

ORIGIN OF MOLECULAR AND ISOMERIC MINIMA IN THE FRAGMENTATION POTENTIAL OF THE ^{296}Lv SUPERHEAVY ELEMENT

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Abstract. The fragmentation potentials for for the ^{296}Lv superheavy element are determined within two formalisms for comparison purposes. One is computed within the Woods-Saxon two center shell model and another within the one center shell model. Within the two center shell model two isomeric minima and a molecular minimum for alpha decay are obtained in the potential landscape. The second isomeric minimum located close to the Pb channel and the molecular minimum belonging to the alpha-decay are not evidenced by calculations made with the one center shell model. We conclude that the two center shell model is more appropriate for the treatment of very asymmetric disintegration processes and of the scission configurations than the one center shell model.

Key words: Macroscopic-microscopic model, shell effects, isomer states, super-heavy nuclei.

1. INTRODUCTION

During the synthesis of superheavy elements by heavy ion collisions, the fast fission processes are inherent. The mass distributions in fast fission processes can be explained by the formation of several isomeric and molecular states, evidenced by minima in the fragmentation potential [1, 2]. The macroscopic-microscopic model based on the Woods-Saxon two center shell model was used for this purpose [3]. The deformation energy was determined in a configuration space spanned by five degrees of freedom, extrapolating the fusion trajectory of the ^{48}Ca projectile. A wide range of mass asymmetries were explored beginning with the alpha channel and reaching the symmetric fission. It was considered that the minima obtained in the potential energy are responsible for the fast fission mass distribution structures because they behave as centers of re-emission. The experimental quasifission structure distributions obtained experimentally [4] were reproduced. So, metastable states of the compound nucleus are formed and decay through cold fission. We considered

that mostly the cold fission governs the fast fission process because the final states reach the ground state deformations of the fragments. The neutron emission [5] is not considered here. The origin of these isomeric minima were investigated in Ref. [6, 7]. The importance of the shell effects was emphasized for the existence of the metastable minima. However, a problem remains questionable. The behavior of the energy surface was obtained within a double center potential and no similar results were obtained to date within one center potentials. We are interested to investigate whether such results can be also be obtained within a one center shell model.

2. THE TWO CENTER SHELL MODEL

The calculations of single-particle levels for very deformed one-center potentials are limited the theoretical study of binary disintegration processes. Indeed, for very large prolate deformations the sum of single-particle energies obtained from the level scheme reaches an infinite value. The shapes for the passage of one nucleus to two separated nuclei is not described in a correct manner if we don't consider a very large number of multipolar deformations. By using a double center potential, these difficulties are surpassed. A two-center model allows the description of single-particle evolutions from the ground-state up to the formation of two separated fragments.

In our previous calculations, we used a two center Woods-Saxon potential that include the Coulomb interaction and the spin orbit term. A complete analytical eigenvectors basis can be only obtained for the semi-symmetric two-center oscillator:

$$V_o(\rho, z) = \begin{cases} \frac{1}{2}m\omega_{z1}^2(z - c_1)^2 + \frac{1}{2}m\omega_\rho^2, & z < 0, \\ \frac{1}{2}m\omega_{z2}^2(z - c_2)^2 + \frac{1}{2}m\omega_\rho^2, & z \geq 0, \end{cases} \quad (1)$$

where ω denotes the stiffness of the potential along different directions as follows, $\omega_{z1} = \omega_0 \frac{R_0}{a_1}$, $\omega_{z2} = \omega_0 \frac{R_0}{a_2}$, $\omega_\rho = \omega_0 \frac{R_0}{b_1}$, $\omega_0 = 41A_0^{-1/3}$, $R_0 = r_0A_0^{1/3}$. The asymmetric two center shell oscillator provides an orthogonal eigenvectors basis for only one Hermite space [8–11]. An analytic system of eigenvectors can be obtained for V_0 by solving the Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m_0}\Delta + V_o(\rho, z) \right] \Psi(\rho, z, \varphi) = E\Psi(\rho, z, \varphi) \quad (2)$$

