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ON THE EXACT SOLUTIONS OF NONLINEAR LONG-SHORT WAVE RESONANCE EQUATIONS

H. JAFARI[1,a], R. SOLTANI[1], C.M. KHALIQUE[2], D. BALEANU[3,4,5,b]

[1]Department of Mathematics, University of Mazandaran, Babolsar, Iran
[2]International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, South Africa
[3]Department of Chemical and Materials Engineering, Faculty of Engineering, King Abdulaziz University, Saudi Arabia
[4]Department of Mathematics and Computer Sciences, Cankaya University, 06530 Ankara, Turkey
[5]Institute of Space Sciences, Magurele-Bucharest, Romania
[a] *E-mail*: jafari@umz.ac.ir; [b] *E-mail*: dumitru@cankaya.edu.tr

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Abstract. The long-short wave resonance model arises when the phase velocity of a long wave matches the group velocity of a short wave. In this paper, the first integral method is used to construct exact solutions of the nonlinear long-short wave resonance equations. One-soliton solutions are also obtained using the travelling wave hypothesis.

Key words: first integral method, long-short wave resonance equations, exact solutions, travelling wave solutions.

1. INTRODUCTION

Long-short (LS) wave resonance is a special case of three wave resonances that can occur [1] when the second order nonlinearity arises in the process. Benney [2] presented a general theory for deriving nonlinear partial differential equations that permit both long- and short-wave solutions. The LS-type equations, first derived by Djordjevic and Redekopp [3], describing the resonance interaction between the long wave and the short wave, can be written as

$$\begin{cases} iS_t + \alpha S_{xx} - LS = 0, \\ L_t + \beta(|S|^2)_x = 0, \end{cases} \tag{1}$$

where S is the envelope of the short wave and is a complex function, while L is the amplitude of the long wave and is a real function, and α and β are real constants. As pointed out in [3], the physical significance of the Eqs. (1) is that the dispersion of the short wave is balanced by nonlinear interaction between the long wave and the short wave, while the evolution of the long wave is driven by the self-interaction of the short wave. These equations also appear in the analysis of internal waves [4] and

Rossby waves. In plasma physics [5], similar equations were proposed that describe the resonance between high-frequency electron plasma oscillations and associated low-frequency ion density perturbations. There have been many works on the qualitative research of the global solutions for the long-short wave resonance equations [6–12]. In Ref. [9], Chaw found that (1) can be rewritten in Lax's formulation and solved the Cauchy problem for them by the inverse scattering method. In Ref. [10], Laurencot confirmed that the solitary wave solution of (1) is stable. In Ref. [12], Guo and Chen considered the orbital stability of the solitary waves for the long-short wave resonance equations.

Recently, various powerful mathematical methods such as the inverse scattering method [13, 14], the bilinear transformation [15], the tanh-sech method [16, 17], the extended tanh method [18, 19], the sine-cosine method [20, 21], the homogeneous balance method [22], the sub-equation method [23] and other techniques have been proposed to obtain exact and/or numerical solutions for nonlinear evolutions problems [24–30].

In this paper, we use the *first integral method* for solving the long-short wave resonance equations, which was first proposed by Feng [31] for a reliable treatment of the nonlinear partial differential equations (NPDEs) [32–36]. The remaining portion of this article is organized as follows: Section 2 is a brief introduction to the first integral method. In Section 3, the first integral method will be implemented and some new exact solutions for long-short wave resonance equations will be reported. Additionally, in this section, the travelling wave solution will also be obtained to retrieve the corresponding soliton solution. The conclusions and future directions for research will be summarized in the last section.

