

PHASE DIAGRAM IN QUANTUM CHROMODYNAMICS

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Abstract. It is suggested that the hadronization of the quark-gluon plasma is a first-order phase transition described by a critical curve in the temperature-(quark) density plane which terminates in a critical point. Such a critical curve is derived from the van der Waals equation and its parameters are estimated. The main assumption is that quark-gluon plasma created by high-energy nucleus-nucleus collisions is a gas of ultrarelativistic quarks in equilibrium with gluons (vanishing chemical potential, indefinite number of quarks). This plasma expands, gets cool and dilute and hadronizes at a certain transition temperature and transition density. The transition density is very close to the saturation density of the nuclear matter and, it is suggested that both these points are very close to the critical point $n \simeq 1 fm^{-3}$ (quark density) and $T \simeq 200 MeV$ (temperature).

Key words: Quark-gluon plasma; phase diagram; hadronization; van der Waals equation.

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As it is well known, the Quantum Chromodynamics (QCD) developed in the past 50 years describes the quark-gluon strong interaction. The QCD lagragian is

$$L = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^{a} + \bar{\psi}_{f\alpha} \left[i\gamma^\mu (\partial_\mu \delta^{\alpha\beta} + ig t_a^{\alpha\beta} A_\mu^a) - m_f \delta^{\alpha\beta} \right] \psi_{f\beta} , \quad (1)$$

where

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c \quad (2)$$

and

$$[t_a, t_b] = i f_{ab}^c t_c. \quad (3)$$

A_μ^a are gluon potentials, with μ the Lorentz label and $a = 1 \dots 8$ the gluon label; $G_{\mu\nu}^a$ are the gluon fields, g is the coupling constant and f_{bc}^a is the structure factor of the SU(3) group. The eight 3×3 matrices t_a ($\alpha, \beta = 1, 2, 3$) are the SU(3) generators. $\psi_{f\alpha}$ are the quark fields (bispinors), labelled by flavour $f = 1 \dots 6$ and color $\alpha = 1, 2, 3$, with mass m_f ; γ^μ are the Dirac matrices. (For a review of QCD the reader can consult the recent Refs. [1]-[3]).

The Lagrangian given by equation (1) is constructed by close analogy with the Quantum Electrodynamics (QED), with two major differences: the Yang-Mills fields (quadratic term in the gluon fields in equation (2)) and the underlying SU(3) symmetry (color group). Perturbation-theory calculations indicate that effective (renormalized) coupling strength becomes weak for high-energy processes (and short distances), a phenomenon known as quark de-confinement; the quarks and gluons are free at high energy and get confined at low energy; for instance, they are bound in hadrons (mesons and baryons), which are color-singlet states (zero color charge).[4, 5] The confinement phenomenon, which is opposite to QED, is due to the non-linear Yang-Mills contribution. The usual type of calculations in QCD is the lattice-gauge theory calculations,[6] the results being often of a more qualitative nature. Various simplifications are customary in calculations, in particular the limitation to only (the lightest) u and d quarks, whose mass is set equal to zero. In this case, the chiral symmetry (handedness) of the theory should be broken at low energies. Broken symmetries, associated phase transitions and, in general, methods borrowed from condensed matter physics, are employed with the hope of getting more quantitative results in QCD.[2]

In nucleus-nucleus collisions high energy can be transferred to the internal structure of the nucleons (*e.g.*, 1 TeV per nucleon as compared with the nucleon binding energy 1 GeV), such that we may expect the liberation of quarks and gluons for a short while, followed by a quick hadronization.[7]-[11] An ultrarelativistic gas of quarks can be formed in such collisions, reaching quickly the thermal equilibrium at a temperature produced by the collision energy; we may speak of a quark-gluon plasma, with a threshold (ignition) temperature (of cca 125 – 180 MeV), and a hadron-quark-gluon plasma transition.[12]-[14] In a high-energy collision the plasma expands, its volume and number of quarks and gluons (at local equilibrium) increase, the quark density and temperature decrease, and the quarks in the outer shell hadronize.[15] It is tempting to assign a second kind to such a transition, corresponding to the broken chiral symmetry with zero-mass quarks, but the real situation involves non-vanishing masses.[16]-[19] We may only assume that the hadron-quark-gluon plasma is first order, involving a hadron binding energy, very similar with the van der Waals liquid (solid)-gas transition.[20, 21] As it is well known, the van der Waals isotherms are given by

$$(p + an^2)(1 - bn) = nT, \quad (4)$$

where p is the pressure, n is the density, T is the temperature and a, b are constants. Since the van der Waals characteristic pressure is $\sim an^2$, we may also assume $p = cn^2$ at transition, where c is a constant. The van der Waals isotherms become

