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LOCAL FRACTIONAL HOMOTOPY PERTURBATION METHOD FOR SOLVING FRACTAL PARTIAL DIFFERENTIAL EQUATIONS ARISING IN MATHEMATICAL PHYSICS

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Abstract. In this article, we propose and apply a local fractional homotopy perturbation method, which is and extended form of the classical homotopy perturbation method. We discuss convergence aspect of the technique and present two illustrative examples to show the efficiency of the proposed method in order to find the approximate solutions for some local fractional differential equations arising in mathematical physics.

Key words: local fractional homotopy perturbation method, convergence, approximate solution, diffusion equation, wave equation, local fractional derivative.

1. INTRODUCTION

The homotopy perturbation technique, which was systematically structured by He [1–3], was applied to solve several nonlinear problems in science and engineering. For examples, Ganji and Kachapi [4] discussed the nonlinear equations in fluid mechanics, Abbasbandy [5] presented the solutions for certain functional integral equations, Khan and Wu [6] derived the solutions for the homogeneous and nonhomogeneous advection equations, and Xu [7] obtained the solution for the boundary layer equation in unbounded domain.

Recently, the local fractional calculus was successfully used to describe the non-differentiable problems arising in mathematical physics, such as the diffusion equations on Cantor space-time [8], the linear and nonlinear local fractional Korteweg-de Vries equation [9], the fractal heat-conduction equation [10, 11], wave equation in fractal strings [12] and Laplace equation [13].

The local fractional derivative operator is given as follows [8–13]:

$$D^{\alpha} f\left(x_0\right)=\frac{\mathrm{d}^{\alpha} f\left(x_0\right)}{\mathrm{d} x^{\alpha}}=\frac{\Delta^{\alpha}\left(f\left(x\right)-f\left(x_0\right)\right)}{\left(x-x_0\right)^{\alpha}}, \tag{1}$$

where

$$\Delta^{\alpha}\left(f\left(x\right)-f\left(x_0\right)\right)\cong\Gamma\left(1+\alpha\right)\left[f\left(x\right)-f\left(x_0\right)\right]. \tag{2}$$

Moreover, the local fractional partial derivatives of higher order is defined as follows [8–13]:

$$\frac{\partial^{k\alpha}}{\partial x^{k\alpha}} f\left(x, y\right)=\overbrace{\frac{\partial^{\alpha}}{\partial x^{\alpha}}\cdots\frac{\partial^{\alpha}}{\partial x^{\alpha}}}^{k \text{ times}} f\left(x, y\right). \tag{3}$$

It is an interesting research topic to find solutions of various families of local fractional differential equations. There are some analytical and approximate techniques, such as the local fractional decomposition method [8, 12, 13], the local fractional Laplace variational iteration method [10], the Yang-Laplace transform method [11], the local fractional function decomposition method [13, 14], the Sumudu transform method [15], and so on. The aim of this article is to extend the homotopy perturbation method in the sense of local fractional derivative in order to solve some local fractional differential equations arising in mathematical physics. The structure of the paper is as follows. In Section 2, the local fractional homotopy perturbation method is analyzed. In Section 3, two illustrative examples are given. Finally, the conclusions are presented in Section 4.

2. ANALYSIS OF THE METHOD

In order to show the local fractional homotopy perturbation method, we consider the local fractional differential equation of the type

$$L_{\alpha}\left(u^{\alpha}\right)=0,\ u\in R, \tag{4}$$

where L_{α} is a local fractional differential operator.

We define a convex non-differentiable homotopy $H_{\alpha}\left(u, p\right)$ by

$$H_{\alpha}\left(u, p\right)=\left(1-p^{\alpha}\right)\left(L_{\alpha}\left(u^{\alpha}\right)-L_{\alpha}\left(u_0^{\alpha}\right)\right)+p^{\alpha} L_{\alpha}\left(u^{\alpha}\right),\ u\in R,\ p\in\left[0, 1\right]. \tag{5}$$

Or, equivalently, by

$$H_{\alpha}\left(u, p\right)=L_{\alpha}\left(u^{\alpha}\right)-L_{\alpha}\left(u_0^{\alpha}\right)+p^{\alpha} L_{\alpha}\left(u_0^{\alpha}\right),\ u\in R,\ p\in\left[0, 1\right], \tag{6}$$

where p is an imbedding parameter and u_0 is an initial approximation of (5).