The analytic solution of Eq. (2) is obtained using the ansatz

$$\Psi(\rho, z, \varphi) = Z(z)R(\rho)\Phi(\varphi) \quad (3)$$

with

$$\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} \exp(im\varphi) \quad (4)$$

$$R_{nm}(\rho) = \sqrt{\frac{2n!}{(n+m)!}} \alpha_\rho \exp\left(-\frac{\alpha_\rho^2 \rho^2}{2}\right) (\alpha_\rho \rho)^m L_n^m(\alpha_\rho^2 \rho^2) \quad (5)$$

$$Z_\nu(z) = \begin{cases} C_{\nu_1} \exp\left(-\frac{\alpha_{z_1}^2 (z-c_1)^2}{2}\right) \mathbf{H}_{\nu_1}[-\alpha_{z_1}(z+c_1)], \\ z < 0; \\ C_{\nu_2} \exp\left(-\frac{\alpha_{z_2}^2 (z-c_2)^2}{2}\right) \mathbf{H}_{\nu_2}[\alpha_{z_2}(z-c_2)], \\ z \geq 0, \end{cases} \quad (6)$$

where $L_n^m(x)$ is the Laguerre polynomial, $\mathbf{H}_\nu(\zeta)$ is the Hermite function, $\alpha_l = (m_0 \omega_l / \hbar)^{1/2}$ ($l = z_1, z_2, \rho$) are length parameters, and C_{ν_i} ($i = 1, 2$) denote the normalization constants. The quantum numbers n and m are integers while the quantum numbers ν_i along the z -axis are real and have different values for the intervals $(-\infty, 0]$ and $[0, \infty)$. Imposing conditions for the continuity of the wave function and its derivative, together with those for the stationary energy and orthonormality, the values of ν_1 , ν_2 , and of the normalization constants C_{ν_1} , and C_{ν_2} could be obtained. Details concerning these solutions and expressions for the normalization constants are found in Refs. [8, 9].

The solutions of the one center potential differ from the previous one by replacing Eq. (6) by

$$Z_n(z) = C_n \exp\left(-\frac{\alpha_z^2 z^2}{2}\right) H_n(-\alpha_z z) \quad (7)$$

where H is the Hermite polynomial and n is the integer quantum number along the axis z .

The macroscopic energy (liquid drop one) is obtained in the framework of the Yukawa-plus-exponential model [12] extended for binary systems with different charge densities as detailed in Ref. [13, 14]. The parameters of the model are taken from Ref. [15]. The macroscopic energy contains several terms: the surface energy, the Coulomb energy and its diffusivity, the volume energy, the Wigner term and the A_0 -one. When the nucleus changes the deformation, the single particle levels are rearranged. The internal structure of the nucleus is translated in shell and pairing corrections by means of the Strutinsky prescriptions [16]. These corrections represent the varying parts of the total binding energy caused by the shell structure. The single-particle-level diagrams are computed within the Woods-Saxon superasymmetric two-center shell model [3]. The Woods-Saxon potential, the spin-orbit term and the electrostatic potential are diagonalized within the two-center semi-symmetric oscillator wave functions [8–11]. This parametrization was widely used by the Bucharest group in the calculations addressing the cluster and alpha decay [17–24], the fission [25–29] and its dissipation, the pair breaking [30, 31], the heavy element synthesis [32, 33],

and the double beta decay [34]. Being able to treat extreme asymmetries of the fission process, the two center approach emerges as a competitive model for the alpha decay which is usually treated with preformation models [36–38].

It is important to note, that our previous calculations involving the formation and the disintegration of the superheavy elements were done by taking into account a gradual modification of the shell effects of the parent nucleus to the sum of the shell effects of the two partners nuclei in the region of the scission configuration. So, the shell and pairing corrections were computed by using the whole level scheme in the overlapping region and were determined separately for each nucleus by using their appropriate level schemes in the external region. This procedure was allowed by the two center shell model because it is able to identify precisely the single particle states located in the two potential wells formed after the scission. This identification is not possible with an one center potential. In order to be consistent in the comparison made between the two formalisms, the shell and pairing effects are calculated in the following by taking in consideration the whole level scheme even in the external region, no matter if the single particle states are located in the heavy or the light fragment. In other words, the shell corrections are calculated with a superposition of the single particle level schemes even if the fragments are separated. Consequently, the actual fragmentation potential will differ from that calculated previously within the two center shell model, in the external region.

