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A STRAIGHTFORWARD REALIZATION OF A QUASI-INVERSE SEESAW MECHANISM AT TEV SCALE

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Abstract. In this letter we address the issue of generating appropriate tiny neutrino masses within the framework of a particular $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge model by adding three singlet exotic Majorana neutrinos to the ones included in the three lepton triplet representations. The theoretical device is the general method of treating gauge models with high symmetries $SU(3)_c \otimes SU(N)_L \otimes U(1)_X$ proposed by Cotăescu more than a decade ago. When it is worked-out in the 3-3-1 model it supplies a unique free parameter (a) to be tuned in order to get a realistic mass spectrum for both the boson and charged-fermion sectors. Its most appealing feature (of special interest here) is that it contains all the needed ingredients to realize the inverse seesaw mechanism for neutrinos. The mandatory couplings leading to the lepton number soft violation in pure Majorana terms result without invoking any element outside the model (such as GUT scales, as one usually does in the literature). The overall breaking scale in this particular model can be set around 1 TeV so its phenomenology is quite testable at present facilities.

Key words: inverse seesaw mechanism, right-handed neutrinos, extensions of the SM.

1. INTRODUCTION

It is well-known that the Standard Model (SM) ([1] - [3]) - based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ undergoing a spontaneous symmetry breaking (SSB) in its electro-weak sector up to the universal $U(1)_{em}$ - is not a sufficient device, at least for some stringent issues in the particle physics today. When it comes to generating neutrino tiny masses [4, 5], the framework of the SM is lacking the needed ingredients, so one should call for some extra considerations which are less natural in the context. One of the ways out seems to be the enlargement of the gauge group of the theory as to include naturally among its fermion representations some right-handed neutrinos - mandatory elements for some plausible mass terms in the neutrino sector Yukawa Lagrangian density (Ld) of the theory.

Among such possible extensions of the SM, the so called "3-3-1" class of models [6] - [9] - where the new gauge group is $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ - has meanwhile established itself as a much suitable candidate. A systematic classification [10] - [12] and phenomenological study of these models (especially those which don't include exotic electric charges [13] - [23]) have been done during the last two decades. Some of the studies address the neutrino mass issue [24] - [35] with viable results within the framework of such models.

Here we propose a slightly different approach from the canonical one, in the sense that we apply the prescriptions of the general method [36] of treating gauge models with high symmetries. Proposed initially by Cotăescu, it essentially consists of a general algebraical procedure in which electro-weak gauge models with high symmetries ($SU(N)_L \otimes U(1)_Y$) achieve their SSB in only one step up to the residual $U(1)_{em}$ by means of a special Higgs mechanism. The scalar sector is organized as a complex vector space where a real scalar field φ is introduced as the norm for the scalar product among scalar multiplets. It also ensures the orthogonality in the scalar vector space. Thus, the survival of some unwanted Goldstone bosons is avoided. This leads to a one-parameter mass spectrum, due to a restricting trace condition that has to hold throughout. The compatibility of this particular method with the canonical approach to 3-3-1 models in the literature was proved in a recent paper by the author [37] where an appealing outcome with only two physical massive Higgses with non-zero interactions finally emerged. This will be precisely the framework of our proceedings here. Furthermore, we exploit the realization of a kind of quasi-inverse seesaw mechanism [38] - [46] in the framework of 3-3-1 gauge models with 3 right-handed neutrinos (ν_R) included in the fermion triplets and 3 exotic sterile Majorana singlets (N_R) , in which the free parameter (let's call it a) is tuned in order to obtain the whole mass spectrum. An apparently unused up to now parameter η_0 in the general method proves itself here as the much needed "lepton number violating" coupling to achieve the Majorana mass terms for N_R in the neutrino sector.

The letter is divided into 5 sections. It begins with a brief presentation of the model and its parametrization supplied by the general Cotăescu method (in Sec.2) and continues with the inverse seesaw mechanism worked out within this framework (Sec. 3) and the tuning of the parameters (Sec. 4) in order to obtain phenomenologically viable results for the neutrino masses. Some conclusions are sketched in the last section (Sec. 5).