Let $H_\alpha(u,p)=0$. Then it is obvious that

$$H_\alpha(u,0)=L_\alpha(u^\alpha)-L_\alpha(u_0^\alpha)=0, \tag{7}$$

$$H_\alpha(u,1)=L_\alpha(u_0^\alpha)=0. \tag{8}$$

In the non-differentiable homotopy, this is called the non-differentiable deformation and $L_\alpha(u^\alpha)-L_\alpha(u_0^\alpha)$ and $L_\alpha(u_0^\alpha)$ are called the non-differentiable homotopies. By applying the non-differentiable perturbation method, the solution of (7) can be expressed as follows:

$$u^\alpha=u_0^\alpha+p^\alpha u_1^\alpha+p^{2\alpha}u_2^\alpha+p^{3\alpha}u_3^\alpha+....=\sum_{i=0}^{n}p^{i\alpha}u_i^\alpha\ . \tag{9}$$

Substituting (9) into (5), we obtain

$$H_\alpha\left(\sum_{i=0}^{n}p^iu_i,p\right)=\left(1-p^\alpha\right)\left(L_\alpha\left(\sum_{i=0}^{n}p^iu_i\right)-L_\alpha(u_0)\right)+p^\alpha L_\alpha\left(\sum_{i=0}^{n}p^iu_i\right). \tag{10}$$

In order to get the approximate solution of (4), we can expand $L_\alpha(u)$ into a local fractional Taylor series in the form:

$$L_\alpha(u^\alpha)=L_\alpha\left(\sum_{i=0}^{n}p^{i\alpha}u_i^\alpha\right)=$$

$$=L_\alpha(u_0^\alpha)+\frac{\mathrm{d}^\alpha\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^\alpha}\frac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^\alpha}{\Gamma(1+\alpha)}+\frac{\mathrm{d}^{2\alpha}\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^{2\alpha}}\frac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^{2\alpha}}{\Gamma(1+2\alpha)}+.... \tag{11}$$

$$+\frac{\mathrm{d}^{n\alpha}\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^{n\alpha}}\frac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^{n\alpha}}{\Gamma(1+n\alpha)}+....$$

Thus, upon substituting (10) into (7), we have

$$H_\alpha\left(\sum_{i=0}^{n}p^iu_i,p\right)=$$

$$=\left(1-p^\alpha\right)\left(\begin{array}{l}\dfrac{\mathrm{d}^\alpha\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^\alpha}\dfrac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^\alpha}{\Gamma(1+\alpha)}+\dfrac{\mathrm{d}^{2\alpha}\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^{2\alpha}}\dfrac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^{2\alpha}}{\Gamma(1+2\alpha)}\\+\dfrac{\mathrm{d}^{n\alpha}\left(L_\alpha(u_0^\alpha)\right)}{\mathrm{d}u^{n\alpha}}\dfrac{\left(\sum_{i=0}^{n}p^iu_i-u_0\right)^{n\alpha}}{\Gamma(1+n\alpha)}\end{array}\right)+\ ,$$

$$+p^{\alpha}\left(\begin{array}{l} L_{\alpha}\left(u_0^{\alpha}\right)+\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{\alpha}}{\Gamma(1+\alpha)}+\frac{\mathrm{d}^{2\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{2\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{2\alpha}}{\Gamma(1+2\alpha)} \\ +\frac{\mathrm{d}^{n\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{n\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{n\alpha}}{\Gamma(1+n\alpha)}+.... \end{array}\right), \tag{12}$$

which reduces to

$$\begin{aligned} &H_{\alpha}(u,0)=L_{\alpha}\left(u^{\alpha}\right)-L_{\alpha}\left(u_0^{\alpha}\right)= \\ &=\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{\alpha}}{\Gamma(1+\alpha)}+\frac{\mathrm{d}^{2\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{2\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{2\alpha}}{\Gamma(1+2\alpha)}+ \\ &+\frac{\mathrm{d}^{n\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{n\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{n\alpha}}{\Gamma(1+n\alpha)}+...=0, \end{aligned} \tag{13}$$

and

$$\begin{aligned} &H_{\alpha}(u,1)=L_{\alpha}\left(u^{\alpha}\right)= \\ &=L_{\alpha}\left(u_0^{\alpha}\right)+\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{\alpha}}{\Gamma(1+\alpha)}+\frac{\mathrm{d}^{2\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{2\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{2\alpha}}{\Gamma(1+2\alpha)} \\ &+\frac{\mathrm{d}^{n\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{n\alpha}}\frac{\left(\sum_{i=0}^{n}p^{i}u_i-u_0\right)^{n\alpha}}{\Gamma(1+n\alpha)}+...=0. \end{aligned} \tag{14}$$

