

## EVOLUTION OF PLASMA TURBULENCE BEYOND THE QUASILINEAR REGIME; A SEMI-ANALYTICAL STUDY\*

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*Abstract.* The interdependence of drift turbulence evolution and of ion diffusion is studied by developing a semi-analytical method, which is able to describe the complex processes that appear beyond the quasilinear regime.

*Key words:* fusion physics, plasma turbulence, drift instabilities.

### 1. INTRODUCTION

The turbulence generated in magnetically confined plasmas has a complex behaviour with generation of large scale correlations, increase of order and appearance of zonal flow modes [1, 2], which leads to the nonlinear damping of turbulence. Most of these conclusions are based on numerical simulations or on simplified models. The theoretical studies apply to the quasilinear regimes characterized by random behaviour with negligible degree of coherence. They are not able to describe turbulence evolution and transport in the nonlinear stage where quasi-coherent structures are generated.

Analytical results for test modes on turbulent plasmas were recently obtained [3, 4] starting from the first principle description of plasma. A Lagrangian approach is developed, which extends the type of methods initiated by Dupree [5] to the nonlinear regime, which is characterized by ion trapping or eddy in the stochastic potential. We have shown that there is a sequence of effects, which appear at different stages of evolution as transitory processes and that the drift turbulence saturates in an oscillatory state. Zonal flow modes are self consistently generated, but they do not have (as generally considered) the main role in the turbulence decay. We have shown that drift turbulence damping is produced by the ion flows, which appear due to ion trapping combined with the diamagnetic velocity.

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The present study is a development of the results presented in [4], which focuses on the effects of ion diffusion on the test modes. We show that a complex interaction between transport and turbulence evolution appears in drift turbulence.

The paper is organized as follows. Sections 2 and 3 present a short review of the problem of test modes on turbulent plasmas and of the approach developed in reference [4]. The effects of ion diffusion at each stage in the evolution of drift turbulence are analyzed in Section 4. The last section contains the conclusions drawn from this work.

## 2. TEST MODES ON TURBULENT PLASMAS

Low-frequency drift type turbulence has the strongest influence on the energy and particle transport in magnetically confined plasmas. These instabilities are generated due to the non-uniformity of density and/or temperature that always exists in plasmas. The universal drift instability is analyzed in the collisionless limit, in a constant magnetic field (along  $z$  axis,  $\mathbf{B} = B\mathbf{e}_z$ ). Plasma average density  $n_0(x)$  has a gradient with characteristic length  $L_n = n_0/|dn_0/dx|$  much larger than the wave length of the drift modes. Drift modes are represented by wave type potential  $\delta\phi(x, y, z, t) = \phi_{k\omega} \exp(ik_x x + ik_y y + ik_z z - i\omega t)$ , where  $k_i$  are the components of the wave number and  $\omega$  is the frequency (with imaginary part  $\gamma$ ). They have  $k_z \ll k_x, k_y$  and  $v_{Ti} \ll \omega/|k_z| \ll v_{Te}$ , where  $v_{Te}$ ,  $v_{Ti}$  are the thermal velocities of electrons and ions. The solution of the dispersion relation, which is the quasineutrality condition, is

$$\bar{\omega} = \frac{\bar{k}_y}{1 + \bar{k}_\perp^2}, \quad (1)$$

$$\bar{\gamma} = \gamma_0 \bar{\omega} (\bar{k}_y - \bar{\omega}), \quad (2)$$

where [6]:  $\bar{k}_i = k_i \rho_s$ ,  $\bar{k}_\perp = \sqrt{\bar{k}_x^2 + \bar{k}_y^2}$ ,  $\bar{\omega} = \omega L_n / c_s$ ,  $\bar{\gamma} = \gamma L_n / c_s$ ,  $\gamma_0 = \sqrt{\pi/2} (c_s / L_n) / |k_z| v_{Te}$ ,  $\rho_s = c_s / \Omega_i$ ,  $c_s = \sqrt{T_e / m_i}$ ,  $T_e$  - the electron temperature,  $m_i$  - the ion mass,  $e$  - the absolute value of electron charge. Drift modes are unstable ( $\gamma > 0$ ) if  $\bar{\omega} < \bar{k}_y$ , which is always true as seen in Eq. (1). The maximum growth rate is for  $\bar{\omega} = \bar{k}_y / 2$ , which corresponds to  $k_\perp \rho_s = 1$ , and for  $\bar{k}_y > \bar{k}_x$ . The  $\rho_s$  dependence that appears in (1) yields from the ion