2. THE FIRST INTEGRAL METHOD

Consider a general NPDE in the form

$$P(u,u_t,u_x,u_y,u_{xx},u_{tt},u_{yy},u_{xt},u_{xy},u_{yt},u_{xxx},...)=0. \tag{2}$$

Using the wave variable $\eta = x - 2\alpha kt$, Eq.(2) carries into the following ordinary differential equation (ODE):

$$Q(U,U',U'',U''',...)=0. \tag{3}$$

where prime denotes the derivative with respect to the variable η. Next, we introduce new independent variables $x = u$, $y = u_\eta$; thus we obtain a dynamical system of ODEs:

$$\begin{cases} x' = y, \\ y' = f(x,y). \end{cases} \tag{4}$$

According to the qualitative theory of differential equations [37], if one can find two first integrals to equations (4) under the same conditions, then analytic solutions to Eq.(4) can be found directly. However, in general, it is difficult to realize this even for a single first integral, because for a given autonomous system in two spatial dimensions, there does not exist any general theory that allows us to extract its first integrals in a systematic way. A key idea of our approach here to find first integrals is to use the *division theorem*. For convenience, first let us recall the division theorem for two variables in the complex domain **C** [38].

Theorem. (Division theorem) (See [39]) Suppose $P(x,y)$ and $Q(x,y)$ are polynomials of two variables x and y in $\mathbf{C}[x,y]$ and $P(x,y)$ is irreducible in $\mathbf{C}[x,y]$. If $Q(x,y)$ vanishes at all zero points of $P(x,y)$, then there exists a polynomial $G(x,y)$ in $\mathbf{C}[x,y]$ such that $Q(x,y) = P(x,y)G(x,y)$.

3. EXACT SOLUTIONS OF LONG-SHORT WAVE RESONANCE EQUATIONS

In order to seek exact solutions of equations (1), we suppose

$$S(x,t) = u(x,t)\exp[i(kx+\lambda t+l)], \tag{5}$$

where k and λ are constants to be determined later, l is an arbitrary constant. Substituting equation (5) into system (1) we obtain

$$\begin{cases} i(u_t+2\alpha k u_x)+\alpha u_{xx}-(\lambda+\alpha k^2)u-Lu=0, \\ L_t+\beta(u^2)_x=0. \end{cases} \tag{6}$$

Using the transformation

$$u=u(\eta), \qquad L=L(\eta), \qquad \eta=x-2\alpha kt, \tag{7}$$

where α is a constant, equations (6) further reduce to

$$\begin{cases} \alpha u''-(\lambda+\alpha k^2)u-Lu=0, \\ -2\alpha kL'+\beta(u^2)'=0, \end{cases} \tag{8}$$

where prime denotes the derivate with respect to η. Integrating the second part of equation (8) with respect to η and taking the integration constant as zero yields

$$L=\frac{\beta}{2\alpha k}u^2. \tag{9}$$

Substituting Eq. (9) into the first part of (8) yields

$$\alpha u''-(\lambda+\alpha k^2)u-\frac{\beta}{2\alpha k}u^3=0. \tag{10}$$

3.1. APPLICATION OF DIVISION THEOREM

In order to apply the division theorem, we introduce new independent variables $x = u$, $y = u_\eta$ which change Eq. (10) to the dynamical system given by

$$\begin{cases} x' = y, \\ y' = \frac{\lambda+\alpha k^2}{\alpha}x + \frac{\beta}{2k\alpha^2}x^3. \end{cases} \tag{11}$$

Now, we apply the division theorem to seek the first integral to (11). Suppose that $x = x(\eta)$ and $y = y(\eta)$ are nontrivial solutions to (11), and

$$P(x,y) = \sum_{i=0}^{m} a_i(x)y^i,$$

is an irreducible polynomial in $\mathbf{C}[x,y]$ such that

$$P(x(\eta),y(\eta)) = \sum_{i=0}^{m} a_i(x(\eta))y(\eta)^i = 0, \tag{12}$$

where $a_i(x)$ $(i = 0,1,...,m)$ are polynomials in x and all relatively prime in $\mathbf{C}[x,y]$, $a_m(x) \neq 0$. Equation (12) is also called the first integral to (11). We start our study by assuming $m = 1$ in (12). Note that $\frac{dP}{d\eta}$ is a polynomial in x and y, and $P[x(\eta),y(\eta)] = 0$ implies $\frac{dP}{d\eta} = 0$. By the division theorem, there exists a polynomial $H(x,y) = h(x)+g(x)y$ in $\mathbf{C}[x,y]$ such that

