Cosmic ray particles at very high energies are generated in supernova remnants, on the stars, in Galactic Active Nuclei (AGN), in quasars and in astrophysical compact object.

This paper was designed in two parts: part 1 is dealing with the research for the muon energy spectrum at sea level, and part 2 deals with the calculation of the number density of charged particles at the sea level. I calculated the muon shower size and the number of muons per square meter as a function of the lateral distance from the center of the shower. The data interpretation provides the most significant physical information for understanding the main problems in cosmic ray physics. The problem of the energy dependence of the muon intensity at sea level is in this respect a specific goal to which the present article is devoted. Calculations were made in the laboratory system. In addition, the properties of air shower (with  $10^6$  size at sea level) induced by primary cosmic ray proton were examined.

## 2. AN ESTIMATE OF THE MUON SPECTRUM AT SEA LEVEL

Cosmic rays energies range from  $10^8$  to  $10^{20}$  eV, following an approximate power law. The flux of the highest energy cosmic rays is very low. Therefore high – energy cosmic rays are only detectable by their interaction with the Earth atmosphere [3]. Only nucleons greater that about 4 GeV have sufficient energy to generate a muon cascade capable of penetrating the atmosphere and reaching the Earth's surface [4]. The subject of atmospheric muons is one of the most interesting topic since the muons are the most numerous charged particles at the ground level

in the atmosphere and at sea level. Most muons are produced high in the atmosphere and lose about 2 GeV to ionization before reaching the ground [5].

According to [5], the differential intensity of the muon cosmic rays at sea level can be expressed by:

$$\frac{dN_{\mu}}{dE_{\mu}} \approx \frac{0.14 E_{\mu}^{-2.7}}{cm^2 \cdot s \cdot sr \cdot GeV} \cdot \left[ \left( \frac{1}{1 + \frac{1.1 E_{\mu} \cos \theta}{115 GeV}} \right) + \left( \frac{0.054}{1 + \frac{1.1 E_{\mu} \cos \theta}{850 GeV}} \right) \right], \quad (2)$$

where  $E_{\mu}$  is the muon energy at sea level, (in GeV), and  $\frac{dN_{\mu}}{dE_{\mu}}$  is the differential intensity of the muons at sea level.

The two terms, by (2), gives the contribution of pion and the electrically charged kaon to the spectrum of muons. Approximately 115 GeV is the critic energy of the pion in the atmosphere and approximately 850 GeV is the critic energy of the charged kaons in the atmosphere.

Conclusions can be made about the anisotropies in the muon flux at sea level. The overall angular distribution of muons at the ground is  $\sim \cos^2 \theta$ , which is characteristic of muons with  $E_{\mu} \sim 3$  GeV. At lower energy the angular distribution becomes increasingly steep, while at higher energy it flattens, approaching a sec  $\theta$  distribution for  $E_{\mu} \gg e_{\pi}$  and  $\theta < 70^{\circ}$  [5]. Fig. 1a shows the calculated angular dependence for muons (calculated results in this paper) together with proton curve given by Flickiger [2]. A systematic error of 5% has to be added for the results.

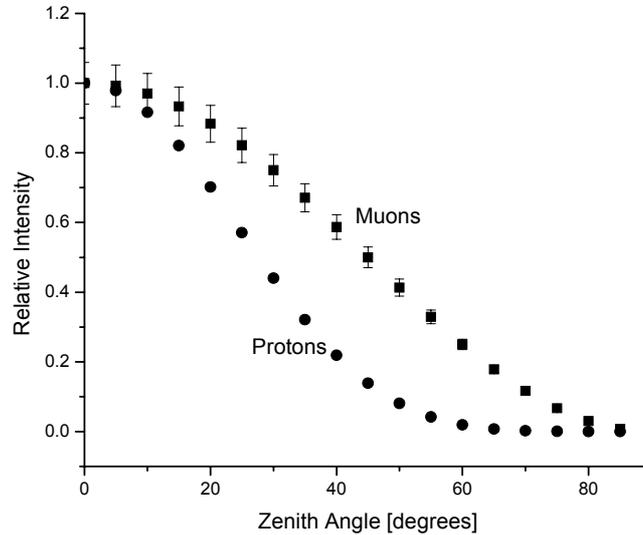


Fig. 1a – Zenith angular distribution of muons (calculated results in this paper) and protons

(•, Flickiger, [2]) at sea level ( $1033 \frac{g}{cm^2}$ ).