Using the expression (12), we find that

$$p^{0\alpha}:L_{\alpha}\left(u^{\alpha}\right)-L_{\alpha}\left(u_0^{\alpha}\right)=0, \tag{15}$$

$$p^{1\alpha}:\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}\frac{u_1^{\alpha}}{\Gamma(1+\alpha)}+L_{\alpha}\left(u_0^{\alpha}\right)=0, \tag{16}$$

$$p^{2\alpha}:\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}\frac{u_2^{\alpha}}{\Gamma\left(1+\alpha\right)}+\frac{\mathrm{d}^{2\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{2\alpha}}\frac{u_1^{2\alpha}}{\Gamma\left(1+2\alpha\right)}=0\,. \tag{17}$$

In view of (16), we have

$$u_1^{\alpha}=-\frac{\Gamma\left(1+\alpha\right)L_{\alpha}\left(u_0^{\alpha}\right)}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}\,. \tag{18}$$

Hence, its first approximate formula is given by

$$u^{\alpha}=u_0^{\alpha}-p^{\alpha}\frac{\Gamma\left(1+\alpha\right)L_{\alpha}\left(u_0^{\alpha}\right)}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_0^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}\,. \tag{19}$$

When $p=1$, applying (19) gives us the following iterative formula:

$$u_{n+1}{}^{\alpha}=u_n^{\alpha}-\frac{\Gamma\left(1+\alpha\right)L_{\alpha}\left(u_n^{\alpha}\right)}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_n^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}\,, \tag{20}$$

which is the well-known local fractional Newton iteration formula [14] and it is known to be convergent. Using (20), we get the local fractional Newton-like iteration formula with second-order approximation in the following form:

$$u_{n+1}^{\alpha}=u_n^{\alpha}-\frac{\Gamma\left(1+\alpha\right)L_{\alpha}\left(u_n^{\alpha}\right)}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_n^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}-\frac{\Gamma\left(1+\alpha\right)}{\Gamma\left(1+2\alpha\right)}\frac{\frac{\mathrm{d}^{2\alpha}\left(L_{\alpha}\left(u_n^{\alpha}\right)\right)}{\mathrm{d}u^{2\alpha}}}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_n^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}\left(\frac{\Gamma\left(1+\alpha\right)L_{\alpha}\left(u_n^{\alpha}\right)}{\frac{\mathrm{d}^{\alpha}\left(L_{\alpha}\left(u_n^{\alpha}\right)\right)}{\mathrm{d}u^{\alpha}}}\right)^{2\alpha}. \tag{21}$$

When $p\rightarrow 1$, we have the approximate solution of the form:

$$u^{\alpha}=\lim_{p\rightarrow 1}\sum_{i=0}^{n}p^{i\alpha}u_i^{\alpha}=\sum_{i=0}^{n}u_i^{\alpha}\,. \tag{22}$$

We notice that the classical homotopy perturbation method presented in [1–7] is obtained when the fractal dimension α is set equal to 1.

3. TWO ILLUSTRATIVE EXAMPLES

In this section, we present two illustrative examples of the local fractional partial differential equations arising in mathematical physics.

Example 1. Let us consider the following linear local fractional diffusion equation in one-dimensional case [8]:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \ t > 0, \ x \in R, \ 0 < \alpha < 1, \tag{23}$$

subject to the initial condition:

$$u(x,0) = E_{\alpha}\left(x^{\alpha}\right). \tag{24}$$

According to the local fractional homotopy perturbation method, we structure the non-differentiable homotopy given as follows:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = p^{\alpha}\left(\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}\right), \ 0 \le p \le 1, \tag{25}$$

and we have the following set of linear local fractional partial differential equations:

$$p^{0\alpha}: \quad \frac{\partial^{\alpha} u_0(x,t)}{\partial t^{\alpha}} = 0, \ u_0(x,t) = E_{\alpha}\left(x^{\alpha}\right), \tag{26}$$

$$p^{1\alpha}: \quad \frac{\partial^{\alpha} u_1(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2\alpha} u_0(x,t)}{\partial x^{2\alpha}}, \ u_1(x,0) = 0, \tag{27}$$

$$p^{2\alpha}: \quad \frac{\partial^{\alpha} u_2(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2\alpha} u_1(x,t)}{\partial x^{2\alpha}}, \ u_2(x,0) = 0, \tag{28}$$

$$p^{3\alpha}: \quad \frac{\partial^{\alpha} u_3(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2\alpha} u_2(x,t)}{\partial x^{2\alpha}}, \ u_3(x,0) = 0, \tag{29}$$

$$p^{4\alpha}: \quad \frac{\partial^{\alpha} u_4(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2\alpha} u_3(x,t)}{\partial x^{2\alpha}}, \ u_4(x,0) = 0, \tag{30}$$

....

and so on.