$$\begin{aligned} \frac{dP}{d\eta} &= \left[\frac{\partial P}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial P}{\partial y}\frac{\partial y}{\partial \eta}\right]_{(11)} \\ &= \sum_{i=0}^{1} a_i'(x)y^{i+1} + \sum_{i=0}^{1} i a_i(x)y^{i-1}\left[\frac{\lambda+\alpha k^2}{\alpha}x + \frac{\beta}{2k\alpha^2}x^3\right] \\ &= (h(x)+g(x)y)(\sum_{i=0}^{1} a_i(x)y^i), \end{aligned} \tag{13}$$

where prime denotes differentiation with respect to the variable x. On equating the coefficients of y^i $(i = 2,1,0)$ on both sides of (13), we have

$$a_1'(x) = g(x)a_1(x), \tag{14}$$

$$a_0'(x) = h(x)a_1(x) + g(x)a_0(x), \tag{15}$$

$$a_1(x)\left[\frac{\lambda+\alpha k^2}{\alpha}x + \frac{\beta}{2k\alpha^2}x^3\right] = h(x)a_0(x). \tag{16}$$

Since, $a_1(x)$ is a polynomial in x, from (14) we conclude that $a_1(x)$ is a constant and $g(x) = 0$. For simplicity, we take $a_1(x) = 1$, and balancing the degrees of $h(x)$ and

$a_0(x)$ we conclude that deg $h(x) = 1$. Now suppose that $h(x) = Ax + B$, then from (15), we find

$$a_0(x) = \frac{1}{2}Ax^2 + Bx + D,$$

where D is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$, and $h(x)$ in (16) and setting all the coefficients of powers x to zero, we obtain a system of nonlinear algebraic equations and by solving this system, we obtain

$$A = \pm\sqrt{\frac{\beta}{k\alpha^2}}, \quad B = 0, \quad D = \pm(\lambda + \alpha k^2)\sqrt{\frac{k}{\beta}}. \tag{17}$$

Using (17) in (12), we obtain

$$y \pm \frac{1}{2}\sqrt{\frac{\beta}{k\alpha^2}}x^2 \pm (\lambda + \alpha k^2)\sqrt{\frac{k}{\beta}} = 0. \tag{18}$$

Combining Eq. (18) with the first part of (11), we obtain the exact solutions of Eq. (10) given by

$$u_1(\eta) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda + \alpha k^2)}\tan\left[\sqrt{\frac{\lambda + \alpha k^2}{2}}(\eta + \eta_0)\right],$$

and

$$u_2(\eta) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda + \alpha k^2)}\cot\left[\sqrt{\frac{\lambda + \alpha k^2}{2}}(\eta + \eta_0)\right],$$

where η_0 is an arbitrary constant. Therefore, the exact solutions to (10) can be written as

$$u_1(x,t) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda + \alpha k^2)}\tan\left[\sqrt{\frac{\lambda + \alpha k^2}{2}}(x - 2\alpha kt + \eta_0)\right],$$

and

$$u_2(x,t) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda + \alpha k^2)}\cot\left[\sqrt{\frac{\lambda + \alpha k^2}{2}}(x - 2\alpha kt + \eta_0)\right].$$

