Romanian Reports in Physics, Vol. 68, No. 3, P. 1060–1068, 2016

HYDRODYNAMIC FLOW AND PHASE TRANSITIONS IN RELATIVISTIC NUCLEAR COLLISIONS REFLECTED BY HUBBLE TYPE FIREBALL EVOLUTION

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Received September 14, 2015

Abstract. In this work we present some estimations for the so-called "microscopic" Hubble constant for relativistic and ultrarelativistic nuclear collisions similar to cosmological Hubble constant, based on temporal connections between the evolution of nuclear matter produced in relativistic heavy ion collisions and the Universe evolution after the Big Bang. The energy dependence of the evolution time and "microscopic" Hubble constant at chemical and thermal freeze-out has been investigated too. Estimations for Pb-Pb collisions at 2.76 TeV are also given.

Key words: relativistic nuclear collisions; quark-gluon plasma; collective flow; "microscopic" Hubble constant.

1. INTRODUCTION

Based on different cosmological scenarios [1–3], a specific phase of the nuclear matter, "the quark-gluon plasma", is supposed to be formed around 10^{-6} s after Big Bang. This primordial "soup" has a very short lifetime, followed by the process of the hadron formation (hadronisation). To explore these early moments of the Universe in the laboratory, we can study relativistic heavy ion collisions, due to the fact that in the overlapping region of the two colliding nuclei a very high compressed, hot nuclear matter region is formed (participant region or fireball). The characteristics of this region resemble with the supposed features of the matter that filled the early Universe.

A systematic analysis of the data gathered at SPS, RHIC and LHC during the last decade has given an optimistic perspective of the formation of the QGP and its experimental evidences. Experimental data collected at RHIC have shown that in central heavy-ion collisions it was produced a novel state of matter, a strongly interacting quark-gluon liquid instead of a weakly interacting gas of quarks and gluons called the sQGP [4]. Therefore, we can learn how the QGP becomes confined nuclear matter during the hadronization process and extrapolate the properties of QGP to the conditions predicted to the early Universe. However, the experimental environment created in relativistic heavy ion collisions differs from what we know about the early Universe. In these circumstances, we introduced some connections and extrapolations to take into account initial conditions accordingly. Previously, in refs. [5–8] connections were made between the evolution of a relativistic nuclear heavy ion collision and the Universe.

In the present work, taking into account the hydrodynamic evolution of the fireball, we estimate the microscopic Hubble constant and the evolution time using experimental results published by scientific collaborations from AGS-BNL, SPS-CERN, RHIC-BNL and LHC-CERN [4, 9–11].

2. COSMOLOGICAL *VS.* "MICROSCOPIC" HUBBLE CONSTANT

During the early evolution of the Universe the expansion rate, as measured by the Hubble parameter H, is determined by the total energy density. The relation between the energy density and the temperature corresponds to the Stefan-Boltzmann law $\epsilon(t) \sim T^4(t)$ and the Hubble constant is

$$H = \left(\frac{8\pi G_N \epsilon}{3}\right)^{1/2} = \left(\frac{8\pi G_N \frac{\pi^2}{30} g_* T^4}{3}\right)^{1/2} = 1.66\sqrt{g_* G_N} T^2 \tag{1}$$

where g_* is the effective number of degrees of freedom, T is the medium temperature, $G_N = \hbar c/m_{Pl}^2$ is the Newton's gravitational constant and m_{Pl} is the Planck mass. During the radiation era, the density and the temperature decrease as the universe expands and the evolution time can be estimated with the following equation $t = \frac{1}{2H}$. Using $g_* \sim 62$, a temperature around 200 MeV would have been reached at time of 7.7 microseconds after the Big Bang [12]. This is the time when hadrons were born in the Universe. When the dominant component of the energy density changes from radiation to matter, the evolution time changes to $t = \frac{2}{3H}$.

In contrast to the Big Bang, nuclear collisions produce less gravitational attraction and allow plasma to expand rapidly. For T >Tc the relevant degrees of freedom are quarks and gluons, while below Tc hadrons are relevant. When the temperature is dropping below Tc the quark-hadron phase transition occurs. Since experimental measurements take place after the phase transition in the hadronic stage, direct evidence of deconfined matter cannot be addressed.