polarization drift

$$\mathbf{u}_p = \frac{m_i}{eB^2} \partial_t \mathbf{E}_\perp. \quad (3)$$

The solution in the limit  $\rho_s = 0$  is  $\omega = k_y V_{*e}$ ,  $\gamma = 0$ , which represents the stable drift waves. For an arbitrary initial condition  $\phi_0$ , this solution is

$$\phi(x, y, z, t) = \phi_0(x, y - V_{*e}t, z), \quad (4)$$

which shows that the initial potential  $\phi_0$  does not change its shape, but it only moves with the diamagnetic velocity  $\mathbf{V}_{*e} = V_{*e} \mathbf{e}_y$ ,  $V_{*e} = T_e / (eBL_n) = \rho_s c_s / L_n$ .

Drift instabilities appear for a large range of wave numbers and produce a turbulent potential. Consequently, the results (1–2) obtained for a quiescent plasma are not relevant. Test modes on a turbulent plasma have to be considered, taking a background potential  $\phi_b(\mathbf{x}, z, t)$ . The growth rates  $\gamma$  and the frequencies  $\omega$  of the test modes are determined as functions of the statistical characteristics of  $\phi_b$ . The potential  $\phi_b$  is taken as the  $\rho_s = 0$  solution (4). The modification of potential shape and amplitude appears due to polarization drift on a larger time scale of the order  $1/\gamma$ . The test mode studies of turbulence are based on this time scale separation.

The frequency and the growth rates as functions of the statistical characteristics of the background turbulence were determined in [4] starting from the first principle description of the drift turbulence, based on the drift kinetic equation. The ion response depends on the turbulence, while the electron perturbation is the same as in a quiescent plasma. The dispersion relation for test modes on turbulent plasma is

$$-\left(k_y V_{*e} - \omega \rho_s^2 k_\perp^2\right) \bar{\Pi}^i = 1 + i \sqrt{\frac{\pi}{2}} \frac{\omega - k_y V_{*e}}{|k_z| v_{Te}}, \quad (5)$$

where the propagator is

$$\bar{\Pi}^i = i \int_{-\infty}^t d\tau M(\tau) \exp[-i\omega(\tau - t)] \quad (6)$$

and  $M(\tau)$  is

$$M(\tau) \equiv \left\langle \exp \left[ i \mathbf{k} \cdot (\mathbf{x}(\tau) - \mathbf{x}) - \int_\tau^t d\tau' \nabla \cdot \mathbf{u}_p(\mathbf{x}(\tau')) \right] \right\rangle. \quad (7)$$

The integral in this equation and the average  $\langle \dots \rangle$  are along the characteristics of

the ion equation, which are the ion trajectories obtained from

$$\frac{d\mathbf{x}(\tau)}{d\tau} = -\frac{\nabla\phi(\mathbf{x} - \mathbf{V}_{*e}t) \times \mathbf{e}_z}{B}, \quad (8)$$

integrated backwards in time with the condition  $\mathbf{x}(t) = \mathbf{x}$  at  $\tau = t$ . The last term in the argument of the exponential in Eq. (7) accounts for the compressibility effects in the background turbulence produced by the polarization drift. This term has zero average, but its correlation with the displacements is important in the nonlinear evolution of the turbulence.

According to reference [4], the function  $M(\tau)$  defined by Eq. (7) embeds all the effects of the background turbulence. It depends on the background potential through the statistical characteristics of the trajectories (8), which determine the average, and through the compressibility term. In the case of quiescent plasmas  $M = 1$ .

### 3. STATISTICS OF THE CHARACTERISTICS

The main difficulty in determining the function  $M(\tau)$  is due to the complexity of the trajectories obtained from Eq. (9). They appear as stochastic sequences of trapping (or eddying) events and large jumps. This behaviour is the consequence of the Hamiltonian type of the equations (8) with  $x$  and  $y$  conjugated variables. The Lagrangian potential is invariant for  $V_{*e} = 0$  and infinite correlation time. The trapping or eddying events are the consequence of this invariance that is approximately maintained when the perturbations are small enough. The first statistical methods that are in agreement with potential invariance are the decorrelation trajectory method DTM [7] and the nested subensemble approach NSA [8]. These methods reduce the problem of determining the statistics of the stochastic trajectories to the calculation of weighted averages of some smooth, deterministic trajectories determined from the Eulerian correlation (EC) of the stochastic potential. NSA is the development of DTM as a systematic expansion that validates DTM and yields much more statistical information. These are semi-analytic methods, which need PC level computations of few minutes.