Calculating from the above equations for $u_0, u_1, u_2, u_3,$ and $u_4,$ we derive some components of the local fractional homotopy perturbation solution for (23) as follows:

$$
\begin{aligned}
& u_0(x,t)=E_\alpha\left(x^\alpha\right), \\
& u_1(x,t)=\frac{t^\alpha}{\Gamma(1+\alpha)}E_\alpha\left(x^\alpha\right), \\
& u_2(x,t)=\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha\left(x^\alpha\right), \\
& u_3(x,t)=\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha\left(x^\alpha\right), \\
& u_4(x,t)=\frac{t^{4\alpha}}{\Gamma(1+4\alpha)}E_\alpha\left(x^\alpha\right), \\
& \ldots \\
& u_i(x,t)=\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}E_\alpha\left(x^\alpha\right),
\end{aligned} \tag{31}
$$

and so on.

Finally, the non-differentiable approximation for (23) becomes

$$
u(x,t)=\sum_{i=0}^{n}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}E_\alpha\left(x^\alpha\right)=E_\alpha\left(t^\alpha\right)E_\alpha\left(x^\alpha\right) \tag{32}
$$

and its graph is illustrated in Fig. 1.

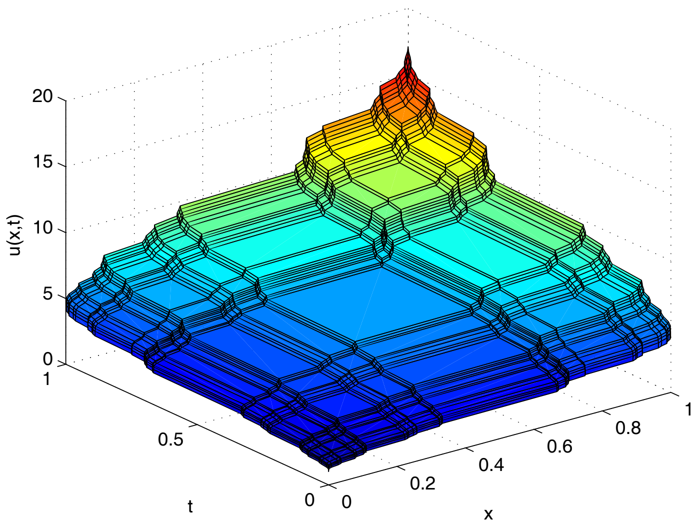

Fig. 1 – Plot of the non-differentiable solution of (23) with $\alpha=\ln 2/\ln 3$.

Example 2. Let us consider the following wave equation in fractal strings [12]:

$$\frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}-\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}=0,\ t>0,\ x\in R,\ 0<\alpha<1, \tag{33}$$

subject to the initial condition:

$$u(x,0)=0,\ \frac{\partial^{\alpha}u(x,0)}{\partial t^{\alpha}}=E_{\alpha}\left(x^{\alpha}\right). \tag{34}$$

According to the local fractional homotopy perturbation method, we structure the non-differentiable homotopy given as follows:

$$\frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}=p^{\alpha}\left(\frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}-\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}\right),\ 0\le p\le 1, \tag{35}$$

and we get the following set of linear local fractional partial differential equations:

$$p^{0\alpha}:\ \ \frac{\partial^{2\alpha}u_0(x,t)}{\partial t^{2\alpha}}=0,\ u_0(x,0)=0,\ \frac{\partial^{\alpha}u_0(x,0)}{\partial t^{\alpha}}=E_{\alpha}\left(x^{\alpha}\right), \tag{36}$$

$$p^{1\alpha}:\ \frac{\partial^{2\alpha}u_1(x,t)}{\partial t^{2\alpha}}=\frac{\partial^{2\alpha}u_0(x,t)}{\partial x^{2\alpha}},\ u_1(x,0)=0,\ \frac{\partial^{\alpha}u_1(x,0)}{\partial t^{\alpha}}=0, \tag{37}$$

