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THEORETICAL PHYSICS

TELEPARALLEL KILLING MOTIONS OF BIANCHI TYPE V SPACETIMES

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Abstract. The aim of this paper is to explore teleparallel Killing vector fields for Bianchi type V spacetime in teleparallel theory of gravitation. For this purpose some algebraic and direct integration techniques are used. Although, curvature of the spacetime under consideration is zero in teleparallel theory, the presence of torsion restrict Bianchi type V spacetime to admit only five or six teleparallel Killing vector fields. It also comes out that torsion has a weaker effect on Killing symmetry of Bianchi type V spacetime than curvature.

1. INTRODUCTION

The general theory of relativity unifies mass and energy elegantly with geometric structure of spacetime through Einstein's field equations. At classical level the geometric description of this theory has been proved through experiments and seems true in nature while at quantum level interaction among the particles do not follow such description.

In order to get a consistent and unified theory for classical and quantum levels, Einstein made an attempt and introduced the notion called absolute parallelism. Although the attempt of Einstein did not succeed but the modern researchers are trying to unify other known interactions with gravitation. Different attempts made it possible to give a structure to gravitation other than metric tensor. In these investigations the metric tensor can be written as by-product of these structures. One of these structures known as tetrad field is now a basic entity in teleparallel theory [1]. In this theory the gravitational interaction of particles are described only by torsion in the spacetime and no curvature is formed due to mass in the spacetime. Unlike general relativity, which is based on Riemannian geometry, teleparallel theory is based on Weitzenböck geometry [2]. Although the scope of relativity theory is very wide and researchers are working on both theoretical and experimental side of the theory, the study of symmetries in a spacetime has got a great attention. In general relativity theory symmetries played a vital role in the exploration of some critical aspects of the spacetime physics the details of which can be found in [3–4]. Symmetries are so much important because they provide us the laws of conservation [5] and physical information about the spacetime. For example, self-similarity solutions are extensively used for cosmological perturbations, star formation, gravitational collapse, primordial black holes, cosmological voids and cosmic censorship [6]. Many authors classified well known spacetimes according to Killing vector fields in general relativity. Static spherically symmetric spacetime was classified according to its Killing vector fields in [7]. In this paper they found that the spacetime under consideration admits minimum four linearly independent Killing vector fields. Killing symmetry for static cylindrically symmetric spacetime and non static spherically symmetric spacetime was studied in [8–9]. They classified these spacetimes completely by their metrics and isometries. In [10] the non static plane symmetric spacetime was classified according to Killing vector fields and metrics.

Symmetries in the presence of torsion were ignored until it got the attention of M. Sharif and M. J. Amir [11]. They introduced Lie derivative which included the terms governed by torsion of the spacetime. They used teleparallel Lie derivative to obtain Killing equations of Einstein universe. The definition of teleparallel Lie derivative opened a door for the study of symmetries in teleparallel theory. Later on G. Shabbir *et al.* classified some well known spacetimes according to its teleparallel Killing vector fields [12–17]. The idea of Killing vector fields in teleparallel theory of gravitation was also extended to study proper homothetic vector fields in [18–22]. Recently the idea of symmetries in teleparallel theory is also extended to find conformal vector fields [23–24]. Keeping in mind the importance of symmetries in the presence of torsion our purpose in this paper is to investigate teleparallel Killing vector fields in Bianchi type V space-times and to analyze the effect of torsion on Killing symmetry for this spacetime. In the following a brief introduction of the basic entities in teleparallel theory is given.