$$(a + c)n(1 - bn) = T, \quad (5)$$

or

$$T = -Bn^2 + An, \quad (6)$$

where A and $B > 0$ are constants. We can see that equation (6) is equivalent with the van der Waals equation (4) for zero pressure. Since $\partial T/\partial n < 0$ for a physical transition, we can see that we should consider the above equation from $n = A/2B$ up to $n = A/B$ ($T > 0$), so we should have $A > 0$ (descending branch of the second-order trinomial in equation (6)). Under these conditions the above equation (6) gives the curve corresponding to the hadron-quark-gluon plasma transition in the (n, T) plane, the point $n_c = A/2B$, $T_c = A^2/4B$ being the critical point. Equation (6) can also be written as $T = An[1 - (B/A)n]$, where we can see that the ratio B/A is a limiting volume, which may be viewed as corresponding to a nominal "volume" v_n of the quarks, $B/A = v_n$; the quark density n can be written as $n = N/V = 1/v_q$, where v_q is the mean volume assigned to a quark in the volume V occupied by N quarks. For a mixture of various quark species, the density n can be generalized to the mean density involving partial densities.

Now we describe briefly the theoretical approach given in Ref. [15], because it gives us access to the parameters A and B in equation (6). We consider a nucleus with N_n nucleons in a volume $V_0 = R_0^3$, where R_0 is, approximately, the radius of the nucleus (for simplicity, we leave aside the numerical factor $4\pi/3$); the nucleus is subjected to a high-energy collision, with an energy (per nucleon) $E/N_n = 1$ TeV, for instance. We limit ourselves to the lightest quarks u and d , for which we may neglect their mass at these energy values ($m_u \simeq 2$ MeV, $m_d \simeq 5$ MeV). In general, the energy is dominated by gluons, except for assuming an ultrarelativistic gas of an indefinite number of quarks (vanishing chemical potential) in equilibrium with gluons, *i.e.* a quark-gluon plasma. In this case, the plasma energy (quarks plus gluons, almost equal energy) is given by

$$E = VT^4/(\hbar c)^3 \quad (7)$$

and the mean number of quarks is

$$N = VT^3/(\hbar c)^3 \quad (8)$$

(up to some immaterial numerical factors). (The pressure is $p = E/3$, the entropy is $S \simeq 4E/3T \simeq N$ and the density is given by $T = \hbar cn^{1/3}$; \hbar is Planck's constant and c is the speed of light). At the initial moment we have $E_0 = V_0 T_0^4/(\hbar c)^3$ and $N_0 = V_0 T_0^3/(\hbar c)^3$; for an energy $E/N_n = 1$ TeV we get $T_0 = 1$ GeV and $N_0 = 10^3 N_n$ ($N_n \simeq 100$), assuming a nucleon radius $a = 2$ fm and $R_0 = aN_n^{1/3}$ ($\hbar c =$

200 MeV · fm). This plasma expands in time according to the laws*

$$\begin{aligned} R &= R_0(1 + ct/R_0), & V &= V_0(1 + ct/R_0)^3, \\ T &= T_0(1 + ct/R_0)^{-3/4}, & N &= N_0(1 + ct/R_0)^{3/4}, \end{aligned} \quad (9)$$

its density goes like

$$n = N/V = n_0(1 + ct/R_0)^{-9/4} = n_0(T/T_0)^3 \quad (10)$$

or, using $N_0 = V_0 T_0^3 / (\hbar c)^3$ (i.e. $n_0 = (T_0 / \hbar c)^3$),

$$T = \hbar c n^{1/3}; \quad (11)$$

if we put here the quark density in the cold nucleus ($n \simeq N_n/V_0$, or $n \simeq 3N_n/V_0$) we get the threshold (ignition) temperature 100 – 150 MeV (for $a = 2$ fm; the values 125 – 180 MeV given above are obtained for $a = 1.5$ fm). We can see that, during expansion, plasma gets cool and the quark density decreases according to equations (9); at the same time, the energy is conserved and the entropy increases. Equation (11) defines also the chemical potential of a degenerate ultrarelativistic gas of quarks ($\mu = \hbar c n^{1/3}$).

