Romanian Reports in Physics, Vol. 67, No. 2, P. 700–703, 2015

PHYSICS EDUCATION

ON THE PHYSICAL ORIGIN OF THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

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Received June 17, 2014

Abstract. A simple physical insight into the origin of the anomalous magnetic moment of electron is presented. The basic idea of this new approach is that the electromagnetic mass (electrostatic self-energy) of the electron which is outside the sphere of diameter equal to the electron Compton wavelength does not contribute to the magnetic moment of the electron. A formula is derived which is similar to that obtained by quantum electrodynamics calculus of one loop contribution to anomalous part of the magnetic moment (Schwinger approximation). We give arguments that this explanation of the physical origin of the magnetic moment anomaly is compatible with the quantum electrodynamics approach.

Key words: magnetic moment of Dirac electron, electromagnetic self-energy of electron, physical origin of the anomalous magnetic moment.

1. INTRODUCTION

The anomalous magnetic moment of the electron is calculated with very high accuracy in quantum electrodynamics (QED) and agrees with the experimentally measured value to more than 10 significant figures [1]. On the other hand, there is no simple physical insight into the origin of the moment anomaly [2, 3].

It is suggested [2] that a way to qualitatively understand this effect may be found by remembering that the magnetic moment $e\hbar/2m$ of the Dirac electron is due to circular currents of the radius equal to the electron Compton wavelength $\lambda_e = h/mc$. The zero-point oscillations of the electromagnetic field and the current fluctuations induced in the "vacuum" influence these currents to a certain extent and thus cause the slight change of the magnetic moment [2]. The first calculus of the anomalous magnetic moment of the electron was made by Schwinger [4] who obtained a new formula for the magnetic moment in very good agreement with the experiment: $e\hbar/2m\cdot(1+\alpha/2\pi)$, where α is the fine structure constant.

Starting from the analysis of the magnetic moment of Dirac electron and of the electromagnetic self-energy (mass) of the electron, we propose a simple explanation for the magnetic moment anomaly of electron. Based on this explanation a straightforward calculus of the Schwinger approximation is obtained.

2. THE PHYSICAL ORIGIN OF THE MAGNETIC MOMENT ANOMALY OF THE ELECTRON

The physical interpretation of the magnetic moment $e\hbar/2m$ of the Dirac electron, originating in currents localized in a region of linear dimension $\sim \bar{\lambda}_e$, where $\bar{\lambda}_e = \hbar/mc$ is the reduced Compton wavelength, is largely treated in literature. In the non-relativistic limit the Dirac electron appears as a distribution of charge and current extended over a region of linear dimension $\bar{\lambda}_e$ [5–11]. This explains the appearance of an interaction term characteristic to the presence of a magnetic moment (interaction –μH, spin-orbit interaction) and the appearance of the Darwin term.

On the other hand, the electrostatic self-energy of the electron, due to the electric field generated by the electron, has a spatial distribution extended towards infinite. There is definite experimental evidence that some of the mass of a charged particle is electromagnetic in origin [12]. It is meaningless to separate a point charge, in particular that of electron, from its Coulomb field. Neither can be observed without the other; a charge at rest is always surrounded by a coulomb field and conversely every Coulomb field has a source [13]. Coulomb field means electrostatic energy and consequently electrostatic mass for electron. For instance the electrostatic self-energy (mass) of an electron in the region outside of a sphere of diameter 2r (where $r > \bar{\lambda}_e$) is equal to $e^2/2r$ [12, 13, 14].

Based on the above discussion we propose the following mechanism to explain the anomaly in the magnetic moment: only the mass of the electron which is inside the sphere of diameter λ_e participates at the "dynamics" (current) that generates the magnetic moment. The electron mass outside the sphere of diameter λ_e does not contribute to the magnetic moment of the electron [15]. This mass, Δmc^2 is equal to the electrostatic self-energy e^2/λ_e in the electric field generated by the electron charge in the exterior of the sphere of diameter λ_e. Instead of $e\hbar/2m$, the new expression of the magnetic moment is:

$$\mu = \frac{e\hbar}{2(m-\Delta m)} = \frac{e\hbar}{2m\left(1-\frac{\Delta m}{m}\right)} = \frac{e\hbar}{2m\left(1-\frac{\frac{e^2}{\lambda_e}}{mc^2}\right)}. \tag{1}$$

But:

$$\frac{\frac{e^2}{\lambda_e}}{mc^2} = \frac{\frac{e^2}{\bar{\lambda}_e}}{2\pi \cdot mc^2} = \frac{r_e}{2\pi \cdot \bar{\lambda}_e} = \frac{\alpha}{2\pi}\,, \tag{2}$$

where r_e is the classical electron radius $\left(r_e = e^2/mc^2\right)$ and $r_e = \alpha \cdot \bar{\lambda}_e$. Replacing (2) in relation (1) it results:

$$\mu = \frac{e\hbar}{2m\left(1 - \frac{\alpha}{2\pi}\right)} \cong \frac{e\hbar}{2m}\left(1 + \frac{\alpha}{2\pi} + \ldots\right). \tag{3}$$

The relation (3) is similar to that derived by Schwinger [4].

3. DISCUSSION

The attempts to evaluate radiative corrections to electron phenomena have encountered difficulties due to divergences attributable to self-energy and vacuum polarization effects [16]. To avoid these difficulties, at moderate energies, the Hamiltonian of current hole theory electrodynamics was transformed to exhibit explicitly the logarithmically divergent self-energy of a free electron. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron [16]. The only meaningful statements of the theory involve the sum of these two masses, which is the experimental mass of a free electron. It is underlined that however the transformation of the Hamiltonian is based on the hypothesis of a weak interaction between matter and radiation, which means that the electromagnetic mass must be a small correction $\sim \alpha \cdot m_0$ to the “mechanical” mass m_0 [16].

The above description of the electron mass structure (distribution) given in [16], in particular the weak interaction between the small electromagnetic mass of the electron and the “mechanical” mass of the electron, supports the basic idea of the present work: the electromagnetic mass outside the sphere of diameter λ_e does not participate at the localized “dynamics” which takes place inside the sphere of diameter λ_e and which generates the magnetic moment of the electron. From relation (2) it results that the electromagnetic mass outside the sphere of diameter λ_e , which is of electrostatic origin, is equal to $(\alpha/2\pi)\cdot m$. It is very probable that there is not a sharp frontier but a transition region between the “mechanical” mass, which contains the “dynamics” and this electrostatic self-energy (mass). This could explain why in the present calculus of the anomalous magnetic moment the characteristic distance is λ_e and not the reduced Compton wavelength $\bar{\lambda}_e$.

To a certain extent a similar argument is given in [17] to explain qualitatively the origin of the anomalous magnetic moment of the electron. The “dressed” electron wave function already contains its quantum corrections (interactions with virtual photons). These virtual photons carry off some of the mass of the electron while leaving its charge unaltered and this can affect the magnetic moment generated by the electron during interactions.

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