The aim is to predict some crucial informations for understanding main features of the cosmic rays, from the observed that the flux of the highest energy cosmic ray.

The calculated results of the differential energy spectra of muons at sea level, determined for three zenith angles:  $0^\circ$ ,  $30^\circ$  and  $45^\circ$  with the theoretical model by [5], shows that the upper limit to the variation with direction is low at low energy.

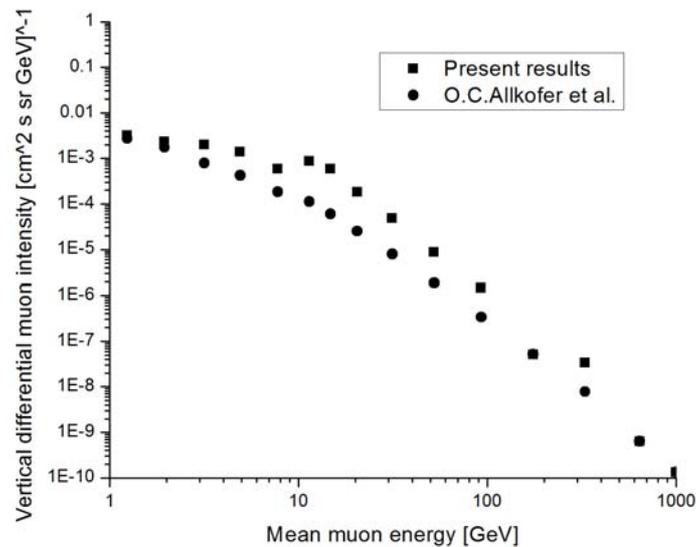


Fig. 1b – The vertical differential muon intensity expected at sea level as a function of mean muon energy. It is interesting to note that there is good agreement with [7] (the Allkofer's result), and the calculated results in this paper.

The aim is to predict some crucial informations for understanding main features of the cosmic rays, from the observed that the flux of the highest energy cosmic ray particle is very low comparing with the flux of the lower energy cosmic rays, to the end product: the energy spectrum. The calculated muon results are presented in Table 1.2.2.2 [11]. On examining Fig. 1b we see that the differential muon intensity falls steeply with rising energy.

Theoretical aspects of the fundamental particles and properties of the fundamental particles have been used as information in the present paper [2, 8, 9, 15]. At ground level the abundance ratios of particles are roughly:  $\gamma : e^{+/-} : \mu^- : \text{hadrons} = 10^3 : 10^2 : 10 : 1$ , and the typical particle energies are 1–10 MeV for photons and electrons and 0.1 – 10 GeV for muons and hadrons [3].

Particles with mean lives shorter than  $10^{-12}$  s have almost no chance at all to survive down to sea level unless they are locally produced [2, 15]. In the present paper muon energy threshold is assumed to be 1 GeV; present results are consistent with the characteristics of the relativistic particle [6, 16].

Particle energy spectrum at sea level for muon cosmic rays together with the excellent valid data of the other authors, for muon and other particles at sea level are given in Fig. 2.

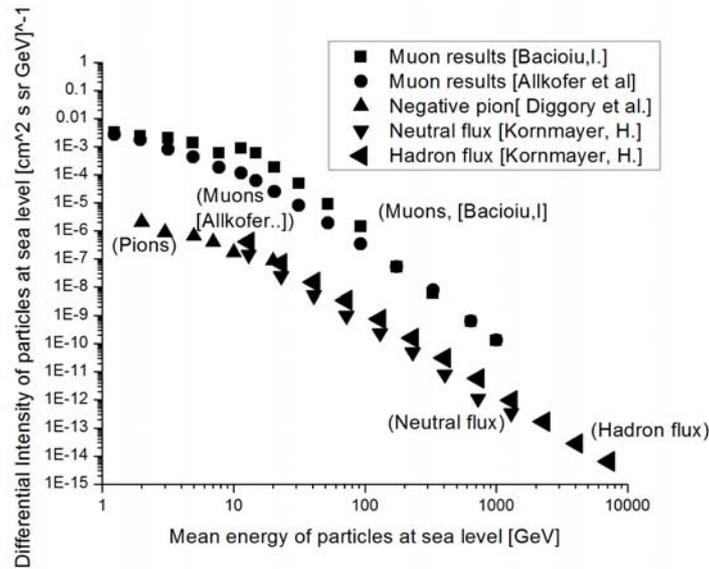


Fig. 2 – Particle energy spectrum at sea level. The results of the authors: Diggory *et al.* [2], H. Kormmayer [8] and O.C.Allkofer [7] are shown as well as results obtained in the present paper.

