EXPLORING THE GRAPHIC FACILITIES OF EXCEL SPREADSHEETS IN THE INTERACTIVE TEACHING AND LEARNING OF DAMPED HARMONIC OSCILLATIONS

I. GRIGORE\textsuperscript{1,2}, CRISTINA MIRON\textsuperscript{1*}, E.S. BARNA\textsuperscript{1}

\textsuperscript{1}Faculty of Physics, University of Bucharest, Bucharest-Magurele, Romania
\textsuperscript{2}“Lazar Edeleanu” Technical College, Ploiești, Romania
*Corresponding author: cmiron_2001@yahoo.com

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\textbf{Abstract.} This article presents the use of Excel spreadsheets in the process of teaching-learning of damped harmonic oscillations. The frictional force was considered to be linearly proportional with the velocity of the oscillator. Under these conditions, a didactic tool is described, made with the help of interactive spreadsheets and highlighting the under-damping oscillations. The parameters characteristic for the damped oscillatory motion have been calculated with the input data, while the law of motion and the kinetic, potential and total energy according to displacement and time have been graphically rendered. Also, the graph for the oscillator motion in the phase space has been drawn. By exploring the graphic facilities of the spreadsheet we can demonstrate how the motion of the oscillator is simulated both in the “potential hole” and in the phase space. By making the resistance coefficient equal to zero, we obtain the particular case of the undamped oscillator and the preservation of the total mechanical energy is graphically highlighted. If the presented tool is used in the classroom, students will be able to grasp aspects connected to the energy dissipation of the damped oscillator and clarify certain links between concepts specific to the harmonic oscillatory motion. By reorganizing spreadsheets, the tool can be adapted so that it encompasses the initial conditions in the input data and considers the over-damping oscillations.

\textbf{Key words:} Spreadsheets, Excel, harmonic oscillations, damping, physics education.

\section{1. INTRODUCTION}

For a wide range of problems of classical Physics and quantum Physics, the harmonic oscillator can represent an exact or approximate model. Therefore, the study of this system is of particular importance for Physics. The mechanical harmonic oscillations can be described together with the oscillating electrical circuits or the energy states of quantum systems. Thus, the development of interactive didactic tools for the teaching and learning harmonic oscillations proves to be beneficial for any introductory course in Physics.
The literature offers a variety of articles on study of the harmonic oscillator [1–3]. Part of the work devoted to this topic approaches oscillations through the use of Excel spreadsheets. Excel has the advantage that is accessible to a wider audience than applications written in other programs [4]. Other facilities of this program have been presented by authors in their previous papers [5, 6]. It has been shown that the development of interactive computing modules with graphics and animations can integrate mathematical concepts in different areas. Consequently, the spreadsheet was used to interpret the solutions for the differential equation of motion of the harmonic oscillator based on different initial conditions [7]. Using the acceleration sensor from the mobile phone and the associated application “Accelerometer Monitor” for Android has shown the way in which the data from experiments with mechanical oscillations can be acquired and processed [8]. Students’ analysis of the pendulum motion with the help of a spreadsheet has proven that they can reach a more firm understanding of mechanical concepts [9].

Three spreadsheet models have been comparatively presented for the motion of the gravitational pendulum. For each model there was graphically presented the time dependence of the pendulum angle to the vertical and the formula to calculate the oscillation period was deduced. It has also been graphically rendered the error in the calculation of the gravitational acceleration depending on the angle amplitude starting from the oscillation period of the pendulum [10]. It was developed a model to describe the motion of one-dimensional oscillator of variable mass [11].