A series of studies ([8] and the references there in, [9, 10]) have shown that trapping strongly influences the statistics of trajectories leading to memory effects, anomalous diffusion regimes, quasi-coherent behaviour and non-Gaussian distribution. The trapped trajectories form structures similar to fluid vortices.

The function  $M(\tau)$  was determined in [4] using the results of DTM and NSA as function of the EC of the background potential. The latter was initially

modelled for a very small amplitude of the turbulence, according to the frequencies and the growth rates of drift modes in quiescent plasma (1–2)

$$E(\mathbf{x}, t) = \beta^2 \partial_y \left[ \exp \left( -\frac{x^2}{2\lambda_x^2} - \frac{y'^2}{2\lambda_y^2} - \frac{t}{\tau_c} \right) \frac{\sin k_0 y'}{k_0} \right], \quad (9)$$

where  $\beta$  is the amplitude of potential fluctuations,  $\lambda_i$  are the correlation lengths,  $k_0$  is the dominant wave number and  $\tau_c$  is the correlation time. The drift of the background potential with the diamagnetic frequency appearing in Eq. (5) leads to  $y' = y - V_{*e} t$  in the Eq. (9). The EC of the potential determines the Eulerian correlations of the velocity components and their amplitudes [7], which are

$$\begin{aligned} V_x^2 = \langle v_x^2 \rangle &= -\frac{\partial^2 E(\mathbf{0}, 0)}{\partial y^2} = \left( \frac{\beta}{B\lambda_y} \right)^2 (k_0^2 + 3), \\ V_y^2 = \langle v_y^2 \rangle &= -\frac{\partial^2 E(\mathbf{0}, 0)}{\partial x^2} = \left( \frac{\beta}{B\lambda_y} \right)^2 \frac{1}{a}, \end{aligned} \quad (10)$$

where  $a = (\lambda_x / \lambda_y)^2$  is the anisotropy parameter.

We have shown that the parameters are changed during the evolution of the turbulence, but the shape of the EC is approximately maintained. It essentially represents the property of drift instability that cannot develop modes with  $k_y = 0$ . The shape of the total EC is modified only in the strongly nonlinear regime, where a different type of modes, the zonal flow modes, are generated. The modification consists of adding to the drift mode contribution (9) of a term  $E_{zf}$  representing the EC of the zonal flow modes. These modes have  $k_x \neq 0$  and their correlation  $E_{zf}$  is similar to Eq. (9), but with the coordinates  $x$  and  $y$  inverted.

The parameters in Eq. (9) determine several characteristic times. The motion of the potential with the diamagnetic velocity defines the diamagnetic time  $\tau_* = \lambda_y / V_{*e}$ . The correlation time describes the change of the shape of the potential and it is of the order  $\tau_c \cong \gamma^{-1} \ll \tau_*$ . The time of flight (or eddy time) is  $\tau_{fl} = \lambda_y / V_y$ . Trajectory trapping appears when  $\tau_{fl}$  is smaller than  $\tau_*$ . The trapping parameter for drift turbulence is

$$K_* = \tau_* / \tau_{fl} = V_y / V_{*e}. \quad (11)$$

Trapping occurs when  $K_* > 1$ . The fraction of trapped trajectories  $n_{tr}$  and

the maximum size of trajectory structures are increasing functions of  $K_*$ . The distribution of displacements obtained from Eq. (8) strongly depends on the ordering of the characteristic times of the stochastic process.

#### 4. EFFECTS OF ION DIFFUSION AND TRAPPING

We have analysed in [4] the effects of trapping on the evolution of drift turbulence, neglecting the ion diffusion, which was considered small. We show here that the test modes are influenced both by the quasi-coherent component of the ion motion (represented by the structures of trapped trajectories) and by the random component (represented by trajectory diffusion).

The growth rates and the frequencies of the drift mode are determined as functions of the EC. The EC is then modified corresponding to  $\omega(\mathbf{k})$  and  $\gamma(\mathbf{k})$ , which show the tendencies in the evolution of the turbulence.

