Abstract. Non-locality is an abstract concept with a growing number of practical implementations in encrypted communications and computer technology. The most preeminent effect of non-locality is the entanglement between a number of quantum elements, that manifests itself by a complex number of correlations between the observables of the composing elements. Laser devices, on the other hand, are the main enablers of fast communications and quantum computers. The paper aims to present the quantum locality violation phenomena that appear in laser structures.

Key words: entanglement, four level lasers, Zeeman effect, quantum communications.

1. INTRODUCTION

Quantum communications have reached the point where integration of various devices used in establishing a secure and efficient link has to be as close to maximum as possible [1, 2]. This integration process has to undergo a series of theoretical studies, in order to qualitatively characterize its effect [3–6]. One of the greatest integration was the realization of different devices and link segments out of the same optical material (optical fiber glass) [7–10]. The material was chosen such that the absorption level at the working communication bandwidth should exhibit a minimum. Furthermore, anisotropic devices [11–14] constructed out of the same optical fiber glass gave the possibility to have a control on the phase and polarization of the emitted signal at any time. Only recently, studies have reported the realization of non-linear optical devices [2, 19], that convert the input signal at the working frequency into different signals at some other desired frequencies [8]. The conversion, although quite inefficient – $10^{-7}$ – is the preferred method at the present time, because the input signal power can be adjusted accordingly. Furthermore, among the non-linear effects that occur in non-linear optical devices, one important effect is the creation of entangled states. Entangled states [15–18], although having a “spooky” reputation [1] in the sense that the manifestations hold at arbitrary long distances, have very practical applications in quantum cryptography and quantum computing.
The most important part of the communication system is the device that gives birth to the pump signal, for it is the one that establishes the working parameters (bandwidth, pulse repetition rate, spectral lines etc.). By far the most commonly used device today is the laser, be it gas or solid-state. The extensive studies that have been done on characterizing the laser devices range from classical to quantum descriptions and formalisms [3, 4, 7]. The former successfully describe the population rates on different energy levels, while the latter explain the spin and photon distributions, while providing proof of correspondence to a classical description in a macroscopic assumption. From a non-linear optics point of view, studies on propagation modes have provided proof that entanglement between different modes exists, and can be exploited in holography and parallel processing [4, 7, 8]. However, quantum cryptography and computing applications [1, 2, 5, 6] still rely on separate non-linear crystals in order to achieve polarization-entangled states. The following paper focuses on studying the existence of entanglement between different types of observables in laser structures from a theoretical point of view and is structured into the following sections: section 2 takes into discussion the quantum model of the laser, section 3 outlines the entanglement phenomena that appear in the laser, while section 4 draws conclusions based on the theoretical model and simulation results.

2. QUANTUM FOUR LEVEL LASER FORMALISM

The description of a laser structure has been done from a classical, semi-quantum and fully quantum approach. The best way to account for all the quantum phenomena of the photons coming out of the laser is by employing a fully quantum description. In terms of frequency, the best way of obtaining energy-time entanglement in a laser structure is by considering a 4-level laser structure, in which an intermediary radiative transition energy level is inserted between the pump and ground level. Any non-radiative transition is neglected in the development of the model. For a four level laser structure with energy states $|a\rangle, |b\rangle, |c\rangle$, $|a\rangle$ being the most excited state and $|c\rangle$ being the ground state, the Hamiltonian has the following form [7]:

$$H = \sum_k \hbar \omega_k a^\dagger a + \sum_r \hbar \omega_r \sigma_r a^\dagger a + g_{ab} \sigma_- a^\dagger + g_{bc} \sigma_+ a + g_{ac} \sigma_+ a^\dagger,$$