It is worth to mention that the stability of the superheavy nuclei was predicted within the macroscopic-microscopic approximation [39]. Therefore the model is reliable. Latter on, the optimum choice of the two partners that collide to form a superheavy nucleus was determined in the framework of the fragmentation theory [40] by considering the fusion as a cold rearrangement process. Investigating the total potential energy, it was predicted that the most favorable reactions are connected with the valley of the Pb [41–43]. An analogue most favorable valley was found for cluster decay [44]. In Ref. [42], the ^{48}Ca was proposed as a projectile on various transuranium targets.

3. RESULTS AND DISCUSSION

The differences between the potential energy landscapes obtained with the macroscopic-microscopic model based on one center and two centers potentials will be investigated.

The path in the multidimensional configuration was used for the family of nuclear shapes that begins from the ground state and reach the scission point, as described in Ref. [1]. A trajectory was deduced for the $^{48}\text{Ca}+^{248}\text{Cm}$ system and was extrapolated for all the other mass asymmetries. For this purpose the eccentricities of

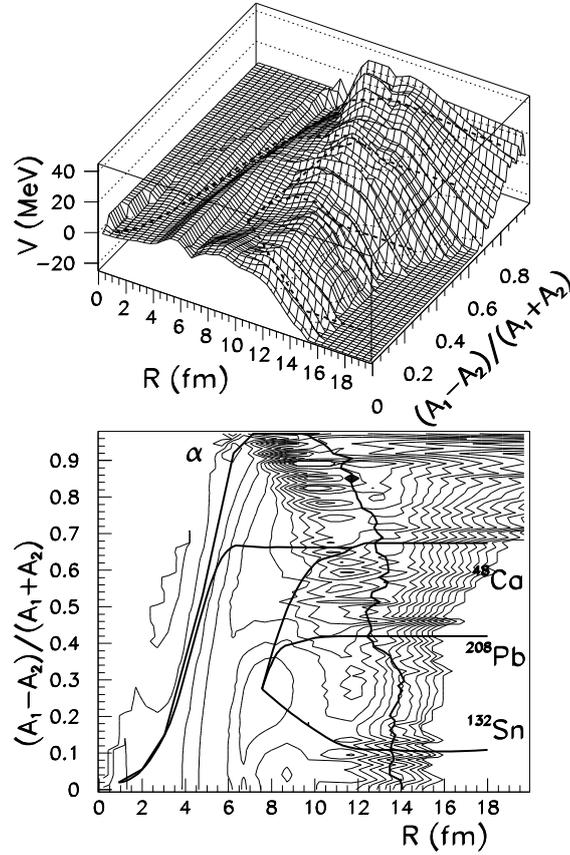


Fig. 1 – Potential energy surface as function of the mass asymmetry $(A_1 - A_2)/(A_1 + A_2)$ and of the elongation R calculated within the two center shell model. In the lower panel, a zig-zag vertical line identifies the scission elongations. The trajectories for the alpha decay, the fusion with Ca projectile, and the fast fission from the isomer well are plotted with full lines.

the ground states were computed for all partners in the reaction $^{A_1}Z_1 + ^{A_2}Z_2 = ^{296}116$.

In the Fig. 1, the energy surface for the fragmentation potential calculated within the two center shell model is represented as function of the mass asymmetry $\eta = (A_1 - A_2)/(A_1 + A_2)$ and the elongation R coordinates. By comparing the actual fragmentation surface and that published in Ref. [1], we can observe differences only close to the scission configurations and in the external regions. These differences are due to the different way used in the calculations of the shell and pairing effects, that does not take into account the localization of the single particle states in one of the two potential wells, as explained in the above section. The major difference is that the second isomeric minimum located along the Pb trajectory has vanished. So, this

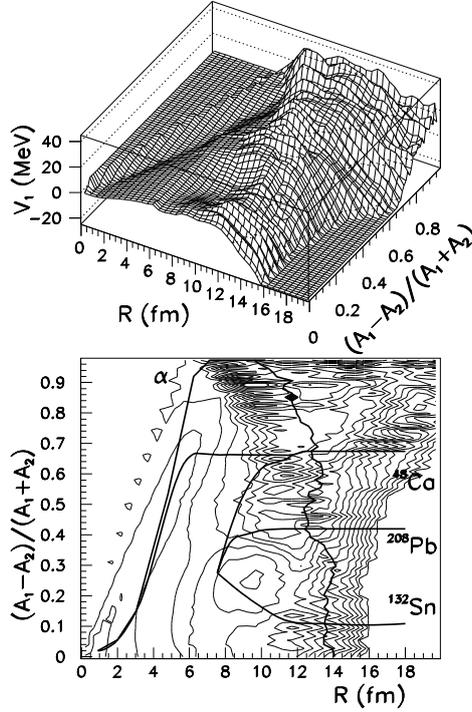


Fig. 2 – Same as figure 1 for the potential energy surface calculated within an one center shell model.

second minimum is due to the influence of the strong shell effects belonging to the doubly magic Pb, that are neglected in the framework of the actual procedure used for microscopic corrections.