Differential equations were investigated for patterns of vibration with dry friction examples of the harmonic oscillator [12]. The nonlinear equation was also discussed for the motion of a body with a given mass attached to an elastic spring [13]. The nonlinear regime of the simple gravitational pendulum was treated by presenting approximate methods for calculating the oscillation period or for the analysis of the solutions for the equation of motion [14, 15]. There was also a description of the simulation for the system with two harmonic oscillators coupled with the aid of the educational math software GeoGebra that allows the simultaneous operation with algebraic objects and graphic representations. By performing the application in the classroom, students were engaged in the construction of virtual, dynamic and interactive models for the oscillatory phenomena studied [16]. There was presented a tool developed with interactive Excel spreadsheets to simulate the composition of perpendicular harmonic oscillations with the same frequency. Changing the time moment in the input data allows the step-by-step visualization of the oscillator motion on the trajectory [17]. For perpendicular oscillation of different frequencies animations with Lissajoux figures were made with the help of spreadsheets [18]. Also with the help of spreadsheets, there was created a simulation model correlated with the chart of the law of motion for the damped harmonic oscillator [19]. Moreover, for the damped oscillations, we have the exact analytic form of the function energy-displacement with explanations of the successive oscillation amplitudes [20]. The same issue was approached by the introduction of the function Lambert W, formulating conclusions regarding the energy dissipation of the oscillator. Numerical examples
with graphic representations have rendered the case of the quadratic damping and that of the linear damping [21]. For the quantic harmonic oscillator, an Excel file was generated for the calculation and graphic representation of the wave functions and the probability densities. The structure of the file was concretely used as a calculation model for the vibration motion of the diatomic molecule of HCl [22].

This article presents an Excel tool with which we can calculate various measures characteristic to the oscillatory motion and visualize certain dependences between measures. The under-damping regime is considered when the frictional force linearly depends on the oscillator velocity. This model can be applied to an elastic pendulum or a gravitational pendulum in a viscous medium. The graphic facilities of the Excel spreadsheet are mainly used to highlight aspects connected to the energy of the oscillator. In particular, making the resistance coefficient equal to zero, we obtain the results for the undamped harmonic oscillator. We have drawn the graphs depicting the law of motion of the oscillator and dependences on displacement and on time of the kinetic, potential and total energy. The energy-displacement graph was obtained without analytically explaining the energy as a function of displacement. For this there were explored the possibilities for handling the data from Excel tables. The energy-displacement graph was correlated with velocity-displacement graph of the phase space. By changing the time moment in the input data we can follow the motion of the oscillator in the potential hole and in the phase space.

2. ORGANIZATION OF SPREADSHEETS

The structure of the tool is similar to that of other tools described by the authors, exploring the facilities of Excel spreadsheets in the process of teaching-learning of Physics [6, 17]. The main spreadsheet contains the sections “Input Data” and “Results”, each of them being divided into several sub-sections, plus the graph area. In the first section we introduce the characteristics of the harmonic oscillation and a time moment at which we calculate and graphically visualize some physical measures for the second section. In the second section we calculate the parameters of the damping and the measures specified at the moment of time introduced in the input data.

The measures introduced in the section “Input Data” are: the mass of the oscillator, \( m \), in cell B4, the resistance coefficient, \( r \), in cell B5, the initial amplitude, \( A_0 \), in cell B6, the undamped period, \( T_0 \), in cell B7, the starting phase angle, \( \varphi_0 \), in cell B8. The moment of time, \( t \), is introduced in cell B10. In the first subsection of the “Results” section we calculate the period, \( T \), in cell B13 and the total initial energy, \( E_0 \), in cell B14. In the second subsection, entitled “Initial Conditions”, we calculate the initial displacement, \( x_0 \), in cell B16 and the initial velocity, \( v_0 \), in cell B17. In the third subsection, entitled “Damping”, we calculate the logarithmic decrement, \( D \), in cell B19, the half time, \( T_{1/2} \), in cell B20 and the time constant, \( \tau \), in cell B21. In the fourth subsection we calculate, for the time
moment, \( t \), the displacement, \( x \), in cell B23, the velocity, \( v \), in cell B24, the kinetic energy, \( E_k \), in cell B25, the potential energy, \( E_p \), in cell B26 and the total energy, \( E \), in cell B27. The measures that appear in the two sections are expressed in S.I. units.