The development of the drift turbulence starting from a very small perturbation with large wave number spectrum is characterized by three regimes. The small amplitude (or quasi-linear) regime corresponds to  $K_* \ll 1$  and to completely random ion trajectories. The weakly nonlinear regime is characterised by the existence of ion trapping or eddying, but with small fraction of trapped trajectories  $n_{tr} \ll 1$ . Trajectory vortical structures are generated in this conditions. The strongly nonlinear regime appears when the fractions of the trapped and free trajectories are comparable. The drift of the potential with the diamagnetic velocity becomes important and it generates fluxes of ions in opposite directions with the absolute value  $n_{tr}V_{*e}$ , which split the distribution of displacements in two parts.

The diffusion of the ions determine in all these regimes a damping contribution in the growth rate, which will be quantitatively analysed in the following subsections.

##### 4.1. THE QUASI-LINEAR REGIME

The trajectories are Gaussian in the quasi-linear regimes with  $K_* \ll 1$  and the trapping does not exist. The solution of the dispersion equation (5) for the growth rate is

$$\bar{\gamma} = \gamma_0 \bar{\omega} (\bar{k}_y - \bar{\omega}) - \bar{k}_i^2 \bar{D}_i, \quad (12)$$

while the frequency is given by Eq. (1), as in quiescent plasma. The diffusion coefficients are normalized,  $\bar{D}_i = D_i / (\rho_s V_{*e})$ . This equation, which is the well-known result of Dupree [5], shows that ion trajectory diffusion in the background turbulence contributes with a negative (damping) term.

The usual evaluation of the diffusion coefficients yields anisotropic diffusive transport with  $D_x(t) \sim |\partial_y E(V_{*e} \tau_c)| / V_{*e}$  and  $D_y = V_y^2 \tau_* = V_y^2 \lambda_y / V_{*e}$ . The diffusive damping in the growth rate (12) is mainly determined by the diffusion along the diamagnetic velocity  $k_y^2 D_y$ , which is much larger than  $k_x^2 D_x$ , because both factors are larger. As the diffusion coefficient increases (quadratically) with the amplitude of the turbulence, the second term in Eq. (12) can compensate the first term leading to saturation of the growth rate. The solution of the equation  $\bar{\gamma} = 0$  determines the amplitude of the potential at saturation,  $\beta_s$ . It is

$$\frac{\beta_s}{B} = \sqrt{\frac{\tau_{ze}}{\tau_*}} \lambda_x V_{*e} \frac{k_\perp}{1 + k_\perp^2}, \quad (13)$$

where  $\tau_{ze}$  is the time of decorrelation of the electrons by the motion along the magnetic field. Due to the large velocity of the electrons,  $\tau_{ze}$  is much smaller than the diamagnetic time and thus  $\beta_s$  corresponds to  $K_* \ll 1$ . This relation implies a simple quasilinear evolution of drift turbulence with low level saturation and absence of trajectory trapping, which is not in agreement with numerical simulations where ion eddy is observed.

We show that the special shape of the EC of the drift turbulence determines much different diffusion coefficients in the quasilinear regime, which cannot saturate the growth rate of the modes and that turbulence reaches the nonlinear stage.

We first note that the drift turbulence has  $\lambda_y < \lambda_x$ , which makes the anisotropy parameter  $a$  larger than one and thus the amplitude of the velocity is smaller along  $y$ , as seen in Eq. (10). This determines the increase of the saturation amplitude of the potential by a factor  $\sqrt{a}$ , as seen in Eq. (13). The ratio of the diamagnetic time and of the correlation time

$$\tau_* / \tau_c = \bar{\gamma} / \bar{\omega} = \gamma_0 (\bar{k}_y - \bar{\omega}) - \frac{\bar{k}_i^2 \bar{D}_i}{\bar{\omega}} \quad (14)$$

is small for drift turbulence and it decreases with the increase of the turbulence amplitude due to the diffusive damping.