The fireball expansion could be characterized by the Hubble law. The expansion rate at chemical and thermal freeze-out provides the nuclear collision analogue of the Hubble constant for the Big Bang. We estimate a "microscopic" Hubble parameter for relativistic nuclear collisions similar to cosmological Hubble constant, based on analogy between the evolution of nuclear matter produced in a collision and the Universe evolution after the Big Bang. Therefore, for the relativistic heavy ion collision case, we replace G_N with an interaction constant that takes into accont the strong interactions between the particles produced in collision, N:

$$G_N = \frac{\hbar c}{m_{Pl}^2} \rightarrow N = C \cdot \frac{\alpha_s \hbar c}{m_\pi^2} \tag{2}$$

where C is a proportionality constant, α_s is the strong coupling constant and m_π is the pion rest mass [13] (see later more considerations on the C constant).

In order to determine the Hubble constant and the evolution time of the collision system based on analogy with cosmology we need the equation of state for the nuclear matter formed in relativistic heavy ion collisions. Since QCD is asymptotically free, it is expected that at high temperature a perturbative evaluation of the equation of state in terms of a weakly interacting gas of quark and gluons should be reliable. However near the hadronization phase transition the nature of the degrees of freedom changes from quarks and gluons to hadrons and QCD becomes non-perturbative. The region of small ϵ corresponds to the hadron gas (HG) phase and can only be treated today in terms of the hadron degrees of freedom. The lattice gauge theory is the used method to study QCD non-perturbatively in the low temperature range. Therefore occured the idea of using a gas of hadrons and resonances (HRG) in low temperatures and a parametrized lattice EoS in high temperatures.

The simplest aproximation is to consider that the produced system in a relativistic heavy ion collision in the low temperature region is well described by a gas of non-interacting hadrons and resonances. Because most of the produced particles are pions, we can consider that in the hadronic phase the system is well described by a pionic gas. The Stefan-Boltzmann (blackbody) relation, valid for ultrarelativistic particles in equilibrium yields the energy density-temperature relation $\epsilon = \pi^2 T^4/10$.

Using this aproximation the microscopic Hubble constant is

$$H = \left(\frac{8\pi N \epsilon}{3}\right)^{1/2} \tag{3}$$

and the evolution time for the system is:

$$t = \frac{2}{3}\sqrt{\frac{3}{8\pi N \epsilon}} = \frac{2}{3}\sqrt{\frac{3 m_\pi^2}{8 C \pi \alpha_s \hbar c \frac{\pi^2 T^4}{10}}} \tag{4}$$

In order to determine the constant C, we use the value of the freeze-out time for Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV, t = 7.2 Fm/c [14], the thermal freeze-out temperature T=89 MeV [15] and based on the Eq. 4, we obtain the folowing value for this constant, C = 0.025.

2.1. ENERGY DEPENDENCE OF THE TIME EVOLUTION AND THE ESTIMATED MICROSCOPIC HUBBLE CONSTANT FROM EXPERIMENTAL RESULTS

The decoupling time, at chemical freeze-out, for heavy ion collisions at different energies, extracted using Eq. 4, is shown in the Table 1. The particle ratios are fitted by the chemical equilibrium model [16] for each collision system and the extracted chemical freeze-out temperatures are summerized in Table 1 [9, 16]. As the energy increases, the chemical freeze-out temperature is higher.

Table 1

The collision energy, the chemical freeze-out temperature, the chemical freeze-out time and the microscopic Hubble constant at the chemical freeze-out.

