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COOPERATIVE GENERATION OF ENTANGLED STATES BY RAMAN CONVERSION OF PHOTONS IN NANOFIBERS

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Abstract. We propose the cooperative conversion of the photons between pump and anti-Stokes pulses stimulated by the trapped atoms in the evanescent fields of nanofibers. This new type of cooperative conversion creates the quantum correlations between the photons, belonging to pump and anti-Stokes modes of the propagation pulse in nonlinear interaction with excited atoms, trapped in the evanescent zone of nanofiber. The trapped atomic system in this zone of nanofiber generates the second order coherence between the pump and anti-Stokes photons and creates the good phase and amplitude of two-field product. For the description of the quantum properties of this field, we have used the bi-boson operators and we have studied the cooperative and entangled states of the photons from the pump and anti-Stokes pulses.

Key words: cooperative conversion, Stokes and anti-Stokes photons, evanescent field.

1. DESCRIPTION OF COOPERATIVE CONVERSION. HAMILTONIAN OF THE SYSTEM

In this paper, we consider the propagating of the pulse along a nanofiber in the two-photon Raman resonance with the excited atomic subsystem trapped in the evanescent field of this fiber [1]. During the pulse propagation in the fiber wave mode the new pulse is generated due to the scattering conversion of the photons in anti-Stokes (or Stokes) mode becomes possible in the process of relaxation (excitation) of the atoms trapped in the evanescent zone of the fiber. By comparing the pulse propagation through the fiber with atomic propagation through the cavity, studied in micro-maser models [2, 3], we propose to study the cooperative photon conversion in the dynamical regime of photon conversion between the two pulses belonging to two modes of nanofiber during the pulse propagation. Because the pump pulse is regarded as the "moving cavity" along the static atomic line, we replace: a) the propagation of atomic flux through the cavity with trapped atomic system in the evanescent zone of the fiber and b) the cavity standing waves in which the photons are generated by two pulses of the light propagating along the fiber [4, 5] (Figs. 1, 2). Due to the interaction of this pulse with the three-level system trapped in the near field zone of fiber it is possible the cooperative conversion of the photons from the pump field into the new pulse mode (Stokes or anti-Stokes) as function of the preparation of the atomic subsystem concentrated in the evanescent field. In other words, we propose to change the standing waves of the cavities with the cooperative conversion of the photons into anti-Stokes mode during the pulse propagation. This effect helps us to obtain in the output field of nanofiber the new correlated pulse of photons generated in the two entangled state modes of the fiber (Figs. 1, 2). The conversion takes place during the time propagation of the short pump pulse through the fiber and the conversion rate depends on the interaction time of the atoms with the photons from nanofiber [4, 5]. The phase and amplitude of the bi-boson field coherently created in the two fiber modes is discussed in the language of induced Bose condensate.

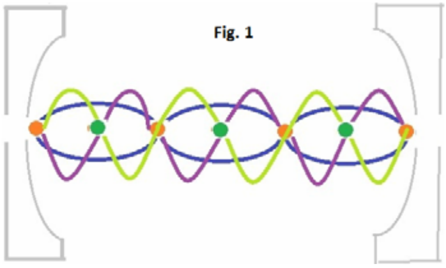

Fig. 1 – Cooperative conversion of the photons from Stokes in anti-Stokes modes stimulated by a stream of atom flying through the micro-cavity two standing waves [4, 5].

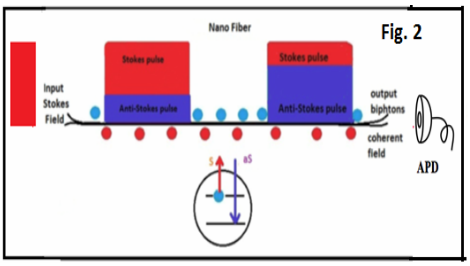

Fig. 2 – Propagation of Stokes pulse through nanofiber and its conversion in anti-Stokes pulse. The red part of the pulse is the Stokes component and the blue part corresponds to the anti-Stokes one.

The anti-Stokes photons are created simultaneously in the same propagation pump pulse during the cooperative scattering process. As a model we consider that *n*-photons from the pump field may be converted into the anti-Stokes photon field, so that after the cooperative time of the conversion the pump and anti-Stokes modes form the new coherent state as a superposition between both modes. This conversion is similar to the *Dicke superradiance* [6], but the cooperation appears between the photons belonging to two propagation modes of nanofiber. The coherence proprieties of this bimodal field may be detected in the output field of fiber using the correlation schemes like Hanbury-Brown and Twiss (HBT) [7] or balanced homodyne detectors [8].