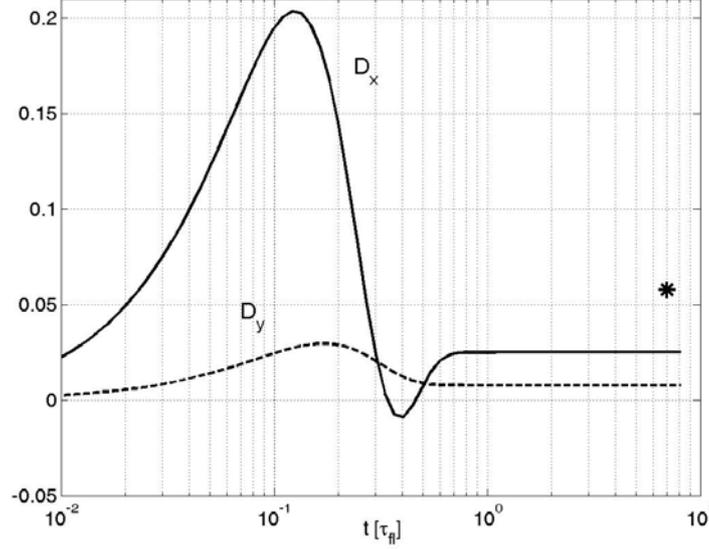


Fig. 1 – The time dependent diffusion coefficients for very small amplitude ( $K_* = 0.1$ ,  $a = 3$ ,  $k_0 = 0$ ).  $D_y$  for a decaying EC is represented for comparison (the star symbol).

The time dependent diffusion coefficients for the special EC (9) and very small amplitude of the turbulence ( $K_* = 0.1$ ) are presented in Fig. 1. These time dependent functions show the “microscopic” characteristics of the diffusion process, while their asymptotic values represent the diffusion coefficients, which determine the macroscopic transport. The anisotropy of the velocity corresponding to the special EC (9) leads to the inversion of the anisotropy of the diffusion compared to the generally accepted evaluation,  $D_y < D_x$ . In particular, in the small time limit  $t < \tau_*$ , the ratio of the diffusion coefficients is  $D_y / D_x = V_y^2 / V_x^2 \ll 1$ .

Most important conclusion from Fig. 1 is that the usual estimation of  $D_y$  (represented by the star) does not apply to this turbulence, which yields much smaller values of the asymptotic diffusion coefficient.

The diffusion along  $x$  has a rather large maximum for  $t \cong \tau_*$ , but the asymptotic value is much smaller due to the large decorrelation time  $\tau_c \gg \tau_*$ , which is in the decay interval.

Thus, the diffusive damping is weak for the quasilinear regime with  $K_* \ll 1$ . Only the large wave number modes are damped, while the dominant part of the spectrum (with  $k_\perp \rho_s$  of the order one) is weakly influenced.

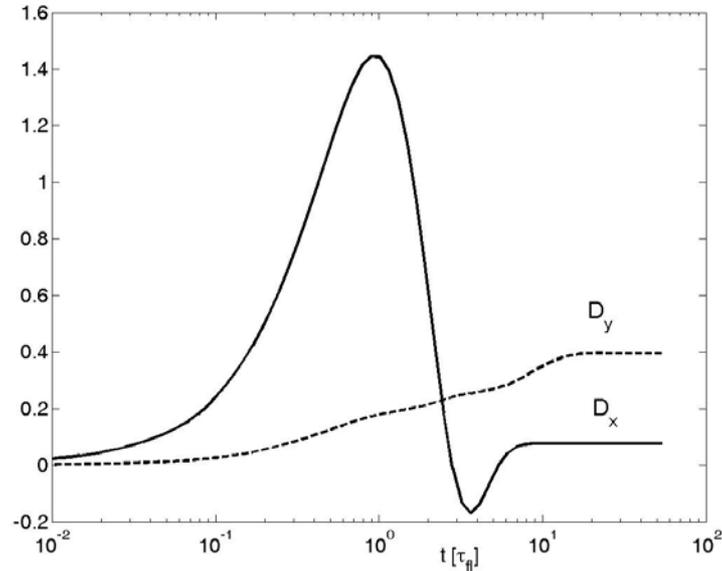


Fig. 2 – Same as in Fig. 1, but for the transition to the nonlinear regime ( $K_* = 1$ ,  $a = 3$ ,  $k_0 = 0$ ).