$\sqrt{s_{NN}}$ [GeV]	T$_{ch}$ [MeV]	t_{ch}^{FO} [Fm/c]	$H_{ch}\cdot 10^{23}$ [s^{-1}]
1.91	49± 3	23.75± 2.91	0.084± 0.010
2.24	54± 2	19.56 ± 1.45	0.102±0.007
2.67	70± 10	11.64± 3.32	0.172±0.049
4.8	125± 3	3.65± 0.18	0.548±0.026
6.27	134± 5	3.18± 0.24	0.630±0.047
7.62	142± 4	2.83±0.16	0.707±0.040
8.8	146± 4	2.67±0.15	0.748±0.041
12.3	153± 5	2.44± 0.16	0.821±0.054
17.3	168± 5	2.02± 0.12	0.990±0.059
62.4	154.4± 9.9	2.39±0.31	0.836±0.107
130	154.2± 9.7	2.40±0.30	0.834±0.105
200	159.3± 5.8	2.25±0.16	0.890 ± 0.065

The energy range is broad and therefore the collision evolution is changing, starting from a stopping mechanism at SIS and AGS energies to transparency in the case of RHIC and LHC collisions. For lower energies, the system evolves slowly and reach the chemical freeze-out later in time. As the energy increases, the system expanded rapidly and reaches chemical freeze-out earlier in time and does not have time to cool down more. There is a change in the system behavior between the SPS and RHIC energies and could be an indication that in this energy range the system is changing from a hadronic gas to a sQGP. At LHC, for Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76 TeV, the chemical freeze-out temperature from [17] is 152 MeV. Therefore, the chemical freeze-out time is t_{ch}^{ALICE} = 2.47 $\pm$ 0.16 Fm/c and the Hubble constant is H_{ch}^{ALICE} = (0.810 $\pm$ 0.053)$\cdot$ 10^{23} s^{-1}.

The microscopic Hubble constant at chemical freeze-out is increasing with the beam energy indicating that the produced particles from fireball interacts more strongly (the expansion rate is stronger) and the system reaches the chemical freeze-out stage earlier in time, at a higher temperature.

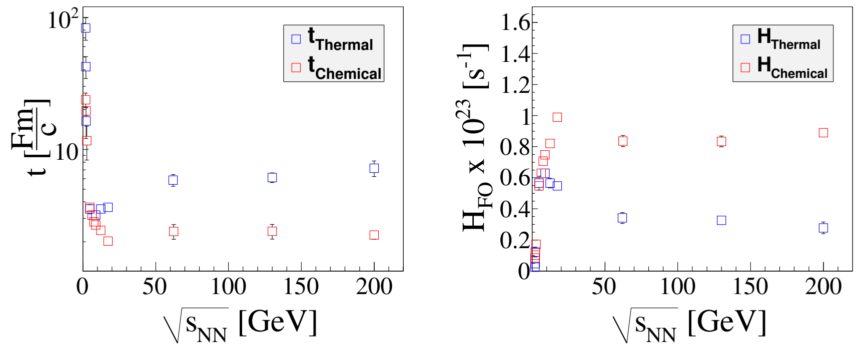

Fig. 1 – Left: The chemical freeze-out time (red) and the thermal freeze-out time (blue) for different heavy ion collisions. Right: The Hubble parameter.

In Table 2 is presented the decoupling time, at thermal freeze-out, from heavy ion collisions at different energies, extracted using Eq. 4. The thermal freeze-out temperatures were obtained from blast-wave fits [18] to transverse momentum distributions for identified particles produced in relativistic heavy ion collisions [10]. The thermal freeze-out time increases with the beam energy.

Table 2

The collision energy, the thermal freeze-out temperature, the thermal freeze-out time and the microscopic Hubble constant at the thermal freeze-out.

$\sqrt{s_{NN}}$ [GeV]	T_{ch} [MeV]	t_{ch}^{FO} [Fm/c]	$H_{ch} \cdot 10^{23}$ [s^{-1}]
1.98	26.2 ± 5.1	83.08 ± 16.17	0.024 ± 0.005
2.05	36.7 ± 7.5	42.34± 8.65	0.047 ± 0.009
2.14	59.0 ± 15.5	16.38 ± 4.30	0.122 ± 0.032
4.8	127 ± 10	3.54 ± 0.28	0.566 ± 0.045
8.8	134 ± 5	3.18 ± 0.12	0.630 ± 0.024
12.3	127 ± 7	3.54 ± 0.19	0.566 ± 0.031
17.3	125 ± 5	3.65 ± 0.15	0.548 ± 0.022
62.4	98.7 ± 10.2	5.85 ± 0.60	0.342 ± 0.035
130	96.5 ± 8.0	6.12 ± 0.51	0.327 ± 0.027
200	89 ± 12	7.20 ± 0.97	0.278 ± 0.038