2. BRIEF REVIEW OF THE MODEL

The particle content of the 3-3-1 gauge model of interest here is the following:

Lepton families

$$l_{\alpha L} = \left(\begin{array}{c} \nu_\alpha^c \\ \nu_\alpha \\ e_\alpha \end{array} \right)_L \sim (\mathbf{1}, \mathbf{3}, -1/3) \qquad e_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, -1) \tag{1}$$

Quark families

$$Q_{iL} = \left(\begin{array}{c} D_i \\ -d_i \\ u_i \end{array} \right)_L \sim (\mathbf{3}, \mathbf{3}^*, 0) \qquad Q_{3L} = \left(\begin{array}{c} U_3 \\ u_3 \\ d_3 \end{array} \right)_L \sim (\mathbf{3}, \mathbf{3}, +1/3) \tag{2}$$

$$d_{iR}, d_{3R} \sim (\mathbf{3}, \mathbf{1}, -1/3) \qquad u_{iR}, u_{3R} \sim (\mathbf{3}, \mathbf{1}, +2/3) \tag{3}$$

$$U_{3R} \sim (\mathbf{3}, \mathbf{1}, +2/3) \qquad D_{iR} \sim (\mathbf{3}, \mathbf{1}, -1/3) \tag{4}$$

with $i = 1, 2$.

The above representations ensure the cancellation of all the axial anomalies (by an interplay between families, although each one remains anomalous by itself). In this way one prevents the model from compromising its renormalizability by triangle diagrams. The capital letters are reserved for the exotic quarks (D_i, D_2 and U_3) in each family. They are heavier than the ordinary quarks known from the SM.

To this fermion content one can add 3 Majorana exotic neutrinos $N_R \sim (\mathbf{1}, \mathbf{1}, 0)$ without the danger of spoiling the renormalizability. The advantage these 3 exotic neutrinos bring is that they can play a crucial role in realizing the inverse seesaw mechanism [38] - [46].

Gauge bosons

The gauge bosons of the model are determined by the generators of the electro-weak $su(3)$ Lie algebra, expressed by the usual Gell-Mann matrices $T_a = \lambda_a/2$. So, the Hermitian diagonal generators of the Cartan sub-algebra are

$$D_1 = T_3 = \frac{1}{2}\text{Diag}(1, -1, 0), \quad D_2 = T_8 = \frac{1}{2\sqrt{3}}\text{Diag}(1, 1, -2). \tag{5}$$

In this basis the gauge fields are expressed by: A_μ^0 (corresponding to the Lie algebra of the group $U(1)_X$) and $A_\mu \in su(3)$, that can be put as

$$A_\mu = \frac{1}{2} \left(\begin{array}{ccc} A_\mu^3 + A_\mu^8/\sqrt{3} & \sqrt{2}X_\mu & \sqrt{2}Y_\mu \\ \sqrt{2}X_\mu^* & -A_\mu^3 + A_\mu^8/\sqrt{3} & \sqrt{2}W_\mu \\ \sqrt{2}Y_\mu^* & \sqrt{2}W_\mu^* & -2A_\mu^8/\sqrt{3} \end{array} \right), \tag{6}$$

where $\sqrt{2}W_\mu^\pm = A_\mu^6 \mp iA_\mu^7$, $\sqrt{2}Y_\mu^\pm = A_\mu^4 \pm iA_\mu^5$, and $\sqrt{2}X_\mu = A_\mu^1 \pm iA_\mu^2$, respectively. One notes that apart from the charged Weinberg bosons (W) from SM, there are two new complex boson fields, X (neutral) and Y (charged) - off-diagonal entries in eq. (6).

The diagonal Hermitian generators will provide us with the neutral gauge bosons A_μ^{em}, $Z_\mu$and Z'_μ. Therefore, on the diagonal terms in eq.(6) a generalized Weinberg transformation (gWt) must be performed in order to consequently separate the massless electromagnetic field from the other two neutral massive fields. One of the two massive neutral fields is nothing but the Z^0-boson of the SM. The details of the general procedure with gWt can be found in Ref. [36] and its concrete realization in the model of interest here in Refs. [19, 26]. In Ref. [19] the neutral currents for both Z_μ and Z'_μ are completely determined, while in Ref. [26] the boson mass spectrum is calculated.

For the sake of completeness we write down the electric charge operator in this particular 3-3-1 model when Cotăescu method is involved. It stands simply as: $Q^\rho = \frac{2}{\sqrt{3}}T_8^\rho + Y^\rho$ for each representation ρ.