2. TELEPARALLEL THEORY

For a covariant tensor having rank 2, covariant derivative in teleparallel theory is defined as [2]

$$\nabla_{\rho} E_{\mu\nu} = E_{\mu\nu,\rho} - \Gamma^{\theta}{}_{\rho\nu} E_{\mu\theta} - \Gamma^{\theta}{}_{\mu\rho} E_{\nu\theta}. \tag{1}$$

Here partial derivative is denoted by a comma and $\Gamma^{\theta}{}_{\rho\nu}$ represents Weitzenböck connections, which are defined as [2]

$$\Gamma^{\theta}{}_{\mu\nu} = U_{a}{}^{\theta} \partial_{\nu} U^{a}{}_{\mu}. \tag{2}$$

where $U_a^{\ \nu}$ stands for non-trivial tetrad field and $U^a_{\ \mu}$ represents the inverse field. This tetrad field must satisfy a relation

$$U^a_{\ \mu} \quad U_a^{\ \nu} = \delta_\mu^{\ \nu}, \qquad U^a_{\ \mu} U_b^{\ \mu} = \delta_b^{\ a} \tag{3}$$

The symbols μ, ν, ρ,...= 0, 1, 2, 3 denote the space-time indices and a, b, c,...= 0, 1, 2, 3 denote the tangent space indices. The above stated tetrad field and its inverse field generate the Riemannian metric as follows

$$g_{\mu\nu} = \eta_{ab}\, U^a_{\ \mu} U^b_{\ \nu}, \tag{4}$$

where η_{ab} = diag(–1, 1, 1, 1) is the Minkowski metric. Torsion component in terms of Weitzenböck connections is defined as

$$T^\theta_{\ \mu\nu} = \Gamma^\theta_{\ \nu\mu} - \Gamma^\theta_{\ \mu\nu} \tag{5}$$

This torsion is anti symmetric in its last two indices. It is the beauty of this interpretation that Riemann curvature tensor vanishes identically when defined through Weitzenböck connection. The teleparallel Killing equation in terms of torsion tensor is defined as [11]

$$\underset{X}{L^T}\, g_{\mu\nu} = g_{\mu\nu,\rho} X^\rho + g_{\rho\nu} X^\rho_{\ ,\mu} + g_{\mu\rho} X^\rho_{\ ,\nu} + X^\rho (g_{\theta\nu} T^\theta_{\ \mu\rho} + g_{\mu\theta} T^\theta_{\ \nu\rho}) = 0. \tag{6}$$

Main Results. The line element for Bianchi type V spacetime in the usual coordinate system is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \Phi(t)\mathrm{d}x^2 + \mathrm{e}^{2q\,x} \quad [\psi(t)\mathrm{d}y^2 + \Omega(t)\mathrm{d}z^2] \tag{7}$$

where Φ, Ψ and Ω are no-where zero functions of t only and $q \neq 0$. The tetrad components and its inverse can be obtained by using the relation (4) as

$$U^a_{\ \mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\Phi(t)} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{q\,x}\sqrt{\Psi(t)} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{qx}\sqrt{\Omega(t)} \end{pmatrix} \qquad U_a^{\ \mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{\Phi(t)}} & 0 & 0 \\ 0 & 0 & \frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{e}^{-qx}}{\sqrt{\Omega(t)}} \end{pmatrix} \tag{8}$$

The non vanishing torsion components by using (5) are

$$\begin{gathered} T^1_{\ 01} = -T^1_{\ 10} = \frac{\dot{\Phi}}{2\Phi}, \quad T^2_{\ 02} = -T^2_{\ 20} = \frac{\dot{\Psi}}{2\Psi}, \quad T^3_{\ 03} = -T^3_{\ 30} = \frac{\dot{\Omega}}{2\Omega}, \\ T^2_{\ 12} = -T^2_{\ 21} = q, \quad T^3_{\ 13} = -T^3_{\ 31} = q. \end{gathered} \tag{9}$$

where “dot” denotes the derivative with respect to t. Now using (7) and (9) in (6) we get the teleparallel Killing equations as follows:

$$X^0_{\ ,0}=0,\quad X^1_{\ ,1}=0,\quad X^2_{\ ,2}=0,\quad X^3_{\ ,3}=0, \tag{10}$$

$$X^0_{\ ,1}-\Phi(t)X^1_{\ ,0}-\frac{1}{2}\ \dot{\Phi}(t)\ \ X^1=0, \tag{11}$$

$$X^0_{\ ,2}-\mathrm{e}^{2qx}\Psi(t)X^2_{\ ,0}-\frac{1}{2}\mathrm{e}^{2qx}\ \ \dot{\Psi}(t)\ \ X^2=0, \tag{12}$$

