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WAVELET AND SHORT TIME FOURIER TRANSFORMATIONS – TWO COMPLEMENTARY METHODS FOR SPECTRAL ANALYSIS OF MUSCLE ELECTRICAL ACTIVITY

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Abstract. In this paper we propose two mathematical methods for more thoroughly analysis of the electrical activity recordings of some human organs: electrocardiograms, electroencephalograms, electromyograms. The spectrograms obtained through short time Fourier transformation and wavelet transformation may be used to evaluate the degree of similarity and difference between two recordings of a healthy and sick human organ. Here we have analyzed a potential electric recording of the triceps brachii muscle contraction.

Keywords: short time Fourier analysis, wavelet spectrogram analysis, electromyogram.

1. INTRODUCTION

The mathematical transformations are applied in the spectral analysis of the signals in order to obtain additional information hidden in the initial signal, in unprocessed condition. The domain of mathematical transformations was continuously improved and enriched both in terms of the theoretical substantiation and of the increase of scientific and engineering applications number. In 1980 it was proposed a new type of transformation to remove the limits of the Fourier transformation, applied to the entire signal or on portions of it [1]. This was called the wavelet transformation. In a short time, its applications increased extraordinarily: harmonic analysis, numerical analysis, processing of signals and images, analysis of the seismic movements, genomics etc.

2. MATHEMATICAL TRANSFORMS FOR SPECTRAL ANALYSIS

2.1. FOURIER TRANSFORMATION

The *Fourier transform* is used in the analysis of the periodical signals (or processes). Most often, a signal represents the periodical variation of a physical quantity, which we will generically call ***amplitude***, depending on time. In this form, which we call initial or unprocessed form, it is known only the information concerning the variation of that quantity depending on time. For this reason, it is called the amplitude-time representation. Many times, it is important to know the frequency of the signal component waves.

By the Fourier transform of the signal, it is found the spectral composition, namely the frequency of the sinusoidal components which compose the signal and their share [2].

The Fourier is defined as:

$$X\left(f\right)=\int_{\infty}^{+\infty}x(t)\,\mathrm{e}^{-2\pi\, fti}\mathrm{d}\,t, \tag{1}$$

where *x(t)* is the function that describes the initial signal (in the domain of time), *f* is the frequency of the component which we assume that is contained by the initial signal, $X(f)$ is the function that describes the signal processed in the frequency domain.

The representation in the frequency domain does not allow the localization in time of the component signals.

If the signal is stationary, namely the spectral composition does not change during the existence of the signal, we know all the information.

The graph of the Fourier transform will represent the amplitude depending on the frequency. Corresponding to the values on the abscissa, appropriate to the frequencies of the component waves, a pick will occur.

For the non-stationary signals, the spectral composition changes, so there is localization in time, which is not sensed by the Fourier transform. In other words, a non-stationary signal contains additional information compared to a stationary signal that is not marked out by the Fourier transform.

The Fourier transform may be used for the analysis of the **stationary or non-stationary signals**, if we are interested only by the spectral composition, not also the localization in time of the components.

However, there are situations when not only the frequency of the spectral components is important, but also the time of their occurrence.

If the signal is not stationary, the most frequently encountered case, then other transformations are used, by means of which the time dependent spectral composition (localized in time) to be obtained: *Short Time Fourier Transform and Wavelet Transform.*

2.2. SHORT TIME FOURIER TRANSFORM

In case of the short time Fourier transform method, it was resorted to an *artifice* often used in physics. The non-stationary signal was divided in a number of equal intervals so as the signal to be stationary on each time interval. Mathematically, this signal division is accomplished by the multiplication of the *signal function* with a function called *window function.* This function is significantly different from zero on an interval, whose support coincides with the width of the time interval in which the signal was divided. The coverage of each component region of the signal is performed through the window function translation along the signal.