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Fig. 1 – Main spreadsheet with the displacement-time graph of the under-damping oscillator. The colored versions can be accessed at http://www.infim.ro/rrp/.

Fig. 2 – The energy-displacement graphs of the under-damping oscillator for two values of the resistance coefficient: a) \( r = 1.60 \) kg/s; b) \( r = 3.20 \) kg/s. The colored versions can be accessed at http://www.infim.ro/rrp/.
In order to do the calculations in Excel we use the following cell names: “Mass” for cell B4, “Coefficient_r” for cell B5, “Amplitude_0” for cell B6, “Period_0” for cell B7, “Phase_0” for cell B8, “Time” for cell B10, “Period_P” for cell B13, “Energy_0” for cell B14, “Displacement_0” for cell B16, “Velocity_0” for cell B17.

We will further exemplify the calculations done in the cells from the main spreadsheet rendered in Fig. 1. For this, first we calculate the undamped angular frequency, \( \omega_0 \), the damping factor, \( b \), and angular frequency, \( \omega \), with the help of the relations presented in the literature [23] and transcribed in Excel. Thus, in the secondary sheet entitled “Annex_Calculations”, taking into account the input data, we write the following Excel formulas in three adjacent cells: 

- \( \text{“=}2\pi/\text{Period_0} \) to calculate \( \omega_0 \)
- \( \text{“=}\text{Coefficient_r} / (2\times\text{Mass}) \) to calculate \( b \)
- \( \text{“=}\text{IF(Factor_b<Frequency_0;SQRT(Frequency_0^2-Factor_b^2);“NO”)} \) to calculate \( \omega \)

The cells in which we calculate \( \omega_0 \), \( b \) and \( \omega \) have been entitled “Frequency_0”, “Factor_b” and “Frequency_P”. The period \( T \), in cell B13 of the main spreadsheet is calculated with the Excel formula \( \text{“=}\text{IF(Factor_b<Frequency_0;(2\times\pi)/Frequency_P;“NO”)} \).

The displacement of the oscillator at the moment \( t \) is calculated according to the relation [23, 24]:

\[
x(t) = A_0 e^{-bt} \cos(\omega t + \varphi_0).
\]

The transcription in Excel of formula (1) in cell B23 becomes:

\( \text{“=}\text{IF(Factor_b<Frequency_0;Amplitude_0*EXP(-Factor_b*Time))*COS(Frequency_P*Time+RADIANS(Phase_0));“NO”)} \).

We have used the logical function IF because in the case in which the input data lead to a value of the damping factor higher than the undamped angular frequency the message “NO” is displayed. In this case the limits of the under-damping regime are exceeded. To calculate the velocity of the oscillator in cell B24 we derive relation (1) according to time and then we transcribe the result in Excel. The initial displacement, in cell B16, is obtained by making \( t = 0 \) in relation (1) and transcribing the result in Excel. Analogously, we calculate the initial velocity in cell B17 starting from the expression of the velocity deduced at any given time moment. The calculation of the measures \( D, T_{1/2} \) and \( \tau \) in cells B19, B20 and B21 is performed according to the measures \( b \) and \( T \) following a similar procedure.

To calculate the kinetic energy, \( E_k \), and the potential energy, \( E_p \), in cells B25 and B26 we transcribe in Excel the relations known in the literature [24–26]. For this, we take into account the values of the oscillator mass and of the undamped period from the input data, as well as the values of the velocity and displacement calculated in cells B23 and B24. The value of the total energy, \( E \), from cell B27 is
calculated as the sum of the values of the kinetic and potential energy from cells B25 and B26.

To calculate the kinetic energy, $E_c$, and the potential energy, $E_p$, in cells B25 and B26 we transcribe in Excel the relations known in the literature [24–26]. For this, we take into account the values of the oscillator mass and of the undamped period from the input data, as well as the values of the velocity and displacement calculated in cells B23 and B24. The value of the total energy, $E$, from cell B27 is calculated as the sum of the values of the kinetic and potential energy from cells B25 and B26.