(1)

where $\sigma_\pm$ is the Pauli operator, $r \in \{a\}, \{b\}, \{c\}$, $a^\dagger$ and $a$ are the creation and annihilation operators of a photon on a certain energy level and $g_{ij} = \frac{eE_\nu d_{\nu}}{\sqrt{2\hbar}}$ is a
coupling coefficient that depends on the bandwidth between energy states \( i \) and \( j \) with \( E_\theta \), the electrical field, \( d_{ij} \) an interaction factor that accounts for the amount of electrical interaction between states \( i \) and \( j \) and \( \hbar \) being the reduced Planck's constant. The model is represented in figure 1a. Invariant to the photon emission, the frequency relation is written as:

\[
\omega_{ca} = \omega_{ab} + \omega_{bc},
\]

with \( \omega_{ab} \) being the frequency detuning between levels \( |a\rangle \) and \( |b\rangle \) and \( \omega_{bc} \) the detuning between levels \( |b\rangle \) and \( |c\rangle \), making all the emitted frequency in the so-called energy-entanglement. If energy state \( |b\rangle \) is half-way between \( |a\rangle \) and \( |c\rangle \), the two frequencies are the same, and there is no way to distinguish on which emission \( |a\rangle \rightarrow |b\rangle \) or \( |b\rangle \rightarrow |c\rangle \) the photons were created. However, the photons are indistinguishable to any other measurement, which from an applied point of view collapses the state into a separable state. This occurs because of the impossibility of coherent post-selection. If energy state \( |b\rangle \) is not located half-way between \( |a\rangle \) and \( |c\rangle \), then by applying filters centered on either \( \omega_{ab} \) or \( \omega_{bc} \), one can measure the frequency of only one emitted photon and infer the other frequency. However, this type of measurement is easy to reproduce in case any interception is conducted in the case of quantum key distribution. Furthermore, once the energy levels are set, they cannot be changed dynamically, and the quantum state \( |\omega_{ca}\rangle \) is collapsed into a classical set of fixed energy levels at first measurement. The most useful observable for quantum key distribution and quantum computing applications is the polarization, due to the fact that polarization states can be made purely quantum, in the sense that the polarization bases can be arbitrarily chosen and manipulated. In order to exploit the polarization at the output of a 4 – level laser, we have exploited the already famous Zeeman effect [9]: application of a magnetic field orthogonal to the wave number of the emitted photons \( B_z \) splits the discrete energy levels \( |a\rangle \), \( |b\rangle \) and \( |c\rangle \) into a discrete spectrum with a maximum detuning of \( \pm \Delta \omega \) centered around each of the initial levels. Any of the ensuing possible transitions will follow the magnetic quantum number selection rules \( \Delta m_l = 0, \pm 1 \). Besides splitting the spectral lines of a laser structure by a detuning \( \Delta \omega \), studies have shown that depending on the selection rules, the photons have linear polarization for \( \Delta m_l = 0 \), and circular left and right for \( \Delta m_l = 1 \) and \( \Delta m_l = -1 \) respectively. The linear polarizations are denoted as
\(\pi\) transitions, while the circular right and left polarizations are denoted as \(\sigma_+\) and \(\sigma_-\) respectively. The model diagram is presented in Fig. 1b.

Fig. 1 – Normal (a) and Zeeman (b) transitions in a four energy level laser structure. The normal transition presumes linearly polarized transitions between the three energy levels, while the Zeeman transitions take into account transitions between the split energy levels induced by the magnetic field \(\vec{B}\) in accordance with the magnetic moment quantum number selection rules \(\Delta m_l = 0, \pm 1\).

In the following framework, linear \(\pi \rightarrow \pi\) transitions are omitted for the cascade transitions from \(|b\rangle\) to \(|c\rangle\). We will consider a \(\sigma_+\) transition having a circular right polarization, and a \(\sigma_-\) transition having a circular left polarization. Following the selection rules of the magnetic dipole, the allowed circular polarizations \(\sigma\) transitions from \(|a\rangle\) to \(|c\rangle\) are written as follows:

\[
|\sigma_+\sigma_+\rangle = \frac{1}{\sqrt{2}} \left( |\sigma_{+ab}^{(1,0)}\sigma_{+bc}^{(0,1)}\rangle + |\sigma_{-ab}^{(0,-1)}\sigma_{+bc}^{(-1,0)}\rangle \right),
\]

with the associated frequency state:

\[
|\omega\rangle^\pm = |\omega_{ab} \pm \Delta \omega; \omega_{bc} - \Delta \omega\rangle.
\]