Thus the exact solutions for Eqs. (1) are

$$\begin{cases} S_1(x,t) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda + \alpha k^2)}\tan\left[\sqrt{\frac{\lambda+\alpha k^2}{2}}(x - 2\alpha kt + \eta_0)\right]e^{i(kx+\lambda t+l)}, \\ L_1(x,t) = (\lambda + \alpha k^2)\sec^2\left[\sqrt{\frac{\lambda+\alpha k^2}{2}}(x - 2\alpha kt + \eta_0)\right] - (\lambda + \alpha k^2 + c_1). \end{cases}$$

and

$$\begin{cases} S_2(x,t) = \pm\sqrt{2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\cot\left[\sqrt{\frac{\lambda+\alpha k^2}{2}}(x-2\alpha kt+\eta_0)\right]e^{i(kx+\lambda t+l)}, \\ L_2(x,t) = (\lambda+\alpha k^2)\text{csc}^2\left[\sqrt{\frac{\lambda+\alpha k^2}{2}}(x-2\alpha kt+\eta_0)\right]-(\lambda+\alpha k^2+c_1), \end{cases}$$

where c_1 and η_0 are arbitrary constants. Now we assume that $m=2$ in (12). By the division theorem, there exists a polynomial $H(x,y)=h(x)+g(x)y$ in $\mathbf{C}[x,y]$ such that

$$\begin{aligned} \frac{dP}{d\eta} &= \left[\frac{\partial P}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial P}{\partial y}\frac{\partial y}{\partial \eta}\right]_{(11)} \\ &= \sum_{i=0}^{2} a_i'(x)y^{i+1}+\sum_{i=0}^{2} ia_i(x)y^{i-1}\left[\frac{\lambda+\alpha k^2}{\alpha}x+\frac{\beta}{2k\alpha^2}x^3\right] \\ &= (h(x)+g(x)y)(\sum_{i=0}^{2} a_i(x)y^i), \end{aligned} \tag{19}$$

On equating the coefficients of y^i $(i=3,2,1,0)$ from both sides of (19), we have

$$a_2'(x) = g(x)a_2(x), \tag{20}$$

$$a_1'(x) = h(x)a_2(x)+g(x)a_1(x), \tag{21}$$

$$a_0'(x) = -2a_2(x)\left[\frac{\lambda+\alpha k^2}{\alpha}x+\frac{\beta}{2k\alpha^2}x^3\right]+h(x)a_1(x)+g(x)a_0(x), \tag{22}$$

$$a_1(x)\left[\frac{\lambda+\alpha k^2}{\alpha}x+\frac{\beta}{2k\alpha^2}x^3\right] = h(x)a_0(x). \tag{23}$$

Since $a_2(x)$ is a polynomial of x, from (20) we conclude that $a_2(x)$ is a constant and $g(x)=0$. For simplicity, we take $a_2(x)=1$, and balancing the degrees of $h(x)$, $a_0(x)$ and $a_1(x)$ we conclude that deg $h(x)=1$ or 0, therefore we have the following two cases:

Case 1:

Suppose that deg $h(x)=1$ and $h(x)=Ax+B$, then from (21) we find

$$a_1(x) = \frac{1}{2}Ax^2+Bx+D,$$

where D is an arbitrary integration constant. From (22) we have

$$a_0(x) = \left(\frac{A^2}{8}-\frac{\beta}{4k\alpha^2}\right)x^4+\frac{AB}{2}x^3+\left(\frac{1}{2}(B^2+AD)-\frac{\lambda+\alpha k^2}{\alpha}\right)x^2+BDx+E,$$

where E is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$ and $h(x)$ in (23) and setting all the coefficients of powers x to zero, we obtain a system of nonlinear algebraic equations and by solving this system of equations, we obtain

$$A=\pm 2\sqrt{\frac{\beta}{k\alpha^2}},\quad B=0,\quad D=\pm 2\frac{k}{\beta}(\lambda+\alpha k^2)\sqrt{\frac{\beta}{k}},\quad E=\frac{k}{\beta}(\lambda+\alpha k^2)^2. \tag{24}$$