2.2.1. Window function

Let us suppose that we have a non-stationary signal composed of a sequence of two monochromatic signals with the frequencies f_1, f_2 ($f_1 > f_2$) having a duration t_1, t_2. Let us imagine that we can see this signal through a window. We chose two windows of different widths. We see very well the signal with the frequency f_1 through the narrower window. When we reach to the signal with the frequency f_2, only a part of the signal can be seen. We see very well the signal with the frequency f_2 with the wider window. The physicists divide the initial signal in two stationary signals, each having the mentioned characteristics.

Two observations have to be mentioned:

1. In order to see correctly each spectral component, a window with adequate width has to be chosen. Two windows have to be chosen in the above example.

2. Each component is located in a time interval.

Mathematically, the signal is described by a function. The window may be defined by means of other function which is different from zero (or significantly different from zero) over a finite interval. The function that defines the signal is multiplied with the window function and a new function is obtained that contains from the signal function only the segment corresponding to the interval on which the window function is different from zero.

For example, if the initial signal is described by the function $x(t)\colon \mathbb{R} \to \mathbb{R}$ and the window function is:

$$F(t) = \begin{cases} w(t) & \text{for } t \in I = [\alpha,\ \beta] \\ 0 & \text{for } t \in \mathbb{R} - I \end{cases}, \tag{2}$$

then, a new function is obtained by multiplying the functions $x(t)$ and $F(t)$:

$$X\left(t\right)=\begin{cases}w(t)x\left(t\right) & \text{for}\ \ t\in I\\ 0 & \text{for}\ \ t\in\mathbb{R}-I\end{cases}. \tag{3}$$

This new function is different from zero only on the interval *I*, which is the support of the function *w*(*t*). In other words, the signal *X*(*t*) contains only the portion of the signal *x*(*t*) corresponding to the values from the interval *I*. The signal sampling is based on this property in the transformations which will be discussed further.

The short time Fourier transform is obtained by performing the Fourier transform to the signal-function *X*(*t*) [2]:

$$\tilde{X}\left(f\right)=\int_{-\infty}^{+\infty}X\left(t\right)\mathrm{e}^{-\mathrm{i}2\pi ft}\mathrm{d}t=\int_{\alpha}^{\beta}x\left(t\right)w^{*}(t)\mathrm{e}^{-\mathrm{i}2\pi ft}\mathrm{d}t. \tag{4}$$

The Fourier transform is performed only on the signal segment corresponding to the interval *I* = [α, β]. The size of the interval *I* may be choose so that the corresponding segment from the signal to be stationary. Therefore, this transformation is called **S**hort **T**ime **F**ourier **T**ransformation (STFT).

2.2.2. Choice of the window function

The choice of the window function is not performed arbitrary. From the mathematical point of view, the function *w*(*t*) must have the following properties:

1) To have a small definition domain (sometimes, it is also called compact support), or to decrease rapidly enough in order to be obtained a good location in space.

2) The average value to be equal to zero ($\int_{-\infty}^{+\infty}w(t)\mathrm{d}t=0$). This property is necessary as the window function to fulfill the admissibility condition in order to be also calculated the inverse transformation. Moreover, this property determines the undulatory (sinuous) character of the window function.

In order to obtain some adequate resolutions in the analysis of the signals, the window function must fulfill the following conditions:

1) To be **translatable** along the signal. The translation of the window function with the “distance” τ is performed by the replacement of the variable *t* with *t* – τ, so with the use of the function *w*(*t*–τ) in the transformation relations

$$\tilde{X}\left(\tau,f\right)=\int_{-\infty}^{+\infty}x\left(t\right)w^{*}\left(t-\tau\right)\mathrm{e}^{-\mathrm{i}2\pi ft}\mathrm{d}t=\int_{\alpha}^{\beta}x\left(t\right)w^{*}\left(t-\tau\right)\mathrm{e}^{-\mathrm{i}2\pi ft}\mathrm{d}t. \tag{5}$$

2) To have a certain **width,** determined by the resolution traced within the signal analysis, and determined by the stationarity condition of the analyzed signal portion in the case of STFT. The window width to be **adjustable.**

The width (support) modification of the window function, $w(x)$, is performed by the introduction of a scale factor.