The Hamiltonian that describes the interaction of the atoms with Stokes and anti-Stokes modes of the cavity electromagnetic field (EMF) can be represented through the atomic and field operators

$$H = \hbar(\omega_a + \omega_b)^{n_{ph}}\!\big/_2 + \hbar(\omega_b - \omega_a)\hat{J}_z + \sum_{j=1}^{N} \hbar\omega_0 \hat{R}_z^j + i\sum_{j=1}^{N} G_j(k_a,\ k_b)\{\hat{R}_j^- \hat{J}^+ - \hat{J}^- \hat{R}_j^+\} \quad (1)$$

in which the last term represents the interaction Hamiltonian part, $\hat{R}_{zj} = (|e\rangle_l \langle e|_l - |g\rangle_j \langle g|_j)/2$ is the population inversion of atom j; $\hat{R}_j^+ = i|e\rangle_j \langle g|_j \mathrm{e}^{-i(\mathrm{k}_b - \mathrm{k}_a)\mathrm{x}_j}$ and $\hat{R}_j^- = -i|g\rangle_j \langle e|_j \mathrm{e}^{i(\mathrm{k}_a - \mathrm{k}_b)\mathrm{x}_j}$ represent the operators, which describe the excitation and lowering operators between the |g>-ground and |e > -excited states of atomic subsystem, respectively (Fig. 3). The operator $\hat{a}(\hat{a}^+)$ is the creation (annihilation) of Stokes photons and $\hat{b}(\hat{b}^+)$ is the creation (annihilation) of anti-Stokes field operators.

The interaction constant $G(k_a, k_b) \approx 2d_{ei}d_{gi}g_a g_s/(\hbar\delta)$ describes the effective nonlinear coupling of the atom with fiber modes k_a and k_b with the energies $\hbar\omega_a$, $\hbar\omega_b$ and polarizations $e_{\lambda s}$ $e_{\lambda a}$, respectively. In order to describe the collective processes, we introduced in Hamiltonian (1) the collective operators for bimodal field $\hat{J}^+ = \hat{a}\hat{b}^+$, $\hat{J}^- = \hat{b}\hat{a}^+$, $\hat{J}_z = (\hat{a}^+\hat{a} - \hat{b}^+\hat{b})/2$, which are represented in Fig. 3.

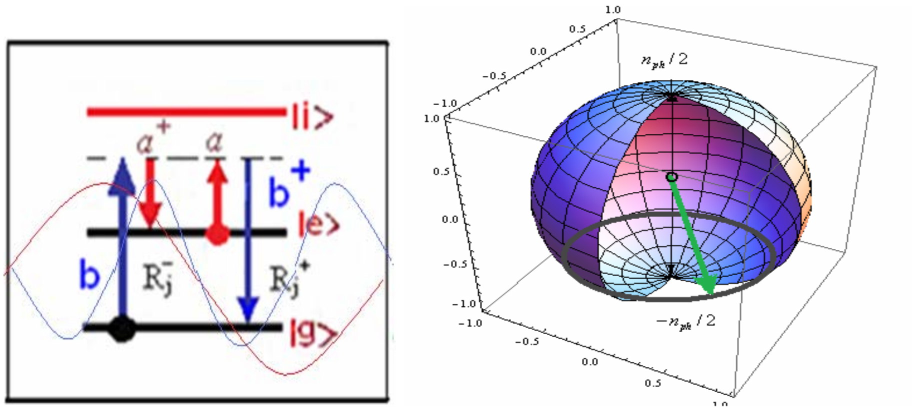

Fig. 3 – The left panel shows the collective excitation of atom with absorption of Stokes photon and generation of anti-Stokes quanta and vice-versa. The geometric representation of the symmetry of such field generation in the phase space of bimodal cavity is shown in the right panel.

The operator of annihilation (generation) for Stokes mode is $\hat{a}^{+}(\hat{a})$ and for anti-Stokes mode is $\hat{b}^{+}(\hat{b})$ belong to SU(2) symmetry (Fig. 3). All Raman processes takes place on the surface of this sphere. This reduction to SU(2) symmetry helps us to solve exactly the superradiant conversion between the Stokes and anti-Stokes photon using the conservation law of Bloch vector.

During the propagation we can use the generalized operator equations for the field components in nanofiber:

$$\frac{d}{dx}\left\langle \hat{O}(x)\right\rangle = i\tilde{\omega}\left\langle \left[\hat{J}_z(x),\hat{O}(x)\right]\right\rangle + \frac{1}{\hbar}\sum_{l=1}^{N}\left\{\left\langle [\hat{J}^{-}(x),\hat{O}(x)]\hat{R}_l^{+}(x)\right\rangle + Hc\right\},$$

where the direction of propagation is $x = ct$, c is the photon velocity in the nanofiber and t is the time of propagation.