$$X^0_{\ ,3}-\mathrm{e}^{2qx}\Omega(t)X^3_{\ ,0}-\frac{1}{2}\mathrm{e}^{2qx}\ \ \dot{\Omega}(t)\ \ X^3=0, \tag{13}$$

$$\Phi(t)X^1_{\ ,2}+\mathrm{e}^{2qx}\Psi(t)X^2_{\ ,1}+q\Psi(t)\mathrm{e}^{2qx}X^2=0, \tag{14}$$

$$\Phi(t)X^1_{\ ,3}+\mathrm{e}^{2qx}\Omega(t)X^3_{\ ,1}+q\Omega(t)\mathrm{e}^{2qx}X^3=0, \tag{15}$$

$$\Psi(t)X^2_{\ ,3}+\Omega(t)X^3_{\ ,2}=0. \tag{16}$$

Now integrating equation (10) we get

$$\begin{aligned} X^0&=W^1(x,y,z), & X^1&=W^2(t,y,z)\\ X^2&=W^3(t,x,z), & X^3&=W^4(t,x,y). \end{aligned} \tag{17}$$

where $W^1(x,y,z)$, $W^2(t,y,z)$, $W^3(t,x,z)$ and $W^4(t,x,y)$ are functions of integration which are to be determined. In the following different obtained results are given and the lengthy details are omitted.

Case I. In this case the teleparallel Killing vector fields for Bianchi type V spacetime are obtained as

$$\begin{aligned} X^0&=xc_4+c_0, & X^1&=\frac{c_4}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t+\frac{c_1}{\sqrt{\Phi(t)}},\\ X^2&=\frac{c_2}{\sqrt{\Psi(t)}}\mathrm{e}^{-qx}, & X^3&=\frac{c_3}{\sqrt{\Omega(t)}}\mathrm{e}^{-qx}, \end{aligned} \tag{18}$$

where $c_0,c_1,c_2,c_3,c_4\in\mathbb{R}$. The line element for Bianchi type V spacetime admitting teleparallel Killing vector fields (18) is given in equation (7). The generators of above teleparallel Killing vector fields can be written as $\frac{\partial}{\partial t}$, $x\frac{\partial}{\partial t}+\Sigma(t)\frac{\partial}{\partial x}$, $\frac{1}{\sqrt{\Phi(t)}}\frac{\partial}{\partial x}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial y}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\Omega(t)}}\frac{\partial}{\partial z}$, where $\Sigma(t)=\frac{1}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t.$ This is the minimal teleparallel Killing symmetry which Bianchi type V spacetime admits. In general relativity this spacetime admits minimal three Killing generators [25] $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial x}-qy\frac{\partial}{\partial y}-qz\frac{\partial}{\partial z}$. It is evident that when Bianchi type V spacetime is governed by torsion only and curvature is zero, teleparallel Killing symmetry generators are more in number than the generators of Killing symmetry in general relativity and three teleparallel Killing vector fields are multiple of the components of the inverse tetrad.

Case II. There are the following three possibilities for the metric functions:
(i) $\Phi=\Phi(t),\ \Psi=\Psi(t),\ \Omega=\text{costan t}$ and $\Phi(t)\neq\Psi(t)$.
(ii) $\Phi=\Phi(t),\ \Omega=\Omega(t),\ \Psi=\text{costan t}$ and $\Phi(t)\neq\Omega(t)$.
(iii) $\Psi=\Psi(t),\ \Omega=\Omega(t),\ \Phi=\text{costan t}$ and $\Psi(t)\neq\Omega(t)$.