In the short time Fourier transform, the modification of the window width is performed by the replacement of the function variable, x, with $f(x)$. If $f < 1$, a space expansion occurs (a widening of the window), and if $f > 1$, a space contraction occurs (a reduction of the window). We used the notation $\underline{\boldsymbol{f}}$ for scaling, related to the frequency analysis from the case of the short time Fourier transform.

The STFT transform may be seen as a signal processing by means of the ***transformation kernel*:**

$$K(\tau, f; t) = w(t-\tau)^{*} \mathrm{e}^{-\mathrm{i}2\pi ft}. \tag{6}$$

2.3. WAVELET TRANSFORM

In the case of this transformation, the role of the window function is fulfilled by a function called *wavelet mother*, $w(t)$ [3].

Wavelet means a small wave (short wave). This implies that this function to have a compact support, which would correspond to a window function of finite width.

The name of *mother* was associated with the fact that the function $w(t)$ will be used for the generation of *two sets of windows functions* by means of two parameters: scale parameter, $\underline{\boldsymbol{s}}$ and translation parameter (or localization parameter), $\underline{\boldsymbol{\tau}}$:

$$w(t) \;\rightarrow\; w\left(\frac{t-\tau}{s}\right). \tag{7}$$

By the introduction of the *scale parameter* $\underline{\boldsymbol{s}}$, it is eliminated the fixed resolution drawback from the STF transformation.

The parameter s is the inverse of the frequency from the transformation kernel $w(t)\cdot\exp(-2\pi ft)$ from STFT. So, one can say that at *low frequencies* a large scale parameter corresponds, equivalent to getting *global information* of the signal, namely an information that refers to the entire signal; at *high frequencies*, namely the small scale parameter, the *detail information* is obtained, information that refers to the short duration transient components.

From mathematical point of view, the parameter s is the equivalent of the space contraction factor: if $s > 1$ it takes place an *expansion of the space and of the function w(t)*; if $s < 1$ we have a *contraction of the space and of the function w(t)*.

The wavelet transform is calculated by the integration of the signal function $X(t)$ [4]:

$$\bar{X} = \int_{\alpha}^{\beta} x(t) w^{*}(t)\,\mathrm{d}t. \tag{8}$$

As it can be seen, the Fourier transform is a frequency function, and the wavelet transform is a number.

For the signal analysis, both the translation and scaling are used:

$$\bar{X}(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t) w^{*}\left(\frac{t-\tau}{s}\right)\mathrm{d}t = \frac{1}{\sqrt{s}} \int_{\alpha}^{\beta} x(t) w^{*}\left(\frac{t-\tau}{s}\right)\mathrm{d}t \quad s > 0. \tag{9}$$

By these operations, both transforms (short time Fourier, wavelet) became functions of two variables. Their graphs will be some surfaces in the 3D space.

3. RESULTS AND DISCUSSIONS

3.1. EXPERIMENTAL MEASUREMENTS

Two electrical signals were recorded at the level of the triceps brachii muscle. The forearm position was parallel to the ground, the movement being performed in a plane parallel to the ground to eliminate the gravitational attraction, from the maximum bending position of the elbow, up to the position in which the forearm is continuing the arm, namely the maximum stretching position of the elbow. That is, the forearm make a rotation around the elbow in horizontal plan. This experiment may be a reference for other types of experiment with the triceps branchii muscle.