In this section the generation of second-order coherence between the pump and anti-Stokes photons is proposed. The bi-boson operators are introduced for the description of these quantum properties of this field. This representation describes the cooperation and entangled states of the photons from the pump and anti-Stokes pulses. As in Dicke superradiance the cooperative effect appears between indistinguishable atoms, thus we decide to export this collective effect between undistinguished photons of Stokes and pump modes during the conversion.

2. ANALOGIES BETWEEN TWO CASES AND A NEW EFFECT

The Dicke superradiance is the emission of an excited atom system of N atoms with intensity of light proportional to the square number of atoms. This effect is attractive because the atoms become indistinguishable, when the dimension of the system is less than the emitted wavelength [6]. In the process of spontaneous emission the cooperative effect appears between indistinguishable atoms so that the system generates a pulse of photons proportional to the square number of atoms. A similar effect can be proposed in the scattering processes between two cavity modes [5]. In this case instead of atoms we have photons. If the photons are scattered in another mode of cavity we may consider like another state of a cavity. In other words, we can associate the photons distributed between two cavity modes as an ensemble of two level system. In fact, if the photons are initially prepared in the Stokes mode (red light, Fig. 4) they can be superradiantly converted into anti-Stokes mode (blue light, Fig. 4). If we have the conversion of photons from red to blue light modes (Fig. 4), it appears the problem: what kind of superradiance is generated in this process? The answer is that the rate of atoms in the ground state firstly increases achieving a maximum value then it decreases like in the case of a superradiance effect. This rate is proportional to the square number of photons prepared in anti-Stokes mode [4, 5] (Fig. 4). According to Fig. 4 this is a rather difficult effect. The state of light can be measured testing the number of atoms in the ground and excited states like in the Walther micromaser experiment [2]. We want to change this procedure of measurements, replacing the two mode cavity with electromagnetic pulse in the fiber. A nanofiber system is a device where the diameter of the evanescent zone is larger than the fiber diameter.

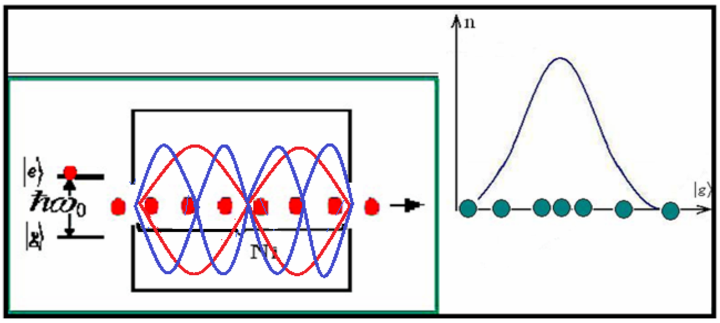

Fig. 4 – The atomic pump of the scattering process with the transformation of *n*-Stokes photon field and the number of atoms in the ground state are presented like in the Walther [2] micromaser experiment (Color online).

An experimental group [10] used a resonant laser for probing the atoms *via* the evanescent field. In this case the atomic beam used in the microcavity can be replaced by trapped atoms along the nanofiber as is presented in Figs. 1, 2. We replace the atomic beam with trapped atoms (or doped fiber), and standing waves with two pulses propagating along the fiber like in Fig. 2. This is equivalent with the studying of moving cavity in the reference system connected with moving atoms through the cavity. If we will pass on this reference system the cavity will move in the opposite direction with velocity $-V$.

Let us now discuss the main differences between the Dicke superradiance and cooperative conversion proposed in this paper. The excited ensemble of N two-level atoms passes to ground state achieving a maximal value of radiation intensity when the inversion takes zero value. The correlation between the atoms from excited and ground state during the emission takes place through vacuum state of EMF. During the atomic superradiance the system emits a pulse of correlated photons. The detection of superradiance is possible if we study the photon pulse shape and photon correlations.

In the case of cooperative conversion of photons we observe that the n photons prepared in the pump mode are converted into the anti-Stokes photons. The cooperative correlation between the pump and anti-Stokes photons take place through the excited state of trapped atoms. In other words, the role of vacuum fields in Dicke superradiance is replaced by "atomic ensemble" in the excited state. The coherence properties of this field depend on the ratio τ_p/τ_c between the propagation time τ_p through the fiber and the cooperative conversion time τ_c. When

$\tau_p/\tau_c < 1$, a number of photons from the pump field was partially converted in the anti-Stokes photons. The opposite case $\tau_p/\tau_c > 1$ can be regarded as a total conversion of pump photons to anti-Stokes photons. According to the theory of superradiance we associated the delay time with τ_p , the time of the formation of superradiance pulse $\tau_p = (\tau_0/n)\ \ln(n)$, where τ_0 is the scattering time. As the similar delay time appears in the process of photon conversion on the pump field into anti-Stokes one, we may consider it large in comparison with superradiance pulse duration. To avoid this discrepancy between the pulse propagation and the delay time of superradiance of cooperative process, it is better to initiate the anti-Stokes pulse to external small pulse. In this situation the cooperative conversion process can start immediately with the entrance the pump pulse in fiber.