Here we shall discuss case II(i) and case II(iii) only. Remember that case II(ii) can be solved the same way as case II(i). In case II(i) the spacetime metric takes the form

$$\mathrm{d}s^2=-\mathrm{d}t^2+\Phi(t)\mathrm{d}x^2+\mathrm{e}^{2qx}\ [\Psi(t)\mathrm{d}y^2+\eta\mathrm{d}z^2], \tag{19}$$

where $\eta\in\mathbb{R}\setminus\{0\}$. The corresponding teleparallel Killing vector fields for (19) are obtained as

$$\begin{aligned} &X^0=xc_4+c_0,\quad X^1=\frac{c_4}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t+\frac{c_1}{\sqrt{\Phi(t)}},\\ &X^2=\frac{c_2}{\sqrt{\Psi(t)}}\mathrm{e}^{-qx},\qquad X^3=\frac{c_3}{\sqrt{\eta}}\mathrm{e}^{-qx}, \end{aligned} \tag{20}$$

where $c_0,c_1,c_2,c_3,c_4\in\mathbb{R}$. The five generators of above teleparallel Killing vector fields can be written as $\frac{\partial}{\partial t}$, $x\frac{\partial}{\partial t}+\Sigma(t)\frac{\partial}{\partial x}$, $\frac{1}{\sqrt{\Phi(t)}}\frac{\partial}{\partial x}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial y}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\eta}}\frac{\partial}{\partial z}$, where $\Sigma(t)=\frac{1}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t$. In case (iii) the spacetime metric after a suitable rescaling of x, takes the form

$$\mathrm{d}s^2=-\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{e}^{2qx}\ [\ \Psi(t)\mathrm{d}y^2+\Omega(t)\mathrm{d}z^2]. \tag{21}$$

In this case the teleparallel Killing vector fields for Bianchi type V spacetime are obtained as

$$X^0 = xc_4 + c_0, \quad X^1 = tc_4 + c_1, \quad X^2 = \frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} c_2, \quad X^3 = \frac{\mathrm{e}^{-qx}}{\sqrt{\Omega(t)}} c_3, \tag{22}$$

where $c_0, c_1, c_2, c_3, c_4 \in \mathbb{R}$. In this case the five generators of (22) takes the form $\frac{\partial}{\partial t}$, $\frac{\partial}{\partial x}$, $t\frac{\partial}{\partial x} + x\frac{\partial}{\partial t}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial y}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\Omega(t)}}\frac{\partial}{\partial z}$.

Case III. In this case the metric functions have the following three possibilities:

a) $\Phi = \Phi(t)$, $\Psi = \Psi(t)$, $\Omega = \Omega(t)$ and $\Psi(t) = \Omega(t)$.

b) $\Phi = \Phi(t)$, $\Psi = \Psi(t)$, $\Omega = \Omega(t)$ and $\Phi(t) = \Omega(t)$.

c) $\Phi = \Phi(t)$, $\Psi = \Psi(t)$, $\Omega = \Omega(t)$ and $\Phi(t) = \Psi(t)$.

Here we shall discuss case III(a) and the other cases III(b) and III(c) can be solved exactly the same way as case III(a). In case III(a) the spacetime (7) takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \Phi(t)\mathrm{d}x^2 + \mathrm{e}^{2qx} \, \Psi(t) \, [\mathrm{d}y^2 + \mathrm{d}z^2]. \tag{23}$$

In this case teleparallel Killing vector fields for the above metric (23) become

$$\begin{aligned} X^0 &= xc_4 + c_0, & X^1 &= \frac{c_4}{\sqrt{\Phi(t)}} \int \frac{1}{\sqrt{\Phi(t)}} \mathrm{d}t + \frac{c_1}{\sqrt{\Phi(t)}}, \\ X^2 &= -z\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} c_5 + \frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} c_2, & X^3 &= y\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} c_5 + \frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}} c_3, \end{aligned} \tag{24}$$

where $c_0, c_1, c_2, c_3, c_4, c_5 \in \mathbb{R}$. The above spacetime (23) admits six linearly independent teleparallel Killing vector fields $\frac{\partial}{\partial t}$, $x\frac{\partial}{\partial t} + \Sigma(t)\frac{\partial}{\partial x}$, $\frac{1}{\sqrt{\Phi(t)}}\frac{\partial}{\partial x}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial y}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial z}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y})$, where $\Sigma(t) = \frac{1}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t$.