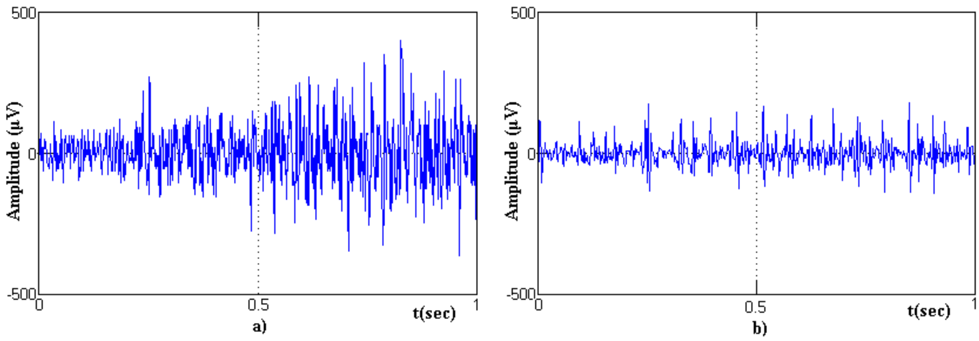

Fig. 1– The electric signal recordings at the level of the triceps brachii muscle: a) the inside electric potential recording at the bone level; b) the outside recording of the electric potential at the skin level. Both recordings represent only a part from the coresponding records of 7 seconds length.

For signal recording, a 24-bit biosignal amplifier was used. The first recording was made outside at the skin level with a surface electrode (Fig. 1a). Surface electrodes have the shape of metal disks (generally of silver) that are applied on the surface of the skin on top of the muscle to be tested by means of a low impedance gel.

The second recording was made inside of the muscle at olecran level (Fig. 1b). For collecting the electrical activity in the insertion area of the triceps brachii muscle on the olecranon, needle electrodes were used. Needle electrodes have three concentric layers. The inner and outer layer are those by means of which the electrical potential is measured, while the middle layer is insulating, allowing the inner layer to collect the electrical activity only from the tip of the needle. Needles with a length of 37 mm and a diameter of 0.45 mm were used.

In Fig. 1 were represented a part of the two records, only for 1 second.

3.2. THEORETICAL ANALYSIS

3.2.1. Short Time Fourier Spectrogram Analysis

From Figs. 2a and 3a, representing the short time Fourier transformation for electric signal registered at the bone surface. One observes the existence of two electrical signals. We have marked with f_1 and f_2 ($f_1 < f_2$). The Fourier transformation gives the simetrical ferquences, but only the first two have physical signification. One of them has low the frequency, f_1 (for s = 0.0039), and the other has a higher frequency, f_2 (s = 0.1451) throughout the recording time.

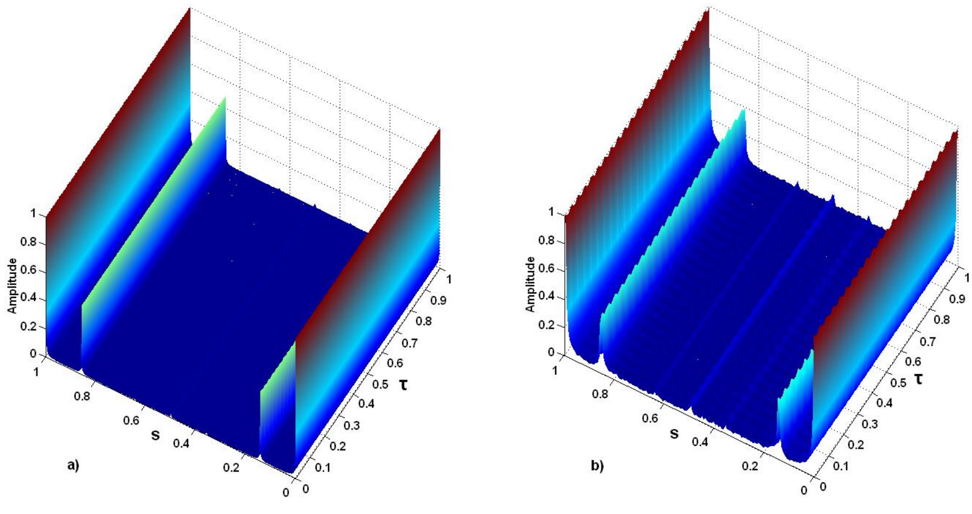

Fig. 2 – 3D representation of the short time Fourier spectrograms for the whole electrical recordings of 7 seconds; a) the outside electrical potential recording; b) the inside electrical potential recording.

The amplitude of electric signal of frequency f_2 is $6 \cdot 10^5$ μV. In the case of internal recording, the signals amplitude remains almost constant in each case. In the case of external recording, according to the notched appearance of the vertical surfaces, it results that the amplitude of the signals changes during the signal recording.