Further on, a mechanism of condensation (hadronization) has been put forward in Refs. [15, 26] (a first-order phase transition). The transition temperature is given by

$$T_t \simeq T_q (T_q / T_m)^{1/2}, \quad (12)$$

where T_q is a characteristic quark temperature and $T_m \simeq m_0 c^2$ is a characteristic temperature given by the average mass m_0 of the condensed quarks (up to some immaterial numerical factors). It is shown in Ref. [15] that only a fraction f of the quark number is affected by hadronization (and dominates the hadronization process, with a classical Boltzmann statistics), so that we have in fact

$$T_t \simeq f^{1/2} T_q (T_q / T_m)^{1/2}. \quad (13)$$

At transition $T_t = T_q$ and

$$T_t = f^{-1} T_m, \quad \hbar c n_t^{1/3} \simeq f^{-1} m_0 c^2, \quad (14)$$

an expected and plausible result. We can see that the transition density

$$n_t = f^{-3} \left(\frac{m_0 c}{\hbar} \right)^3 \quad (15)$$

is related to the Compton wavelength $\hbar/m_0 c$ of the “average” condensed quarks. We can take $m_0 = 4$ MeV (mean mass of the u and d quarks), and get $n_t = f^{-3} (50 \text{ fm})^{-3}$. In Ref. [15] it is suggested that fraction f is given approximately by $f = 1/N_0^{1/3} \simeq 2 \times 10^{-2}$ (an argument derived from the saturation of the nuclear forces), so we

*Compare with the hydrodynamical model of particle production, Refs.[22]-[25].

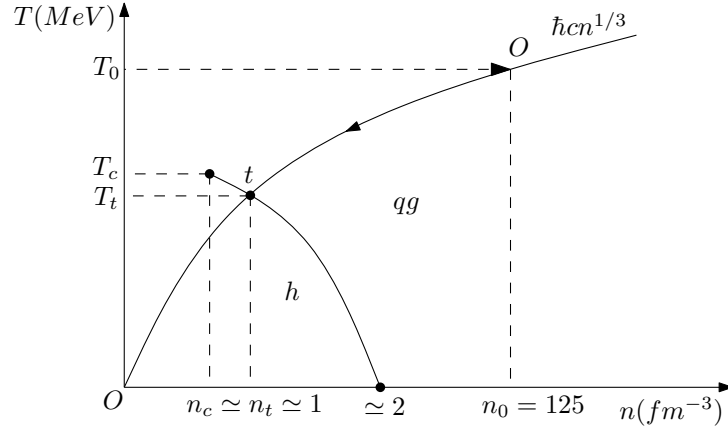


Figure 1 – Hadronization of the quark-gluon plasma. Phase diagram temperature (T) vs quark density (n). Note the hadronization curve $T = \hbar cn^{1/3}$.

get the transition density $n_t \simeq 1 \text{ fm}^{-3}$. It corresponds to a transition temperature $T_t = f^{-1} T_m \simeq 200 \text{ MeV}$ (slightly above the ignition threshold) and a transition radius $R_t = R_0 (n_t/n_0)^{-4/9} = R_0 (T_0/T_t)^{4/3} \simeq 8R_0$. This hadronization happens after $t = 5 \times 10^{-23} \text{ s}$ from the collision and involves $f N_t = f N_0 (R_t/R_0)^{3/4} \simeq 100 N_n$ hadronized quarks. (The transition implies a latent heat, discontinuities of the thermodynamic potentials, etc, as for a first-order (van der Waals) transition). We can see that the transition density $n_t = 1 \text{ fm}^{-3}$ is very close to the saturation density of the nuclear matter.

The transition temperature and density must obey the critical curve give by equation (6). With temperature measured in MeV and density measured in fm^{-3} we get

$$200 = A - B, \quad (16)$$

which gives a relation between the two parameters A and B of the critical curve. Since the transition density is very close to the saturation density of the nuclear matter, we may take tentatively the critical density $n_c = A/2B = n_t = 1 \text{ fm}^{-3}$; we get $A = 400$ and $B = 200$ (and the critical temperature $T_c = A^2/4B = T_t = 200 \text{ MeV}$); the density where the critical curve crosses the n -axis is $A/B = 2 \text{ fm}^{-3}$. It is worth noting that the critical point $n_c = 1 \text{ fm}^{-3}$, $T_c = 200 \text{ MeV}$ derived here can have a universal character; indeed, it lies on the curve $T = \hbar cn^{1/3}$ for n equal to the saturation density of the nuclear matter (e.g., $n = 1 \text{ fm}^{-3}$).

The hadronization of the quark-gluon plasma is shown in Fig. 1. According to the description given above the hadronization process starts with the creation of a quark-gluon plasma at the initial quark density n_0 (e.g., $\simeq 10^3 N_n/V_0 = 125 \text{ fm}^{-3}$) and temperature T_0 (e.g., 1 GeV), followed by a cooling of the plasma along the

curve $T = \hbar cn^{1/3}$ until it encounters the critical curve at the transition point n_t (e.g., 1 fm^{-3}) and T_t (e.g., 200 MeV) where the hadronization occurs. With our units (MeV and fm) the curve $T = \hbar cn^{1/3}$ reads $T = 200n^{1/3}$. The transition temperature is very close to the saturation density of the nuclear matter and, very likely, it is very close to the critical density.[27]

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