The same can be written for:

\[
|\sigma_+\sigma_-\rangle = \frac{1}{\sqrt{2}} \left( |\sigma_{+ab}^{(1,0)}\sigma_{-bc}^{(1,0)}\rangle + |\sigma_{-ab}^{(0,-1)}\sigma_{-bc}^{(-1,0)}\rangle \right),
\]

with

\[
|\omega\rangle^\pm = |\omega_{ab} \mp \Delta \omega; \omega_{bc} + \Delta \omega\rangle.
\]

Also, for the same transition types we have:

\[
|\sigma_-\sigma_-\rangle = \left| \sigma_{-ab}^{(1,0)}\sigma_{-bc}^{(0,-1)} \right\rangle,
\]
Due to the fact that the entangled states must have the same spectrum, the energy level $|\hat{h}\rangle$ is set at the middle of the band gap. It immediately follows that $\omega_{ab} = \omega_{bc}$. The desired entangled states are:

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}} (|\sigma_x\sigma_-\rangle \pm |\sigma_x\sigma_+\rangle),$$

(11)

with the corresponding frequency state:

$$|\omega^\pm\rangle = \frac{1}{\sqrt{2}} (|\omega + \Delta \omega, \omega - \Delta \omega\rangle \pm |\omega - \Delta \omega, \omega + \Delta \omega\rangle),$$

(12)

and

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}} (|\sigma_x\sigma_-\rangle \pm |\sigma_x\sigma_+\rangle),$$

(13)

with the corresponding frequency state:

$$|\omega^\pm\rangle = \frac{1}{\sqrt{2}} (|\omega + \Delta \omega, \omega + \Delta \omega\rangle \pm |\omega - \Delta \omega, \omega - \Delta \omega\rangle).$$

(14)

From all of the above considerations, it follows that for the two-fold photonic transitions in the four level laser structure, a $|\psi\rangle$ state implies that the frequencies are detuned by $2\Delta \omega$, whereas a $|\phi\rangle$ state implies no detuning on the emitted frequencies. Because stabilization and representation can be realized in an easier manner in terms of wavelength, we will express the frequency states in their correspondents – the wavelength states:
\[ |\tilde{\psi}_\lambda\rangle = \frac{1}{\sqrt{2}} (|\tilde{\psi} + \Delta \lambda; \lambda - \Delta \lambda\rangle + |\tilde{\psi} - \Delta \lambda; \lambda + \Delta \lambda\rangle). \] (15)

\[ |\tilde{\phi}_\lambda\rangle = \frac{1}{\sqrt{2}} (|\tilde{\phi} + \Delta \lambda; \lambda + \Delta \lambda\rangle + |\tilde{\phi} - \Delta \lambda; \lambda - \Delta \lambda\rangle). \] (16)

Differentiating from a two-photon and a one-photon emission can be done by post-selecting the output states and retaining only the two-photon emission. However, this operation results in a loss of photons because of filtering. For the detuned emission, the wavelengths can be shifted via wavelength shifting length (WSL) fibers to a reference wavelength in order to eliminate wavelength discrimination.

### 3. EXPERIMENT AND RESULTS

In order to conduct our simulations, we have used the model parameters of a 1.5 mW frequency and intensity stabilized laser diode source running at 1554.94 nm (Thorlabs Pro8000 Laser Diode Module). The source is situated in a magnetic field sufficiently strong in order to cause the splitting of the transition spectrum of the laser atoms by 0.02 nm. The Zeeman splitting being proportional to the frequency splitting, in terms of wavelength, the transition radiations will have a final wavelength of

\[ \lambda_f = \frac{1}{\lambda_0 + k B} c, \] (17)

with

\[ k = \frac{g_i \mu_B m_j}{\hbar}, \]