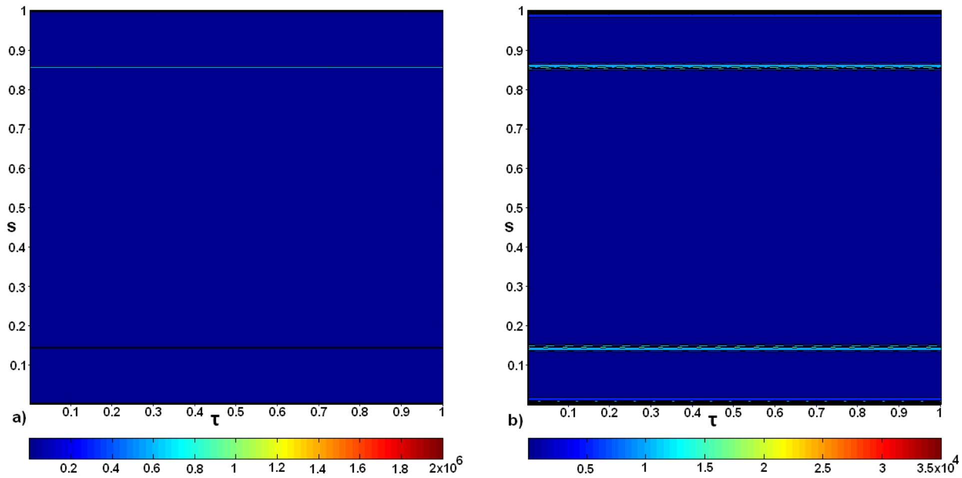

Fig. 3 – 2D representation of the short time Fourier spectrograms for the whole electrical signals recordings of 7 seconds; a) the inside electrical potential recording; b) the outside electrical potential recording.

Both electric signals appear in the recording realized at the skin level with the surface electrod, too. But their amplitudes are different: $(0.4375–0.4832)\cdot10^4$ μV for the signal with frequency f_1, and $(1–1.05)\cdot10^4$ μV for the signal with frequency f_2. The amplitude of the electric signals isn't constant and change succesivelly. For example, the length of the electric signal of frequency f_2 with amplitude of $(1–1.05)\cdot 10^4$ μV is about 208.5 ms and length of the lower one, is 89.36 ms.

Unlike the recording at the bone level, the outside electrical recording contains two suplementary electrical signals. One of them has a low frequency and a length 238 ms, the other has a higher frequency and length of 625 ms. The amplitude of both these electrical signals change succesively. We can easily observe some signals in Fig. 2b. We are thinking that these are spurious signals.

3.2.2. Wavelet Spectrograms Analysis

The wavelet transformation may be applied using different mother wavelet which can give suplementary information about the shape of the spectral components. Using the Shannon function as wavelet mother function, we have found that the electric potential recording both at the skin and bone surface are biphasic. So, the wavelet spectrograms confirm the existence of the two electric signals in both bone and skin level recordings.