As the turbulence level increases, the diffusive damping becomes important (Fig. 2). The shape of  $D_x(t)$  is not changed, but the maximum value increases and the width of this function increases. The diffusion coefficient increases but with a rate that is reduced due to the increase of  $\tau_*/\tau_c$  given by Eq. (14). The evolution of  $D_y(t)$  leads to strong modification of the shape and to strong increase of the asymptotic diffusion coefficient. This yields a strong diffusive decay at the end of the quasilinear regime when  $K_* \cong 1$ . In the absence of a major change in the evolution of the turbulence, the diffusive damping is strong enough to determine the damping of the modes in the dominant part of the spectrum.

#### 4.2. THE WEAKLY NONLINEAR REGIME

Ion trajectory trapping appears when  $K_* \cong 1$  and the fraction of trapped trajectories continuously increases as  $K_*$  increases above this value. Trapping determines the generation of vortical structures, which have average size that increases with  $K_*$ . The probability of displacements becomes non-Gaussian with a narrow maximum due to trapped ions. This changes the propagator of the test modes and the solution of the dispersion relation becomes

$$\bar{\omega} = \frac{\mathbf{F}\bar{k}_y}{1 + \mathbf{F}\bar{k}_\perp^2}, \quad (15)$$

while the growth rate is given by Eq. (12), as in the quasilinear regime. The function  $\mathbf{F}$  is defined by

$$\mathbf{F} \equiv \exp\left(-\frac{1}{2}k_i^2 s_i^2\right), \quad (16)$$

where  $s_i$  is the average size of the vortical trajectory structures. Thus, trajectory trapping determines the decrease of the frequency. The value of  $k_\perp$  corresponding to the maximum of  $\gamma$  is displaced from values  $\bar{k}_\perp \cong 1$  to  $\bar{k}_\perp \cong 1/s$  where  $s = \sqrt{s_x^2 + s_y^2} / \rho_s$ . The unstable mode range is displaced toward small  $k_\perp$  (inverse cascade).

The decrease of the wave numbers of the most unstable modes compensates the increase of the diffusion coefficients and leads to small diffusive damping of these modes. As the amplitude and the correlation lengths of the turbulence continue to increase, the diffusive damping decays (at a small rate).

Thus, the modes developed during the quasilinear stage are progressively damped and replaced by smaller wave number modes that are only weakly influenced by the ion diffusion.

#### 4.3. THE STRONGLY NONLINEAR REGIME

The strongly nonlinear regime is characterized by comparable fractions of trapped and free trajectories. Trapped ions move with the potential and they have an average flux  $n_{tr}V_{*e}$ . As the  $\mathbf{E} \times \mathbf{B}$  drift has zero divergence, the probability of the Lagrangian velocity is the same with the probability of the Eulerian velocity. This means, in particular, that the trapped ion flux has to be compensated by the flux of free particles  $n_f$  that should have an average velocity  $V_f$  such that

$$n_{tr}V_{*e} + n_fV_f = 0. \quad (17)$$

The NSA shows that the probability of displacements splits into two components that move in opposite directions. This modifies the propagator and determines two solutions of the dispersion relation. One is of drift type modes

$$\bar{\gamma}_d = \frac{\gamma_0 (\bar{\omega}_d + n \bar{k}_y) [(1-n) \bar{k}_y - \bar{\omega}_d] - n_f \bar{k}_i^2 \bar{D}_i}{[(1-n) \bar{k}_y - \bar{\omega}_d]^2 (1 + \mathbf{F} \bar{k}_\perp^2) + n \bar{k}_y^2} (\bar{k}_y - \bar{\omega}_d)^2, \quad (18)$$

where  $\bar{\omega}_d$  is the frequency of the modes. It shows that the ion flows determine the damping of the drift modes, first for the small wave number modes and, when  $n \equiv n_r / n_f > 1$ , the growth rate becomes negative for the whole wave number range (see reference [4]). This determines a transitory increase of the amplitude accompanied by the decrease of the correlation lengths, followed by the decay of the drift turbulence. The second solution gives a new type of unstable modes, the zonal flow modes, which have  $k_y = 0$  and very small frequency. They appear due to the combined effect of trapping and of the ion polarization drift

$$\bar{\omega}_{zf} = -\bar{k}_x \frac{2\tau_{fl}}{\Omega_i B^2} \partial_y^2 \Delta E(\mathbf{0}) \frac{1 + \mathbf{F} \bar{k}_x n_f}{1 + \mathbf{F} \bar{k}_x^2}, \quad (19)$$

$$\bar{\gamma}_{zf} = \frac{n_r \mathbf{F} \bar{k}_x^2 \left[ \gamma_0 \bar{\omega}_{zf}^2 - n_f \mathbf{F} \bar{k}_x^4 \bar{D}_x \right]}{(1 + n_f \mathbf{F} \bar{k}_x^2) (1 + \mathbf{F} \bar{k}_x^2)}. \quad (20)$$

There is no causality relation between the generation of the zonal flow modes and drift turbulence damping. Ion diffusion contributes to the damping of both types of modes, as seen in Eqs. (18, 20).