Scalar sector and spontaneous symmetry breaking

In the general method [36], the scalar sector of any $SU(N)_L \otimes U(1)_Y$ electroweak gauge model must consist of n Higgs multiplets $\phi^{(1)}$, $\phi^{(2)}$, ... , $\phi^{(n)}$ satisfying the orthogonal condition $\phi^{(i)+}\phi^{(j)} = \varphi^2\delta_{ij}$ in order to eliminate unwanted Goldstone bosons that could survive the SSB. Here $\varphi \sim (1,1,0)$ is a gauge-invariant real field acting as a norm in the scalar space and n is the dimension of the fundamental irreducible representation of the gauge group. The parameter matrix $\eta = (\eta_0, \eta_1, \eta_2.., \eta_n)$ with the property $Tr\eta^2 = 1-\eta_0^2$ is a key ingredient of the method: it is introduced in order to obtain a non-degenerate boson mass spectrum. Obviously, $\eta_0, \eta_i \in [0,1)$. Then, the Higgs Ld reads:

$$\mathcal{L}_H = \frac{1}{2}\eta_0^2\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\sum_{i=1}^{n}\eta_i^2\left(D_\mu\phi^{(i)}\right)^+\left(D^\mu\phi^{(i)}\right) - V(\phi^{(i)}) \tag{7}$$

where $D_\mu\phi^{(i)} = \partial_\mu\phi^{(i)} - i(gA_\mu + g'y^{(i)}A_\mu^0)\phi^{(i)}$ act as covariant derivatives of the model. g and g' are the coupling constants of the groups $SU(N)_L$ and $U(1)_X$ respectively. Real characters $y^{(i)}$ stand as a kind of hyper-charge of the new theory.

For the particular 3-3-1 model under consideration here the most general choice of parameters is given by the matrix $\eta^2 = \left(1-\eta_0^2\right) Diag\left[1-a\,,\,\frac{1}{2}(a-b)\,,\,\frac{1}{2}(a+b)\right]$. It obviously meets the trace condition required by the general method for any $a,b \in [0,1)$. After imposing the phenomenological condition $M_Z^2 = M_W^2/\cos^2\theta_W$ (confirmed at the SM level) the procedure of diagonalizing the neutral boson mass matrix [19, 26] reduces to one the number of parameters, so that the parameter matrix reads

$\eta^2 = \left(1-\eta_0^2\right) Diag\left[1-a\,,\, a\frac{\left(1-\tan^2\theta_W\right)}{2}\,,\, a\frac{1}{2\cos^2\theta_W}\right].$

With the following content in the scalar sector of the 3-3-1 model of interest here (and based on the redefinition of the scalar triplets from the general method, as in the Ref.[37])

$$\rho = \begin{pmatrix} \rho_1^0 \\ \rho_2^0 \\ \rho_3^- \end{pmatrix}, \chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^- \end{pmatrix} \sim (\mathbf{1},\mathbf{3},-1/3)\,, \quad \phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^0 \end{pmatrix} \sim (\mathbf{1},\mathbf{3},+2/3)\,, \tag{8}$$

one can achieve *via* the SSB the following vacuum expectation values (VEV) in the unitary gauge:

$$\begin{pmatrix} \eta_1 \langle\varphi\rangle + H_\rho \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \eta_2 \langle\varphi\rangle + H_\chi \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ \eta_3 \langle\varphi\rangle + H_\phi \end{pmatrix}, \tag{9}$$

with the overall VEV

$$\langle\varphi\rangle = \frac{\sqrt{\mu_1^2\eta_1^2 + \mu_2^2\eta_2^2 + \mu_3^2\eta_3^2}}{\sqrt{2\left(\lambda_1\eta_1^4 + \lambda_2\eta_2^4 + \lambda_3\eta_3^4\right) + \lambda_4\eta_1^2\eta_2^2 + \lambda_5\eta_1^2\eta_3^2 + \lambda_6\eta_2^2\eta_3^2}} \tag{10}$$

resulting from the minimum condition applied to the potential

$$\begin{aligned} V &= \mu_1^2\rho^+\rho - \mu_2^2\chi^+\chi - \mu_3^2\phi^+\phi \\ &+\lambda_1\left(\rho^+\rho\right)^2 + \lambda_2\left(\chi^+\chi\right)^2 + \lambda_3\left(\phi^+\phi\right)^2 \\ &+\lambda_4\left(\rho^+\rho\right)\left(\chi^+\chi\right) + \lambda_5\left(\rho^+\rho\right)\left(\phi^+\phi\right) + \lambda_6\left(\phi^+\phi\right)\left(\chi^+\chi\right) \\ &+\lambda_7\left(\rho^+\chi\right)\left(\chi^+\rho\right) + \lambda_8\left(\rho^+\phi\right)\left(\phi^+\rho\right) + \lambda_9\left(\phi^+\chi\right)\left(\chi^+\phi\right). \end{aligned} \tag{11}$$