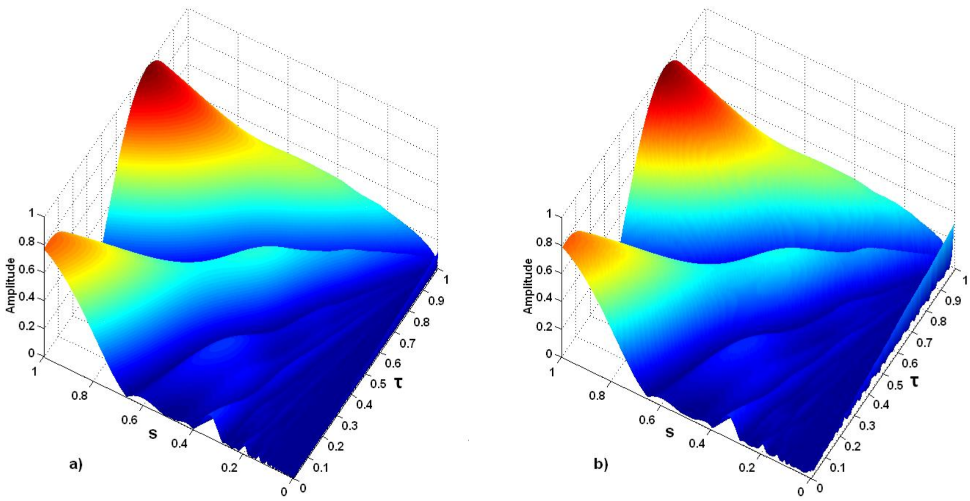

Fig. 4 – 3D representation of wavelet spectrograms for the whole electrical recordings of 7 seconds; a) the outside electrical potential recording; b) the inside electrical potential recording.

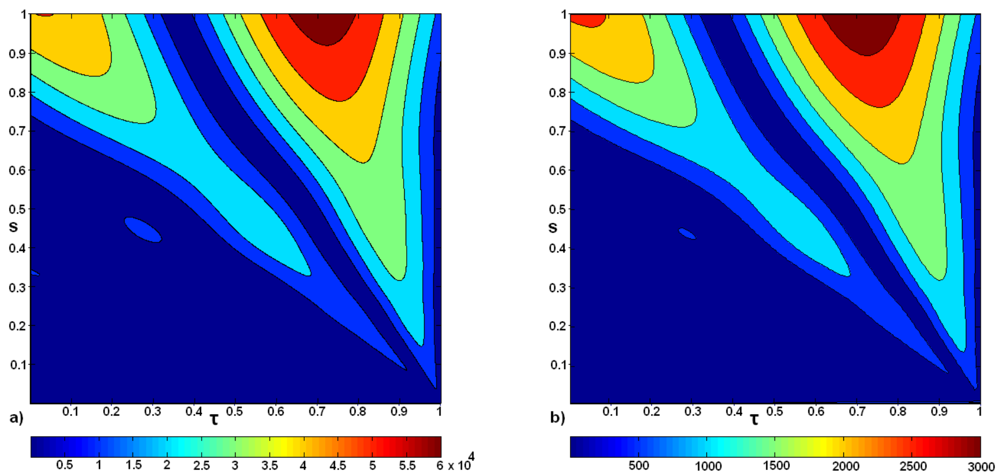

Fig. 5 – 2D representation of wavelet spectrograms for the whole electrical recordings of 7 seconds; a) the inside electrical potential recording; b) the outside electrical potential recording.

The best results were obtained using as mother wavelet the real Shannon wavelet [5]:

$$\psi(x) = \frac{\sin(2\pi x) - \sin(\pi x)}{\pi x}. \tag{10}$$

Because the Shannon wavelet function is a real function, in the eq. (9) the complex conjugate operation becomes useless. On the other hand, the “time” *t* takes discrete values between 1 and *n*, so *s* and *τ* parameters take also the same values as “time” *t*. From this reason, the integral turns into a sum [6, 7]:

$$W(\tau, s) = \sum_{t=1}^{n} \frac{x(t)}{\sqrt{s}} \cdot \psi\left(\frac{t-\tau}{s}\right). \tag{11}$$

4. DISCUSSIONS AND CONCLUSIONS

One of our goals was to find a link between electric potential recorded on the skin level and one registered to the bone surface using the mathematical transformations of electrical signals, in order to avoid the the pain caused by the introduction of needle electrod in muscle. From this reason we used the same space contraction factor, *s*, for both short time Fourier transformation and wavelet transformation.

From the comparison of the signal spectral composition in the case of a healthy person and the case of a sick person, important information may be known about the functional condition of the organ in question (electrocardiogram for the heart’s electrical activity; electroencephalogram for the brain’s electrical activity; electromyogram for the muscles’ electrical activity). One the other hand if one analyses by mathematical transformations both the electric potential recording obtained by the conducting layers of the needle electrod at the bone surface, very usefull information about piezoelectric efect indused by muscle contraction in bone. The piezoelectric effect in human bones may be a method for osteoporosis treatment.

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