In the Fig. 2, the energy surface for the fragmentation potential V_1 is displayed for the one center formalism. There are several major differences compared to the calculation of Fig. 1. A very large mass asymmetries, the molecular minimum for the alpha emission has completely vanished. The isomeric minimum located at the mass asymmetry $(A_1 - A_2)/(A_1 + A_2)$ shifted to larger values of the elongation R and its depth is reduced. In the Fig. 3, we plotted the difference $\Delta V = V_1 - V$ between the potential obtained with the one center formalism and that obtained with the two center one. For configurations close to spherical ones, the differences are very small, close to zero. For large mass asymmetries, at $\eta \approx 0$, the shell effects are more negative for the calculations based on the two center shell model. That explains why the alpha molecular states was observed only within computations based on the two center shell model. In general, the microscopic effect computed within the two center shell model are more negative. The differences are especially large in the

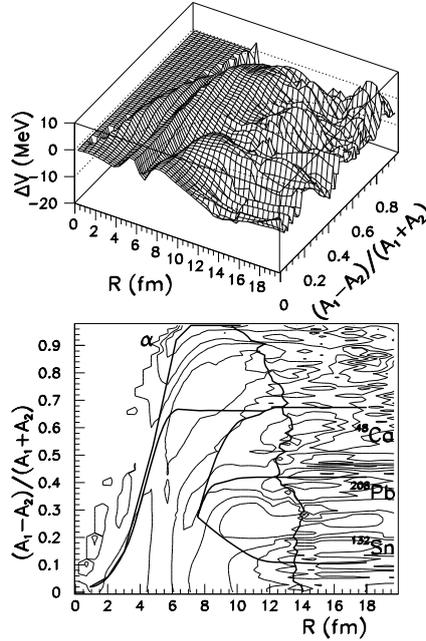


Fig. 3 – The difference ΔV between potential landscapes computed with the one center shell and the two center one.

region of the molecular minimum, $\eta = 0.3$, $R = 7$ fm, amounting to -4 MeV. So, the isomeric minimum should be more evident within the two center formalism. In the external regions, the differences are stabilized and becomes independent of R . However, some fluctuations for large mass asymmetries can be observed even after the scission. These fluctuations are due to the Coulomb polarization, that modifies the energies single particle levels in the external region, especially for the light fragment.

In conclusion, in this work we identified three main causes that prevented observations of the isomeric and molecular minima in works based on one center shell effects: the differences in shell effects that underestimated the stability of the main isomeric configurations, the failure to describe well very asymmetric scission configurations and the impossibility to identify the microscopic states in the two nuclear fragments. These findings suggest a superiority of the two center formalism in the treatment of fusion/fission processes by the ability to properly treat the scission configurations.