3. MASTER EQUATION FOR BIMODAL FIELD

Following the method of elimination of the polarization of atomic subsystem, we can obtain the master equation for the bimodal field in the pulse, represented in Figure 2.

$$\begin{aligned}\frac{\mathrm{d}W(x)}{\mathrm{d}x} &= \beta_1[\hat{J}^+(x)W(x)\hat{J}^-(x) - W(x)\hat{J}^-(x)\hat{J}^+(x) + Hc] - \\ &- \beta_2[\hat{J}^+(x)W(x)\hat{J}^-(x)\hat{J}^+(x)\hat{J}^-(x) - W(x)\hat{J}^-(x)\hat{J}^+(x)\hat{J}^-(x)\hat{J}^+(x) + Hc].\end{aligned} \tag{2}$$

Here $\beta_1 = \tilde{\alpha}_1 / c = G^2(k_a, k_b)N / (c\hbar^2)$ is the analytical expression of emission gain of anti-Stokes photons; G is the interaction constant; the diffusion coefficient is $\beta_2 = \tilde{\alpha}_2 / c = (\tilde{\alpha}_1)^2 / c$.

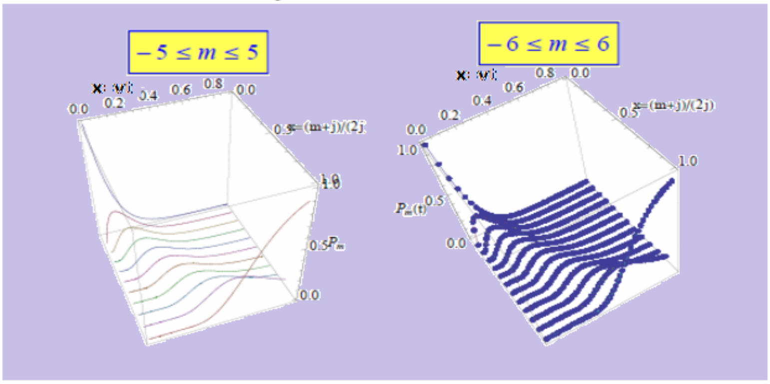

Fig. 5 – The dependence $P_m(t)$ as function of label number m and time t in the relative units $x = (m+j)/(2j)$ and $2\alpha_1 t$ for two cases: $-5 \le m \le 5$ and $-6 \le m \le 6$. The time dependence of probabilities $P_m(t)$ for the following values of the relative parameter $\alpha_1/\alpha_2 = 0.005$ and the number of photons $2j = 10$ and $2j = 12$.

The solution of the master equation (2) can be presented through diagonal bra-ket [4] operators of the angular momentum:

$$W(x)=\sum_{m=-j}^{+j}P_m(x)\left|m,j\right\rangle\left\langle m,j\right|. \tag{3}$$

The set of functions P_m describes the probability of simultaneous existence of $(2j+1)$ Dicke states in the scattering process. The initial condition for the system of equations is $P(-j) = 1$, $P(-j+1) = P(-j+2) = ... = P(j) = 0$, all photons are prepared in the pump pulse, which was named the Stokes mode in Ref. [5]. For example, the distribution of probabilities P_{m} as a function of the photon number 10 and 12 respectively, is shown in Figure 5. The sum of these probabilities is equal to unity:

$$\sum_{m=-j}^{+j}P_m(x)=1.$$

4. CONCLUSIONS

In this paper, it is proposed a new type of cooperative conversion of the photons between pump and anti-Stokes pulses stimulated by the trapped atoms in the evanescent field of nanofibers. It is demonstrated that this type of cooperative conversion creates the quantum correlations between the photons, belonging to pump and anti-Stokes modes of the propagation pulse through the nonlinear interaction with excited atoms, trapped in the evanescent zone of nanofiber. The generation of second order coherence between the pump and anti-Stokes photons is put forward. We introduced the bi-boson operators, which were used for the description of the quantum properties of the system. This representation describes the cooperative and entangled states of the photons from the pump and anti-Stokes pulses.

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