$$p^{2\alpha}:\ \frac{\partial^{2\alpha}u_2(x,t)}{\partial t^{2\alpha}}=\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}},\ u_2(x,0)=0,\ \frac{\partial^{\alpha}u_2(x,0)}{\partial t^{\alpha}}=0, \tag{38}$$

$$p^{3\alpha}:\ \frac{\partial^{2\alpha}u_3(x,t)}{\partial t^{2\alpha}}=\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}},\ u_3(x,0)=0,\ \frac{\partial^{\alpha}u_3(x,0)}{\partial t^{\alpha}}=0, \tag{39}$$

$$\begin{aligned}&p^{4\alpha}:\ \frac{\partial^{2\alpha}u_4(x,t)}{\partial t^{2\alpha}}=\frac{\partial^{2\alpha}u_3(x,t)}{\partial x^{2\alpha}},\ u_4(x,0)=0,\ \frac{\partial^{\alpha}u_4(x,0)}{\partial t^{\alpha}}=0,\\&....\\&p^{i\alpha}:\ \frac{\partial^{2\alpha}u_i(x,t)}{\partial t^{2\alpha}}=\frac{\partial^{2\alpha}u_{i-1}(x,t)}{\partial x^{2\alpha}},\ u_{i-1}(x,0)=0,\ \frac{\partial^{\alpha}u_{i-1}(x,0)}{\partial t^{\alpha}}=0,\end{aligned} \tag{40}$$

and so on.

Evaluating the above equations for $u_0,u_1,u_2,u_3,$ and u_4, some components of the local fractional homotopy perturbation solution for (23) become

$$
\begin{aligned}
&u_0(x,t)=\frac{t^{\alpha}}{\Gamma(1+\alpha)}E_\alpha(x^\alpha),\\
&u_1(x,t)=\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(x^\alpha),\\
&u_2(x,t)=\frac{t^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(x^\alpha),\\
&u_3(x,t)=\frac{t^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(x^\alpha),\\
&u_4(x,t)=\frac{t^{9\alpha}}{\Gamma(1+9\alpha)}E_\alpha(x^\alpha),\\
&\ldots\\
&u_i(x,t)=\frac{t^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}E_\alpha(x^\alpha),
\end{aligned}
\tag{41}
$$

and so on.

Finally, the non-differentiable approximation for (23) assumes the following form:

$$
u(x,t)=\sum_{i=0}^{n}\frac{t^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}E_\alpha(x^\alpha)=\sinh_\alpha(t^\alpha)E_\alpha(x^\alpha)
\tag{42}
$$

and its graph is shown in Fig. 2.

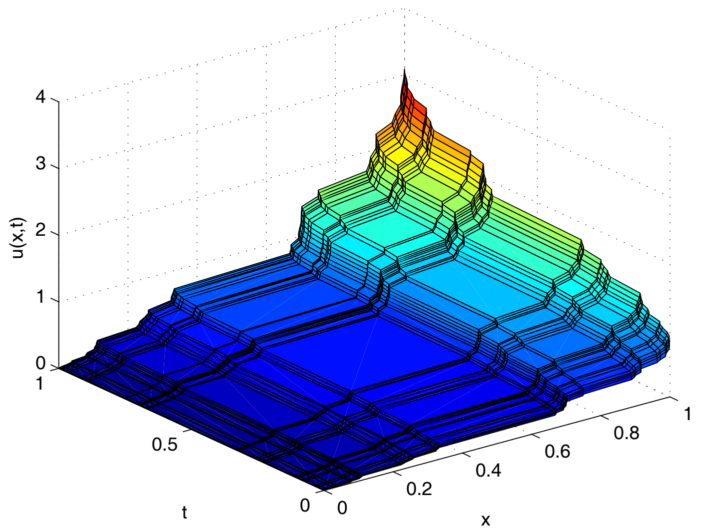

Fig. 2 – Plot of the non-differentiable solution of (33) with $\alpha=\ln 2/\ln 3$.

4. CONCLUSIONS

In this work, a new and novel approach for solving the local fractional differential equations arising in mathematical physics is suggested. The local fractional homotopy perturbation method and its convergence are discussed. Two illustrative examples are also given to show the effectiveness of the tool for finding the local fractional differential equations.

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