Case IV. Let us consider the case $\Phi = \text{constant}$ and $\Psi(t) = \Omega(t)$. With these assumptions, the metric after a suitable rescaling of x, takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{e}^{2qx} \, \Psi(t) \, [\mathrm{d}y^2 + \mathrm{d}z^2]. \tag{25}$$

The solution of equations (10)–(16) becomes

$$
\begin{aligned}
&X^0 = xc_4 + c_0, && X^1 = tc_4 + c_1,\\
&X^2 = -z\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}c_5 + \frac{-\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}c_2, && X^3 = y\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}c_5 + \frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}c_3,
\end{aligned}
\tag{26}
$$

where $c_0, c_1, c_2, c_3, c_4, c_5 \in \mathbb{R}$. The generators of above teleparallel Killing vector fields can be written as $\frac{\partial}{\partial t}$, $\frac{\partial}{\partial x}$, $t\frac{\partial}{\partial x} + x\frac{\partial}{\partial t}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial y}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}\frac{\partial}{\partial z}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\Psi(t)}}(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y})$. Here the metric (25) admits six teleparallel Killing vector fields and only three of them are multiple of the components of the inverse tetrad.

Case V. In this case the metric functions take the form $\Phi = \Phi(t)$ and $\Psi(t) = \Omega(t) =$ constant. For these restrictions, the line element takes the form

$$
\mathrm{d}s^2 = -\mathrm{d}t^2 + \Phi(t)\mathrm{d}x^2 + \mathrm{e}^{2qx}\ \beta\ [\mathrm{d}y^2 + \mathrm{d}z^2], \tag{27}
$$

where $\beta \in \mathbb{R}\setminus\{0\}$. The corresponding teleparallel Killing vector fields are obtained as

$$
\begin{aligned}
&X^0 = xc_5 + c_0, && X^1 = \frac{c_5}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t + \frac{c_1}{\sqrt{\Phi(t)}}\\
&X^2 = -z\frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}c_4 + \frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}c_2, && X^3 = y\frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}c_4 + \frac{c_3}{\sqrt{\beta}}\mathrm{e}^{-qx},
\end{aligned}
\tag{28}
$$

where $c_0, c_1, c_2, c_3, c_4, c_5 \in \mathbb{R}$. The above spacetime (27) admits six linearly independent teleparallel Killing vector fields $\frac{\partial}{\partial t}$, $x\frac{\partial}{\partial t} + \Sigma(t)\frac{\partial}{\partial x}$, $\frac{1}{\sqrt{\Phi(t)}}\frac{\partial}{\partial x}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}\frac{\partial}{\partial y}$, $\frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}\frac{\partial}{\partial z}$ and $\frac{\mathrm{e}^{-qx}}{\sqrt{\beta}}(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y})$, where $\Sigma(t) = \frac{1}{\sqrt{\Phi(t)}}\int\frac{1}{\sqrt{\Phi(t)}}\mathrm{d}t$.

4. CONCLUSION

In this paper we find the teleparallel Killing vector fields for Bianchi type V spacetimes and it turns out that the above spacetimes admit only five or six teleparallel Killing vector fields. It comes out that the minimum number of teleparallel Killing vector fields admitted by Bianchi type V spacetime is five which is greater in number from Killing vector fields in general relativity, as the minimum number of Killing vectors in general relativity is three. This shows that Bianchi type V spacetime exhibit more Killing symmetries in the presence of torsion than in the presence of curvature. At this point it must be kept in mind that in the absence of curvature and torsion a spacetime admit ten Killing vector fields. It is obvious that in the presence of curvature this particular spacetime loses seven generators of its Killing algebra and in the presence of torsion it loses only five generators of Killing algebra. Thus torsion has a weaker effect than curvature for Bianchi type V spacetime.

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