Figure 1 renders the graph for the law of motion. The curve traced in red represents the displacement in relation to time. This curve is modulated by the curves colored in blue which represent the exponential decrease in time of the oscillation amplitude.

Figure 2 renders the dependences of the kinetic, potential and total energy on the displacement of the oscillator for two values of the resistance coefficient $r$. In the left panel we considered $r = 1.60$ kg/s, while in the right panel $r = 3.20$ kg/s. The other input data are in accordance with Fig. 1. By modifying the value of the
time moment in cell B10 we can trace the position of the oscillator in the energy-displacement graph on the motion axis through the black dot. Also, we can notice the motion of the dots associated to the position on the curves of the kinetic, potential and total energy. In the respective figure we rendered the position of the oscillator at \( t = 0.10 \) s.

![The energy of the harmonic oscillator at moment of time t](image)

Fig. 4 – The energy of the oscillator in column and pie charts at a moment of time specified in the input data. The colored versions can be accessed at http://www.infim.ro/rrp/.

The dependence of the kinetic energy on displacement is represented by the family of parabolas opening down and colored in blue. These parabolas are positioned asymmetrically to the ordinate axis. Thus, the maximum values of the kinetic energy are placed on each side of the equilibrium position. As these maximum values decrease, in correspondence with the increasing time values, the asymmetry of the curves is gradually reduced. For \( t \to \infty \), the curve of the kinetic energy tends towards zero in the equilibrium point. Theoretically, we have an infinite number of parabolas, but on the graph the number of curves is limited by the maximum value of the time moment from the source table of the graph.

The dependence of the potential energy on displacement is represented by the family of parabolas opening up and colored in green. The parabolas of the potential energy have the same minimum, \( E_{\text{pmin}} = 0 \), in \( x = 0 \). Due to the symmetry to the axis of the ordinate, these parabolas overlap and we observe, on the graph, only the initial branches of different heights, on each side of the equilibrium position. The other branches, placed over the initial branches, have smaller and smaller heights so that, for \( t \to \infty \), the curve of the potential energy tends towards zero in the equilibrium point.
The total energy is represented by the red curve with the branches supported on the curves of the potential energy. The points of support on the curves of the potential energy, projected on the motion axis, represent the successive extreme points of damped oscillations. The branches of the total energy curves narrow with the oscillator motion so that, for $t \to \infty$, the total energy tends towards zero. Also, the consecutive branches of the total energy curves are closing in as the energy decreases. The curve of the total energy is tangent at the curves of the kinetic energy in point $x = 0$. In this point, the total energy is equal to the kinetic energy because the potential energy is equal to zero. It can be observed that the branches of the total energy curves are all the more open as the resistance coefficient $r$ is bigger.

Through the graphics rendered, Fig. 2 shows how the potential hole of the damped oscillator narrows with the oscillator motion around the equilibrium position. When time tends towards infinite, the potential hole is reduced to zero. The limits of the “initial potential hole” have been represented on the graph by the thick straight line segments colored in brown. Vertical segments have been built – “the walls of the hole” – at $x = \pm A_0$ and the horizontal segment corresponding to the value $E_0$. The segment traced in a brown dotted line highlights the initial conditions on the energy-displacement graph. Thus, the bold points on this segment show the displacement on the motion axis and the values for the kinetic, potential and total energy on the curves of the three measures at the moment $t = 0$. 

Fig. 5 – The particular case $r = 0$: the energy graphs of the undamped harmonic oscillator.
The colored versions can be accessed at http://www.inlim.ro/rrp/.
Exploring the graphic facilities of Excel spreadsheets

Figure 6 – The motion of the under-damping oscillator in the phase space for two values of the resistance coefficient $r$: a) $r = 1.60 \text{ kg/s}$; b) $r = 3.20 \text{ kg/s}$. The colored versions can be accessed at http://www.infim.ro/rrp/.