The diffusive damping term becomes more complicated, but it is essentially determined by the ion diffusion coefficients. They correspond to the stochastic potential that contains both types of modes. The zonal flow modes add a contribution to the EC of the potential that is similar to Eq. (9), but with the coordinates  $x$  and  $y$  interchanged because the growth rate is zero for  $k_y = 0$ . The total EC has a different shape compared to the EC of the drift turbulence. The negative parts are attenuated and eventually eliminated. The effective correlation lengths are changed:  $\lambda_x$  strongly decreases and  $\lambda_y$  weakly increases when the amplitude of the zonal flow modes increases. The effects on the diffusion coefficients are rather important:  $D_x$  and  $D_y$  strongly increases.  $D_y$  contributes sensibly to the decay of the drift modes and amplifies the effect produced by the ion flows.

The zonal flow modes are influenced only by  $D_x$ . A nonstandard effect appears: the diffusive damping term decreases with the increase of the amplitude of

the zonal flow modes. The instability term in Eq. (20) is proportional to  $n_{tr}$  and thus the damping of the drift turbulence determines the decrease of the instability term through the factor  $n_{tr}$ . When values smaller than the damping term are reached, the zonal flow modes are damped.

## 5. CONCLUSIONS

We have shown that a strong interdependence between ion transport and turbulence evolution characterizes the drift turbulence in magnetically confined plasmas. This interaction is rather complex and involves the diffusion coefficients in both directions. The diffusion coefficient perpendicular on the density gradient  $D_y$  has a strong effect on turbulence damping, although it does not produce transport.

The statistics of ion trajectories show both random and coherent aspects. The random motion leads to diffusive behaviour while the coherent motion is associated with trapping or eddying in the structure of the stochastic potential, which generates quasi-coherent trajectory structures and non-Gaussian distributions of displacements. Ion diffusion has a stabilizing effect on turbulence while ion trapping leads to strong nonlinear effects: increase of the correlation lengths, nonlinear damping of the drift modes and generation of zonal flow modes. The strength of each of these processes depends on the stage in the evolution of turbulence (reflected in the parameters of the turbulence). The effects of ion trapping were analyzed in [4] considering the diffusive damping is small. This paper presents a study of the effects of ion diffusion on turbulence evolution.

Ion diffusion determines a damping effect on the drift modes in all evolution regimes. The diffusion coefficients strongly depend on the regime and the damping terms change, but the contribution to the growth rate is always negative. We have shown that the drift turbulence can reach the nonlinear regime due to its special shape of the Eulerian correlation (9), which determines a much smaller diffusion coefficient  $D_y$  across density gradient (Fig. 1). The strong trajectory spreading in this direction, which actually does not produce transport, would saturate the turbulence in the quasilinear regime for usual (decaying) correlations. The maximum values of the diffusion coefficients appear at the transition between the quasilinear and nonlinear regimes (Fig. 2). The damping is strong in this stage, but this process is accompanied by the generation of smaller wave number modes, due to ion trajectory trapping that generates trajectory structures. Trajectory trapping determines, in the weakly nonlinear regime, both the decrease of the wave numbers of the turbulence and of the diffusion coefficients, leading to a strong decrease of the diffusive damping. As the amplitude of the turbulence further increases, the turbulence reaches the strong nonlinear regime characterized by the ion flows.

They determine the nonlinear damping of the drift modes (first for the small  $k$  modes) and the generation of the zonal flow modes. The latter determine the modification of the total EC of the stochastic potential, which leads to a strong increase of the diffusion coefficients, especially for the cross density gradient direction. Thus, there is a contribution of the zonal flow modes to the decay of drift modes, but this is only through the diffusive damping. This damping effect adds a significant contribution to the main effect determined by the direct action of trapping combined with the drift of the potential with the effective diamagnetic velocity.

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