\[ g_i \text{ the gyro-mechanical factor, } \mu_B \text{ the Bohr magneton, } \hbar \text{ Planck's constant and } m_j = 0, \pm 1 \text{ the total kinetic moment quantum number. The transitions of interest for obtaining a } |\psi\rangle \text{ state are represented by } m_j = 1 \text{ for the first transition and } m_j = -1 \text{ for the second transition (and vice-versa), while for obtaining a } |\phi\rangle \text{ state is represented by transitions } m_j = 1, \text{ for the first transition, and } m_j = 1 \text{ for the second (and similar for } m_j = -1) \text{ situation comes into study. The } m_j = 0 \text{ is not of a particular interest as the state is a superposition of entangled states that is no longer entangled. This state is immediately rejected at the output of the laser by a fiber Bragg grating.} \]
For obtaining the desired entangled states, we have conceived different experimental set-ups, presented in Fig. 2. Post-selection of the $|\psi\rangle$ state is achieved by an electro-optical shifting solution, described as follows: In order to achieve the $|\psi\rangle$ state, the detuning induced by the Zeeman effect must be completely compensated by the frequency shifting scheme, but only after separate filtering of the two wavelengths has been realized. The separate filtering is done by means of two narrowband filters (e.g. Yenista XTA 50) having a minimal window of 40 picometers. In order to compensate the splitting from the magnetic field, a RF signal of several GHz must be applied at the input of a polarization-maintaining electro-optic shifter. Recent studies [20] have reported the realization of such devices. Depending on the input state, the wavelength must be either up or down-shifted to the reference wavelength. The wavelength shifting for the splitting and reconstruction stages is presented in Fig. 3.

In our experiment, separate filtering of the $|\psi\rangle$ state components requires adjacent channels with a 0.02 nm detuning from the reference frequency. This corresponds to an approximate magnetic field of 900 Gauss. Reconstruction of the filtered state corresponds to an input RF signal of approximately 2.4 GHz, as the general dependence of the reconstruction signal as a function of the magnetic field is linear:

$$\Delta \nu = \frac{m \gamma_\lambda \mu_\mu \Delta B - \frac{c}{\lambda_i}}{h}.$$  \hspace{1cm} (18)

The processed signal is multiplexed and then sent to Alice and Bob's detectors. The method implies losses of 50% at the beam-splitter. The $|\phi\rangle$ entangled state is obtained simply by placing the two ultra narrowband filters at the up or down shifted frequencies $\lambda \pm \Delta \lambda$. Because both components of the state are
spatially located in the same beam profile, wavelength shifting is no longer necessary. Because the two photons are propagating simultaneously, spatial post-selection cannot be achieved just by using 50/50 beam-splitters, as two-photon interference occurs. The post-selection is simply done by means of time-delay interferometers. As in the $|\psi\rangle$ state case, 50% of the photons will be lost in this post-selection process.

In order to illustrate all of our considerations, we have carried out simulations for the two experimental rigs with the aid of the MATLAB simulation environment. The beam profiles for the magnetic field and RF values were simulated for the construction of the $|\psi\rangle$ state, while standard Bell measurement simulations have been conducted for both states. The results are shown in Figs. 3 and 4: Fig. 3 illustrates the separation and realignment of the laser beam in terms of wavelength, while Fig. 4 illustrates the sinusoidal shape of the recorded coincidences corresponding to a standard Bell measurement: Bob rotates his polarization analyzer in 5 degree steps from 0 to 90 degrees, with respect to Alice's fixed analyzer at different polarization angles. The strong sinusoidal shape together with the low noise counts admitted in the measurement process prove the existence of post-selected polarization-entangled pairs within the 4-level laser.

![Simulation results for the split (left) and reconstructed (right) beam profiles for the $|\psi\rangle$ state.](image.png)
Based on the data obtained from the simulations, we have calculated the degree of entanglement to be 83% for the $|\psi\rangle$ state and 85% for the $|\phi\rangle$ state. Also, the theoretical angles between Alice and Bob's analyzers were chosen to be $\frac{\pi}{8}$, and the experimental graphs were fitted to a series of cosines that yield the Bell parameter as $S = 2.79 \approx 2\sqrt{2}$ which demonstrates the existence of entangled states within the considered structure.

4. CONCLUSIONS

We have theoretically demonstrated the existence of naturally entangled photon pairs in the magnetically modified structure of a laser device. Circular polarization photons can be created by the use of the Zeeman effect and then conveniently manipulated in frequency and time in order to achieve photon identicity. This use of laser devices bypasses the use of non-linear optical equipment such as optical waveguides and optical crystals, together with their small entangled photon pair generation efficiency, and helps with the further integration of the entangled photon pair source.

REFERENCES