3. QUASI-INVERSE SEESAW MECHANISM

With the above ingredients one can construct the Yukawa Ld allowed by the gauge symmetry in the 3-3-1 model with right-handed neutrinos. It simply is:

$$\begin{aligned}-\mathcal{L}_Y &= h_\phi \bar{l}\phi e_R + h_\rho \bar{l}\rho N_R + h_\chi \bar{l}\chi N_R + \tfrac{1}{2} h_\varphi \overline{N_R^c}\eta_0 \varphi N_R \\ &+ \tfrac{1}{2} h \varepsilon^{ijk} \left(\bar{l}\right)_i \left(l^c\right)_j \phi_k + h.c.\end{aligned} \tag{12}$$

where hs are 3×3 complex Yukawa matrices, the lower index indicating the particular Higgs each one connects with.

It leads straightforwardly to the following mass terms:

$$\begin{aligned}-\mathcal{L}_{mass} &= h_\phi \overline{e_L} e_R \langle\phi\rangle + h_\rho \bar{l} N_R \langle\rho\rangle + h_\chi \bar{l} N_R \langle\chi\rangle + \tfrac{1}{2} h_\varphi \overline{N_R} N_R^c \eta_0 \langle\varphi\rangle \\ &+ \tfrac{1}{2}\left(h - h^T\right) \overline{\nu_L}\nu_R \langle\phi\rangle + h.c.\end{aligned} \tag{13}$$

The Yukawa terms allow one to construct the quasi-inverse seesaw mechanism by displaying them into the following 9×9 complex matrix:

$$M = \begin{pmatrix} 0 & \frac{1}{2}\left(h-h^T\right)\sqrt{\frac{1}{2\cos^2\theta_W}} & h_\chi\sqrt{\frac{a}{2}\left(1-\tan^2\theta_W\right)} \\ \frac{1}{2}\left(h^T-h\right)\sqrt{\frac{1}{2\cos^2\theta_W}} & 0 & h_\rho\sqrt{1-a} \\ h_\chi^T\sqrt{\frac{a}{2}\left(1-\tan^2\theta_W\right)} & h_\rho^T\sqrt{1-a} & h_\varphi\eta_0 \end{pmatrix}\langle\varphi\rangle \tag{14}$$

Due to the non-zero h_χ this matrix is slightly different from the traditional inverse seesaw mechanism [38] - [40], but its resulting effects - we prove in the following - are phenomenologically plausible. However, this kind of seesaw matrix appears in the literature, see for instance Refs. [41, 42]. This 9×9 complex matrix can be displayed as:

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix} \tag{15}$$

with $m_D = \begin{pmatrix} \frac{1}{2}\left(h-h^T\right)\sqrt{\frac{1}{2\cos^2\theta_W}} & h_\chi\sqrt{\frac{a}{2}\left(1-\tan^2\theta_W\right)} \end{pmatrix}$ a 3×6 complex matrix and $M_N = \begin{pmatrix} 0 & h_\rho\sqrt{1-a} \\ h_\rho\sqrt{1-a} & h_\varphi\eta_0 \end{pmatrix}$ a 6×6 complex matrix acting in the seesaw formula.

By diagonalizing the above matrix one gets the physical neutrino matrices as: $M(\nu_L) \simeq -m_D\left(M_N^{-1}\right)m_D^T$ and $M(\nu_R, N_R) \simeq M_N$ which yield:

$$\begin{aligned} M(\nu_L) \;&\simeq \tfrac{a\eta_0\langle\varphi\rangle}{8(1-a)\cos^2\theta_W}\left(h-h^T\right)\left(h_\rho^{-1}\right)^T\left(h_\varphi\right)\left(h_\rho^{-1}\right)\left(h^T-h\right) \\ &\quad -\tfrac{a\eta_0\sqrt{(1-\tan^2\theta_W)}\langle\varphi\rangle}{4\sqrt{(1-a)\cos^2\theta_W}}\left[\left(h_\chi\right)\left(h_\rho^{-1}\right)\left(h^T-h\right)+\left(h-h^T\right)\left(h_\rho^{-1}\right)^T\left(h_\chi\right)^T\right] \end{aligned} \tag{16}$$

$$\left(\begin{array}{cc} M(\nu_R) & 0 \\ 0 & M(N_R) \end{array}\right) = \left(\begin{array}{cc} h_\rho\sqrt{(1-a)}+\frac{1}{2}h_\varphi\eta_0 & 0 \\ 0 & -h_\rho\sqrt{(1-a)}+\frac{1}{2}h_\varphi\eta_0 \end{array}\right) \tag{17}$$