Figure 3 renders the time dependences of the kinetic, potential and total energy for the under-damping oscillator. We have utilized the input data from Fig. 1. The curve for the kinetic energy is rendered in blue, the curve for the potential energy in green and the curve for the total energy in red. The curve of the total energy represents the envelope of the kinetic and potential energy curves. The dotted curve colored in brown represents the dependence of the total energy on time according to the relation:

$$E(t) = E_0 e^{-2bt}.$$  \hfill (2)

Relation (2) is obtained in the conditions in which we neglect the variation of the oscillation amplitude in a period. In this case the energy decreases exponentially in time with a damping coefficient of $2b = rm$ [23]. It can be observed that with the growth of the time values, the curve described by equation (2) closes in on the total energy curve so that, for $t \rightarrow \infty$, the two curves coincide. As the resistance coefficient decreases in the input data, it can be verified how the two curves come closer and closer to each other until they overlap.

Figure 4 presents the energy of the oscillator at the moment of time $t$, fixed in the input data, highlighting the comparison between the values. The left panel renders, in a column chart, the values of the kinetic energy, $E_c$, potential energy, $E_p$, and total energy, $E$, and total initial energy, $E_0$. The right panel renders, in a pie chart, the values of the kinetic and potential energy as percentages of the total energy. We have considered $t = 0.10 \text{ s}$ as in Fig. 1.
The energy graphs in Fig. 5 result from making the resistance coefficient, $r$, equal to zero in the input data. In the left panel of the figure we have rendered the energy-displacement graph and in the right panel the energy-time graph. It can be observed that we obtained the energy graphs for the undamped harmonic linear oscillator. The graphs in Fig. 5 highlight the preservation of the total mechanical energy, unlike the graphs from Fig. 2 and Fig. 3 highlighting the dissipation of the energy in the presence of the frictional force.

An efficient way to describe the evolution of a physical system is to use the phase space. That is why we have drawn the graph representing the dependence of the velocity on the displacement of the oscillator. Figure 6 renders the velocity-displacement graph for the under-damping oscillator for two values of the resistance coefficient. It is verified that the trajectory in the phase space is a spiral which narrows asymptotically towards the equilibrium point $x = 0$. The bigger the resistance coefficient, the quicker the spiral narrows towards the equilibrium point. This aspect is observed by comparing the two graphs velocity-displacement in Fig. 6, in the left panel for $r = 1.60 \text{ kg/s}$, and in the right panel for $r = 3.20 \text{ kg/s}$. The starting phase is marked on the graph by the black dot at the end of the spiral. The state of the oscillator at the moment $t$ has been marked by the blue dot on the velocity-displacement curve. We have considered the input data from Fig. 1. By modifying the value of $t$, in cell B10 of the main spreadsheet, we can trace the
evolution of the oscillator through time by the motion of the blue dot on the velocity-displacement curve.

The graphs from Fig. 6 are correlated with the energy-displacement graphs from Fig. 2. With the growth of the damping, a spiral with a bigger “step” in the velocity-displacement graph from Fig. 6 is associated to a total energy curve with the steeper branches in the energy graph in Fig. 2. With the help of the graphs from Fig. 2 and Fig. 6 we can trace, at any time, the correspondence between the position of the oscillator in the “potential hole” and the position of the oscillator in the phase space.

By keeping the input data from Fig. 1, but making the resistance coefficient equal to zero in cell B5 the velocity-displacement graph in Fig. 7 results. The elliptic trajectory of the undamped harmonic oscillator in the phase space has been obtained. We have rendered in Fig. 7 the entire spreadsheet in order to observe the other changes in the cells with results. By modifying the input data, it can be verified that with the growth of the oscillator energy, the surface delimited by the elliptic trajectory in the phase space increases. It can be observed the change in the value of the initial velocity in cell B17 as well as in the velocity at the moment \( t = 0.10 \) s, in cell B24. Also, the change in the values of the kinetic, potential and total energy in cells B25, B26 and B27 can be observed. The Excel formulas written in the “Results” section lead to the appearance of the value zero in cell B19 and of the message “NO” in cells B20, B21 in which the measures \( D \), respectively \( T_1/2 \) and \( T \) are calculated.