It is interesting to note that the differential muon intensity falls steeply with rising energy. On the other hand, high energy muons lose energy by radiative processes and the corresponding photons and electrons initiate cascades [8].

### 3. STUDY OF THE EXTENSIVE AIR SHOWER AT SEA LEVEL

The primary cosmic rays strike the atmosphere and produce extensive air showers (EAS) of secondary particles. The development of a shower in the atmosphere depends on the hadronic and electromagnetic interactions of shower particles with the air, their interaction cross sections, the secondary particle production, decays of unstable particles and the transport through the atmosphere, including energy loss, deflection [3]. An air shower is caused by a single cosmic ray with energy high enough for its cascade to be detectable at the ground. To illustrate the basic result, I briefly discuss some aspects of the Extensive Air Shower components. The shower has a hadronic core, which acts as a collimated source of electromagnetic showers, generated mostly from  $\pi^0 \rightarrow \gamma \gamma$ . In initial hadronic reactions many secondaries are produced. These are mainly pions which in turn decay into muons and gammas, thus giving rise to the electromagnetic and

muonic component of the shower [3]. The purpose of this paper is to calculate the muon size,  $N_\mu$ , at sea level as a function of the total number of charged particles in the atmospheric shower. A study of the number density of charged particles and high accuracy for determined the value of the lateral spread of the muons, at sea level are presented below. The most significant physical information is obtained through  $N_e - N_\mu$  correlation, which depends primarily on the cosmic ray chemical composition and is largely independent of the energy spectrum. According to [4], the usual method of determining primary mass is by way of the “depth of maximum”,  $X_{max}$ , i.e., the depth in the atmosphere at which the number of particles in the shower is greatest. The main problem in determining the primary mass is in knowing the exact position of the theoretical lines, for protons, He...Fe, view of the uncertainty about the interaction model [4].

The number density of charged particles is [5]:

$$\rho_e = C_1(s, d, C_2) x^{s-2} (1+x)^{(s-4.5)} (1+C_2 x^d), \quad (3)$$

where  $s$  is the age parameter. Here,  $s$ ,  $d$ , and  $C_2$  are parameters in terms of which the overall normalization constant  $C_1(s, d, C_2)$  is given by [5]:

$$C_1(s, d, C_2) = \left( \frac{N_e}{2\pi r_1^2} \right) [B(s, 4.5-2s) + C_2 B(s+d, 4.5-d-2s)]^{-1}, \quad (4)$$

where  $B(m, n)$  is the beta function, and  $x$  is  $\frac{r}{r_1}$ . The values of the parameters depend the shower size ( $N_e$ ), depth in the atmosphere, identity of the primary nucleus. For sea level: shower size is  $N_e \approx 10^6$ ; and  $r_1 \approx 78$  m [5], where  $r_1$  is the Moliere radius, which depends on the density of the atmosphere.

According to [5], the number density of charged particles, at sea level, is:

$$\rho_e = \frac{N_e}{2\pi(78^2)} [B(1.25, 2) + C_2 B(2.25, 1)]^{-1} \left[ \left( \frac{r}{78} \right)^{-0.75} \right] \left[ \left( 1 + \frac{r}{78} \right)^{-3.25} \right] \left( 1 + \frac{r C_2}{78} \right), \quad (5)$$

Table 1

The number density of charged particles,  $\rho_e$ , at sea level  
[results calculated with (5) in the present paper]