Using (24) in (12), we obtain

$$y^2\pm\left(\sqrt{\frac{\beta}{k\alpha^2}}x^2+2\frac{k}{\beta}(\lambda+\alpha k^2)\sqrt{\frac{\beta}{k}}\right)y+\frac{\beta}{4k\alpha^2}x^4+\frac{\lambda+\alpha k^2}{\alpha}x^2+\frac{k}{\beta}(\lambda+\alpha k^2)^2=0. \tag{25}$$

Combining Eq. (25) with the first part of (11), we obtain two exact solutions to Eq. (10), which are given by

$$u_1(\eta)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\tanh\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(\eta+\eta_0)\right],$$

and

$$u_2(\eta)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\coth\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(\eta+\eta_0)\right],$$

where η_0 is an arbitrary constant. Therefore, the exact solutions to (10) can be written as

$$u_1(x,t)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\tanh\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right],$$

and

$$u_2(x,t)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\coth\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right].$$

Finally, the exact solutions of Eqs. (1) are

$$\begin{cases} S_1(x,t)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\tanh\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right]e^{i(kx+\lambda t+l)},\\ L_1(x,t)=(\lambda+\alpha k^2)\,\mathrm{sech}^2\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right]-(\lambda+\alpha k^2+c_1),\end{cases}$$

and

$$\begin{cases} S_2(x,t)=\pm\sqrt{-2\frac{\alpha k}{\beta}(\lambda+\alpha k^2)}\coth\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right]e^{i(kx+\lambda t+l)},\\ L_2(x,t)=-(\lambda+\alpha k^2)\,\mathrm{csch}^2\left[\sqrt{\frac{-(\lambda+\alpha k^2)}{2}}(x-2\alpha kt+\eta_0)\right]-(\lambda+\alpha k^2+c_1),\end{cases}$$

where c_1 and η_0 are arbitrary constants.

Case 2:

In this case suppose that deg $h(x) = 0$ and $h(x) = A$. Then from (21) we find $a_1(x) = Ax + B$, where B is an arbitrary integration constant. From (22) we obtain

$$a_0(x) = -\frac{\beta}{4k\alpha^2}x^4 + \left(\frac{A^2}{2} - \frac{\lambda + \alpha k^2}{\alpha}\right)x^2 + ABx + D,$$

where D is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$, and $h(x)$ in (23) and setting all the coefficients of powers x to zero, leads to a system of nonlinear algebraic equations. Solving this system, yields

$$A = 0, \quad B = 0, \quad D = 0. \tag{26}$$

In view of (26) and (12), we obtain

$$y^2 - \frac{\beta}{4k\alpha^2}x^4 - \frac{\lambda + \alpha k^2}{\alpha}x^2 = 0. \tag{27}$$

Combining Eq. (27) with the first part of (11), we obtain the exact solutions of Eq. (10) as

$$u_3(\eta) = \frac{8k\alpha\sqrt{\lambda + \alpha k^2}e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}\eta + 2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{1 - 4e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}\eta + 4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}},$$

and

$$u_4(\eta) = \frac{8k\alpha\sqrt{\lambda + \alpha k^2}e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}\eta + 2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}\eta\right)} - 4\beta e^{\left(4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}},$$

where c_1 is an arbitrary constant. Thus the exact solutions to (10) can be written as

$$u_3(x,t) = \frac{8k\alpha\sqrt{\lambda + \alpha k^2}e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt) + 2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{1 - 4e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt) + 4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}$$

and

$$u_4(x,t) = \frac{8k\alpha\sqrt{\lambda + \alpha k^2}e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt) + 2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)\right)} - 4\beta e^{\left(4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}.$$