One can enforce here the realistic condition

$$\left[\left(h_\chi\right)\left(h_\rho^{-1}\right)\left(h^T-h\right)\right]^T = -\left(h_\chi\right)\left(h_\rho^{-1}\right)\left(h^T-h\right) \tag{18}$$

in order to eliminate the troublesome terms in the left-handed neutrino mass matrix. This condition can be naturally achieved if one takes into consideration the plausible identity

$$h_\chi = h_\rho \tag{19}$$

meaning that the exotic right-handed neutrinos N_R couples similarly with ν_L and ν_R respectively. Consequently, one gets the left-handed neutrino mass matrix as the complex 3×3 matrix:

$$M(\nu_L) \simeq \frac{a\eta_0\langle\varphi\rangle}{8(1-a)\cos^2\theta_W}\left(h-h^T\right)\left(h_\rho^{-1}\right)^T\left(h_\varphi\right)\left(h_\rho^{-1}\right)\left(h^T-h\right) \tag{20}$$

It is evident that it is a pure Majorana mass matrix since $M(\nu_L)^T = M(\nu_L)$ holds. Assuming that all the coupling matrices in the Yukawa sector are of the same order of magnitude, say $\sim O(1)$, one can estimate the order of magnitude of the individual masses in this matrix as

$$TrM(\nu_L) \simeq \frac{3a\eta_0\langle\varphi\rangle}{8(1-a)\cos^2\theta_W} \tag{21}$$

The right-handed neutrinos acquire some pseudo-Dirac masses, since finally one remains with $M(\nu_R)^T \neq M(\nu_R)$ and $M(N_R)^T \neq M(N_R)$ and h_ρ dictates their character.

4. TUNING THE PARAMETERS

Now one can tune the parameters in this particular model in order to get phenomenologically viable predictions. Obviously, both a, $\eta_0 \in (0,1)$. Since η_0 is the parameter responsible with the lepton number violation, one can keep it very small, say $\eta_0 \sim 10^{-8} - 10^{-9}$ in order to safely consider that the global $U(1)_{leptonic}$ symmetry is very softly (quite negligible) violated by the Majorana coupling it introduces.

When comparing the boson mass spectrum in this model - obtained both by using the general Cotăescu method [26] and the SM calculations [1] - one gets a scales connection:

$$\sqrt{\left(1-\eta_0^2\right)a} = \frac{\langle\varphi\rangle_{SM}}{\langle\varphi\rangle} \tag{22}$$

It becomes obviously that η_0 has no part to play in the breaking scales splitting. The later is determined quite exclusively by a. If one takes $\langle\varphi\rangle_{SM} \simeq 246$ GeV and $\langle\varphi\rangle \simeq 1$ TeV then $a \simeq 0.06$.

With these plausible settings the individual neutrino masses come out in the subsequent hierarchy:

$$\sum m\left(\nu_L\right) \simeq 1eV \tag{23}$$

$$\sum m\left(\nu_R\right) \simeq \sum m\left(N_R\right) \simeq 970GeV \tag{24}$$

Furthermore, one can enforce some extra flavor symmetries in the lepton sector in order to dynamically get the appropriate PMNS mixing matrix. Some discrete groups, such as A_4[46, 47], S_4[48] or S_3[49, 50] can be employed in 3-3-1 models with no exotic electric charges, in order to accomplish this task, but this exceeds the scope of this letter.

5. CONCLUSIONS

We have discussed here the possible realization of a quasi-inverse seesaw mechanism in the 3-3-1 class of gauge models with "lepton number violating" exotic Majorana neutrinos added. The Cotăescu general method of treating gauge models with high-symmetries is involved and it successfully provides us not only with the one-parameter mass spectrum but also with the lepton number violating terms needed for a plausible inverse seesaw mechanism due to the possibility of coupling the φ to exotic Majorana neutrinos. To the extent of our knowledge, in low energy models one finds no such terms to give masses to exotic neutrinos, so that some extra assumptions (usually from GUT theories) are invoked. These two characteristics single out our approach from other recent similar attempts [34, 35]. The details of the mixing in the neutrino sector are closely related to the entries in h, h_ρ and h_φ but this lies beyond the scope of this letter and will be presented elsewhere. The framework of this kind of 3-3-1 models is a very promising one. It recovers all the results of the SM and in addition exhibits a lot of assets: it requires precisely 3 fermion generations, its algebraic structure dictates the observed charge quantization, it can predict a testable Higgs phenomenology and, as we presented here, is suitable for neutrino phenomenology.

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