Lateral distance, $r$ [m]	$\rho_e$ [m <sup>-2</sup> ]	Lateral distance, $r$ [m]	$\rho_e$ [m <sup>-2</sup> ]
78	$6.14 \times 10^5$	550	$1.08 \times 10^4$
88	$5.17 \times 10^5$	600	$8.61 \times 10^3$
100	$4.25 \times 10^5$	650	$6.97 \times 10^3$
150	$2.10 \times 10^5$	700	$5.72 \times 10^3$
200	$1.19 \times 10^5$	750	$4.76 \times 10^3$
250	$7.33 \times 10^4$	800	$4.00 \times 10^3$
300	$4.84 \times 10^4$	850	$3.39 \times 10^3$
350	$3.36 \times 10^4$	900	$2.90 \times 10^3$

Table 1 (continued)

400	$2.42 \times 10^4$	950	$2.50 \times 10^3$
450	$1.81 \times 10^4$	1000	$2.17 \times 10^3$
500	$1.38 \times 10^4$		

The calculated results (Table 1) are shown in Fig. 3.

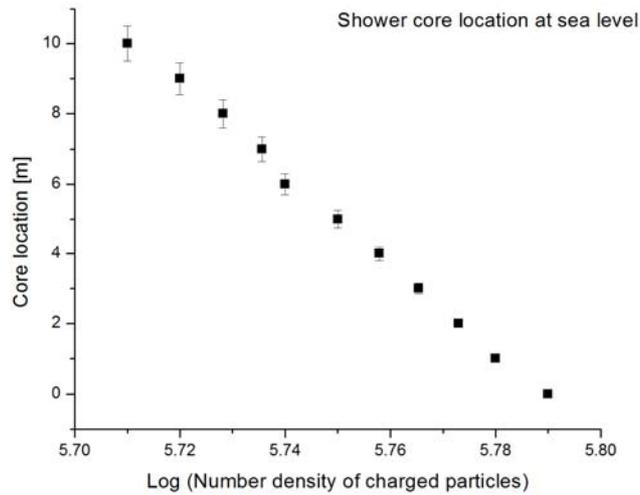


Fig. 3 – The dependence of the shower core location,  $\Delta r$  [1 – 10], vs. the number density of charged particles,  $\rho_g$ , at sea level.

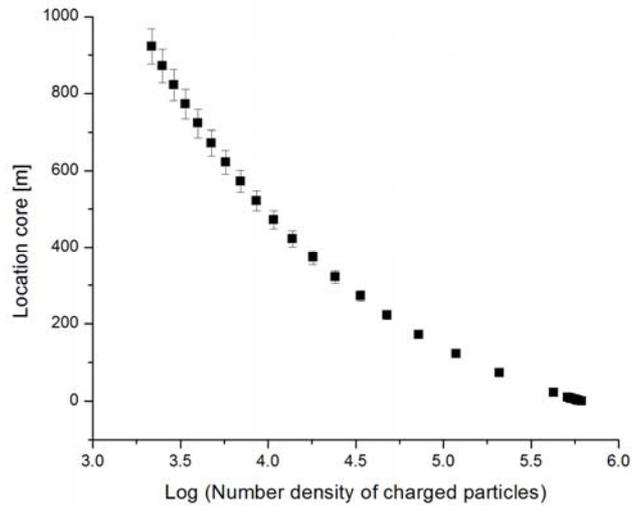


Fig. 4 – The dependence of the core location,  $\Delta r$  [0; 1000], vs. the number density of charged particles,  $\rho_g$ , at sea level.

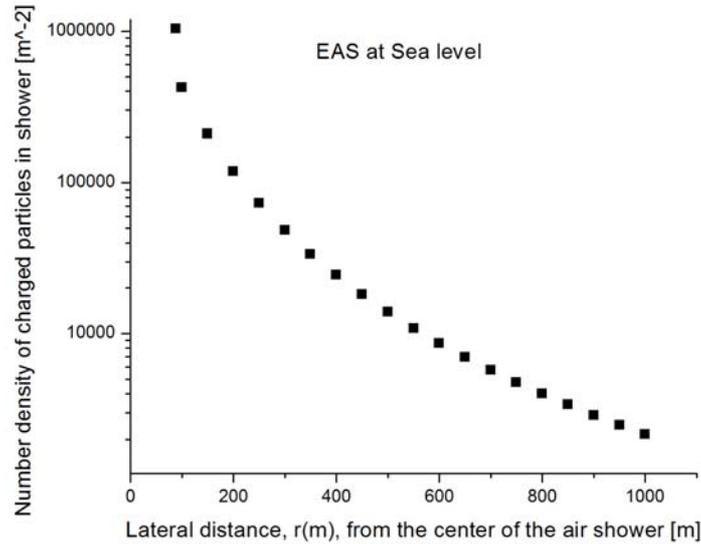


Fig. 5 – The number density of charged particles,  $\rho_{\pm}$ , as a function of the lateral distance,  $r$  [m], from the center of the shower, at sea level.