Hence the solutions of Eqs. (1) are

$$\begin{cases} S_3(x,t) = \dfrac{8k\alpha\sqrt{\lambda+\alpha k^2}\, e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+2k\alpha\sqrt{\lambda+\alpha k^2}c_1+i(kx+\lambda t+l)\right)}}{1-4e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}, \\ L_3(x,t) = \dfrac{32k\alpha\beta(\lambda+\alpha k^2)e^{2\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{\left(1-4e^{(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+4k\alpha\sqrt{\lambda+\alpha k^2}c_1)}\right)^2} \end{cases} \tag{28}$$

and

$$\begin{cases} S_4(x,t) = \dfrac{8k\alpha\sqrt{\lambda+\alpha k^2}\, e^{\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+2k\alpha\sqrt{\lambda+\alpha k^2}c_1+i(kx+\lambda t+l)\right)}}{e^{\left(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)\right)}-4\beta e^{\left(4k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}, \\ L_4(x,t) = \dfrac{32k\alpha\beta(\lambda+\alpha k^2)e^{2\left(\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt)+2k\alpha\sqrt{\lambda+\alpha k^2}c_1\right)}}{\left(e^{(2\sqrt{\frac{\lambda+\alpha k^2}{\alpha}}(x-2\alpha kt))}-4\beta e^{(4k\alpha\sqrt{\lambda+\alpha k^2}c_1)}\right)^2}. \end{cases} \tag{29}$$

3.2. TRAVELLING WAVE SOLUTIONS

The travelling wave hypothesis will be used to obtain the 1-soliton solution to the long-short wave resonance equations (1). It needs to be noted that this equation was already studied by travelling wave hypothesis in 2008 [1], where the power law nonlinearity was considered. Additionally, the ansatz method was used to extract the exact 1-soliton solution to the long-short wave resonance equations in 1994 and that too was studied with power law nonlinearity [7]. In this subsection, the starting point is going to be Eq. (10) that can now be rewritten as

$$u'' = \frac{\lambda+\alpha k^2}{\alpha}u + \frac{\beta}{2k\alpha^2}u^3 \tag{30}$$

Now, multiplying both sides of (30) by u' and integrating while choosing the integration constant to be zero, since we search for soliton solutions, yields

$$\left(u'\right)^2 = bu^2 - au^4 \tag{31}$$

where

$$a = -\frac{\beta}{4k\alpha^2} \tag{32}$$

and

$$b = \frac{\lambda+\alpha k^2}{\alpha}. \tag{33}$$

Separating variables in (31) and integrating gives

$$x-2\alpha kt=-\frac{1}{\sqrt{b}}\,\mathrm{sech}^{-1}\left|u\sqrt{\frac{a}{b}}\right| \tag{34}$$

that yields the 1-soliton solution as

$$u(x-2\alpha kt)=A\,\mathrm{sech}[B(x-2\alpha kt)], \tag{35}$$

where the amplitude A of the soliton is given by

$$A=\sqrt{\frac{b}{a}}=\sqrt{-\frac{4k\alpha\,(\lambda+\alpha k^2)}{\beta}} \tag{36}$$

and

$$B=-\sqrt{b}=-\sqrt{\frac{\lambda+\alpha k^2}{\alpha}} \tag{37}$$

and this leads to the constraints $\lambda+\alpha k^2>0$ and $k\alpha\beta(\lambda+\alpha k^2)<0$. Hence by virtue of (5),

$$S(x,t)=A\,\mathrm{sech}[B(x-2\alpha kt)]e^{i(kx+\lambda t+l)}. \tag{38}$$

Finally from (9), the topological 1-soliton solution is given by

$$L(x,t)=-2(\lambda+\alpha k^2)\,\mathrm{sech}^2[B(x-2\alpha kt)]. \tag{39}$$

Thus, (38) and (39) together constitute the 1-soliton solution of the long-short wave resonance equations given by (1).

4. CONCLUSIONS

We have applied the first integral method to the long-short wave resonance equations and have obtained some new exact solutions. The obtained solutions were expressed in terms of trigonometric, hyperbolic, and exponential functions. In addition, the travelling wave hypothesis was used to obtain the 1-soliton solution of the coupled nonlinear equations, where a topological and non-topological soliton pair is retrieved. These new soliton solutions can be used for the study of some practical nonlinear evolution problems.

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