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REFERENCES

1. A. Sandulescu and M. Mirea, *Eur. Phys. J. A* 50, 110 (2014).
2. R. Budaca, A. Sandulescu and M. Mirea, *Mod. Phys. Lett. A* 30, 1550129 (2015).
3. M. Mirea, *Phys. Rev. C* 78, 044618 (2008).
4. M.G. Itkis, A.A. Bogachev, I.M. Itkis, G.N. Knyazheva, N.A. Kondratiev, E.M. Kozulin, L. Krupa, Yu.Ts. Oganessian, I.V. Pokrovsky, E.V. Prokhorova, A.Ya. Rusanov, V.M. Voskresenski, V.A. Rubchenya and W.H. Trzaska, *Int. J. Mod. Phys. E* 16, 957 (2007).
5. M. Rizea and N. Carjan, *Proc. Rom. Acad. Series A* 16, 176 (2015).
6. D. Aranghel and A. Sandulescu, *Rom. J. Phys.* 60, 147 (2015).
7. D. Aranghel and A. Sandulescu, *Rom. J. Phys.* 60, 1433 (2015).
8. M. Mirea, *Phys. Rev. C* 54, 302 (1996).
9. M. Mirea, *Nucl. Phys. A* 780, 13 (2006).
10. E. Badraxe, M. Rizea and A. Sandulescu, *Rev. Roum. Phys.* 19, 63 (1974).
11. J. Maruhn and W. Greiner, *Z. Phys.* 251, 431 (1972).
12. K.T.R. Davies and J.R. Nix, *Phys. Rev. C* 14, 1977 (1976).
13. M. Mirea, O. Bajeat, F. Clapier, F. Ibrahim, A.C. Mueller, N. Pauwels and J. Proust, *Eur. Phys. J. A* 11, 59 (2001).
14. D.N. Poenaru, M. Ivascu and D. Mazilu, *Comput. Phys. Commun.* 19, 205 (1980).
15. P. Moller, J.R. Nix, W.D. Myers and W.J. Swiatecki, *Atom. Data Nucl. Data Tabl.* 59, 185 (1995).
16. M. Brack, J. Damgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky and C.Y. Wong, *Rev. Mod. Phys.* 44, 320 (1972).
17. M. Mirea, *Eur. Phys. J. A* 51, 36 (2015).
18. A. Sandulescu, M. Mirea and D.S. Delion, *EPL* 101, 62001 (2013).
19. M. Mirea, *Phys. Rev. C* 63, 034603 (2001).
20. M. Mirea, *Rom. J. Phys.* 60, 156 (2015).
21. M. Mirea, A. Sandulescu and D.S. Delion, *Nucl. Phys. A* 870-871, 23 (2011).
22. M. Mirea, A. Sandulescu and D.S. Delion, *Eur. Phys. J. A* 48, 86 (2012).
23. A. Sandulescu and M. Mirea, *Rom. Rep. Phys.* 65, 688 (2013).
24. M. Mirea, *Eur. Phys. J. A* 4, 335 (1999).
25. M. Mirea *Phys. Rev. C* 83, 054608 (2011).
26. M. Mirea, *Phys. Lett. B* 717, 252 (2012).
27. M. Mirea, D.S. Delion and A. Sandulescu, *Phys. Rev. C* 81, 044317 (2010).
28. M. Mirea and R.C. Bobulescu, *J. Phys. G* 37, 055106 (2010).
29. M. Mirea, L. Tassan-Got, C. Stephan, C.O. Bacri and R.C. Bobulescu, *Europhys. Lett.* 73, 705 (2006).
30. M. Mirea, *Phys. Lett. B* 680, 316 (2009).
31. M. Mirea, *Phys. Rev. C* 89, 034623 (2014).
32. M. Mirea, D. S. Delion and A. Sandulescu, *EPL* 85, 12001 (2009).
33. A. Sandulescu and M. Mirea, *Rom. J. Phys.* 58, 1148 (2013).
34. M. Mirea, T. Pahomi, and S. Stoica, *Rom. Rep. Phys.* 67, 872 (2015).
35. A.I. Budaca and I. Silisteanu, *Phys. Rev. C* 88, 044618 (2013).
36. I. Silisteanu and C.I. Anghel, *Rom. J. Phys.* 60, 444 (2015).
37. I. Silisteanu and C.I. Anghel, *Rom. J. Phys.* 59, 724 (2014).
38. D.S. Delion, R.J. Liotta and R. Wyss, *Phys. Rev. C* 76, 044301 (2007)
39. A. Sobczewski and K. Pomorski, *Prog. Part. Nucl. Phys.* 58, 292 (2007).

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40. A. Sandulescu, R.K. Gupta, W. Scheid and W. Greiner, Phys. Lett. B 60, 225 (1976).
 41. R.K. Gupta, C. Parvulescu, A. Sandulescu and W. Greiner, Z. Phys. A 283, 217 (1977).
 42. R.K. Gupta, A. Sandulescu and W. Greiner, Phys. Lett. B 67, 257 (1977).
 43. R.K. Gupta, A. Sandulescu and W. Greiner, Z. Naturforsch 32a, 704 (1977).
 44. A. Sandulescu, D.N. Poenaru and W. Greiner, Sov. J. Part. Nucl. 11, 528 (1980).