The aim is to predict a crucial information for understanding main features of the air shower induced by primary proton. The particle density decreases rapidly with the distance from the shower core. Shower particles typically reach out to about 100 – 200 m at  $E_p = 10^{15}$  eV, and to 1 – 2 km at  $10^{19}$  eV. The air shower analysis aim primarily at the energy spectrum, the arrival direction distribution and the mass composition of the primary cosmic rays [3]. The energy of the primary is deduced from the “size” of the shower, *i.e.* from the number of secondaries produced [3].

#### 4. RESULTS

Correspondence between results of the present paper and the results of the other papers are listed below:

- The shower size and primary energy depends on depth in the atmosphere [5].
- The hard component of meson at sea level is about 72.81 per cent of the total charged component of cosmic rays at sea level [9].
- The  $N_e - N_{\mu}$  correlation depends primarily on the cosmic ray chemical composition and is largely independent of the energy spectrum.
- No evidence has been found for an increase in intensity at low momenta due to multiple scattering in the atmosphere [10].

- Due to the fact that the primary cosmic radiation is essentially positively charged, the secondary component in the atmosphere has a positive charge excess, too [2, 15].

For vertical shower with primary energy  $10^5 < E < 10^8$  GeV, at sea level the shower size is  $N_e \approx 10^6$ , [5]. According to [5] if an energetic proton interacting with an air nucleus creates an air shower with  $10^6$  size at sea level, then the energy of cosmic ray proton initiating the air shower must be of  $3.9 \times 10^6$  GeV. The rather recent model EPOS, for instance, predicts for  $10^{19}$  eV showers more than 30% larger muon numbers at 1 km core distance than the other models [3]. The conclusions of this investigation may be summarized as follows: according [9], the hard component of meson at sea level is about 72.81 per cent of the total component of secondaries cosmic rays at sea level. The physical information obtained through  $N_e - N_\mu$  correlation depends on the chemical composition of the incident cosmic ray particle. Correspondence between the muon shower size and the total shower size is:

$$N_\mu \sim N_e^\alpha, \quad (6)$$

where  $N_\mu$  is the muon shower size at sea level, and  $N_e$  is the total shower size at sea level [12], with  $\alpha = [0.7-0.8]$ .

The calculated results for the number of muons per square meters as a function of the lateral distance  $r$  [m] from the center of the air shower and results for the muon shower size at sea level are tabulated in the Table 2.

Table 2

The muon density and the muon shower size at the ground level and at sea level

Muon densities [m <sup>-2</sup> ]	Log $N_\mu$	Atmospheric depth [g cm <sup>-2</sup> ]	Ranges of distances from the muon detector [m]
$0.03 < \rho_\mu < 0.12$ (in the present paper)	$4.20 < \text{Log } N_\mu < 4.80$ (in the present paper)	1033 (sea level)	$140 < r < 160$ (in the present paper)
$0.05 < \rho_\mu < 0.29$ Ref. [12]	$4.50 < \text{Log } N_\mu < 5.15$ Ref. [12]	810 (2005 m above sea level)	$140 < r < 170$ Ref.[12]

## 5. CONCLUSIONS

The problem of the energy dependence of the muon intensity at sea level and to determine the proportion of muons in the showers are in this respect one of a specific goal to which the present article is devoted. The conclusions of this investigation may be summarized as follows: the data interpretation provides the

most significant physical information for understanding the main problems in cosmic ray physics: in the present study, the calculated results for the muon densities, at sea level, are  $0.03 < \rho_{\mu} < 0.12$ , converted for the muon size in the range  $4.20 < \text{Log}(N_{\mu}) < 4.80$ .

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