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The velocity-displacement graph is useful for the interpretation of the significance of the oscillation phase angle, in particular of the starting phase angle \( \varphi_0 \). Thus, it is verified that the starting phase angle represents the angle between the displacement axis and the straight line connecting the origin of the system of coordinates with the point associated with the initial state. In particular, for \( r = 0 \), putting \( \varphi_0 = 0 \) in cell B8, the point associated to the initial state is placed on the
displacement axis at \( x_0 = 0.20 \text{ m} \), and if we take \( \varphi_0 = 90^\circ \), it is placed on the velocity axis at \( v_0 = -1.26 \text{ m/s} \).

For the graph of the law of motion and the energy graphs, energy-displacement, respectively energy-time, a single source table is used that is placed in a secondary spreadsheet. The drawing of this table, partially rendered in Fig.8, has been done in accordance with a procedure similar to that used by the authors in other papers [6, 17]. In column B there are generated the values of the moments of time, \( t \), with the help of an increasing number, \( n \), with a one-unit step in column A and a time quantum equal to the 100th part of the oscillation period. In column C we calculated the values of the displacement, \( x \), and in columns D and E the values of the variable amplitude, \( A(t) = A_0 e^{-bt} \), respectively, \( A'(t) = -A_0 e^{-bt} \), according to the time values from column B. Also according to the time values from column B we calculated the values of the velocity, \( v \), in column F. In column G we calculated the values of the kinetic energy, \( E_k \), according to the values of the velocity from column F. In column H we calculated the values of the potential energy, \( E_p \), according to the values of the displacement from column C. In column I we calculated the values of the total energy, \( E' \), according to relation (2), using the values of the moments of time from column B. The domain of values for the graph of the law of motion is determined by columns B, C, D and E and for the energy-time graph by columns B, G, H, I and J. 400 lines have been used for both the law of motion graph and for the energy-time graph. The number of lines has been settled so that the maximum value of the time moment in column B is \( 4T \), where \( T \) is the oscillation period. The domain of values for the energy-displacement graph is determined by columns C, G, H, I, plus supplementary columns from K to S to mark the particular elements. In this case, the number of lines is correspondingly extended through the association with supplementary columns. An example of a particular element is represented by the limits of the “initial potential hole”. For graphs in Fig. 4 and Fig. 6 two more source tables have been drawn, placed in separate spreadsheets.

Any of the presented graphs can be placed next to the input data using the option “Freeze panel”, having thus the possibility of tracking the feedback to the change in each and every parameter in the input data.

3. CONCLUSIONS

There are various approaches on how to provide training and development of theoretically acquired knowledge in Physics. A new way of the teaching-learning strategy by using the spreadsheets approach was presented in this article.
With the help of the tool developed students can easily understand the aspects of energy dissipation in damped oscillations. Each energy graph shown addresses the transfer energy from a different perspective. The comparison between the graphs offers the overall picture on the energy of the system. By simulating the motion of the oscillator in the potential hole correlated with the motion in the phase space we can clarify a number of concepts. An example is the notion of phase and, in particular, the link between the starting phase angle and the initial conditions.

The tool can be useful not only for teaching and learning concepts specific to oscillatory phenomena but also for a more complex evaluation. Thus, a theme that would assess higher order skills would be the adaptation of the tool to explore the initial conditions for the oscillator motion. In this case students must reorganize the spreadsheets taking into account the new input data to encapsulate the initial conditions given by initial displacement, \( x_0 \), and the initial velocity, \( v_0 \). The results obtained with the first tool can be used as input data for the newly created instrument. Accordingly, the functionality of the complementary tool can be verified. In this way both knowledge of Physics and Information Technology skills can be assessed. The tool can also be completed to take into account the over-damping oscillations.

The Excel spreadsheet represents a dynamic perspective and an analytical power of the software tool for students in the learning of Physics.

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**REFERENCES**