The time elapsed between the chemical freeze-out and thermal freeze-out for the systems formed in relativistic collisions from SIS to AGS, SPS and RHIC seems to increase with the beam energy. This shows that the system is hotter as the energy increases, lives longer and expands to a larger size at freeze-out as compared to lower energies. It reaches the chemical freeze-out earlier in evolution, then the system expands increasingly more as the energy increases and decouples later in time at a lower temperature. For Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76 TeV, the thermal freeze-out temperature from [11] is 96 MeV. Therefore, the thermal freeze-out time is t_{th}^{ALICE} = 6.2 $\pm$ 0.64 Fm/c and the Hubble constant is H_{ch}^{ALICE} = (0.323 $\pm$ 0.034)$\cdot$ 10^{23} s^{-1}.

The microscopic Hubble constant at thermal freeze-out is decreasing with the beam energy. For all the energies starting from SPS, the microscopic Hubble constant at chemical freeze-out is higher than at thermal freeze-out showing that the expansion rate is stronger at chemical freeze-out than at thermal freeze-out stage.

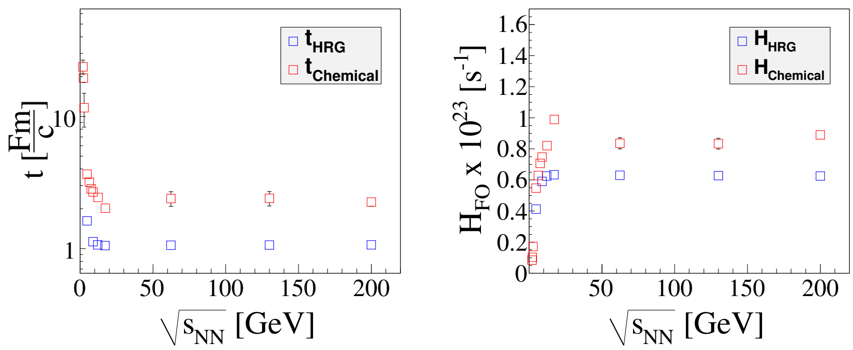

Fig. 2 – Left: The chemical freeze-out time and Hubble constant at the chemical freeze-out stage for the HRG case with excluded volume corrections as a function of the beam energy (blue symbols). Red points are the calculations made for the ideal pionic gas.

Based on the entropy density dependence on the system temperature, s $\sim$ T^3, in ref [19] it is suggested that the system size at thermal freeze-out is at least a factor of T_{ch}/T_{th} of the size at chemical freeze-out and the time difference from chemical freeze-out to thermal freeze-out is: $\Delta t \approx (\frac{Tch}{Tth} - 1)R_{ch}/\beta_s$, where R_{ch} is the system size at chemical freeze-out and β_s is the surface transverse flow velocity. We can estimate the size of the system produced in heavy ion collisions at SPS and RHIC energies at chemical freeze-out. As can be seen in Table 3, because of the flow velocities increasingly higher, the system size at chemical freeze-out also increases with energy. At LHC, in 2.76 TeV Pb-Pb collisions, the system size at chemical freeze-out is R_{CH} = 5.947 Fm.

So far we have considered the fireball acts as a ideal pionic gas characterized by a Stefan-Boltzmann type equation of state. But the pions are not the only particles produced in the collision and there are strong interactions between the fireball constituents, therefore we need a modified state equation that describes the system.

Table 3

The energy, the transverse collective flow velocity, the surface collective flow velocity and the system size at chemical freeze-out for different systems produced in heavy ion collisions at various energies.

$\sqrt{s_{NN}}$ [GeV]	$< \beta_{tr} >$	β_s	R_{ch} [Fm]
8.8	0.450 ± 0.020	0.675 ± 0.020	3.844 ± 0.151
12.3	0.470 ± 0.020	0.705 ± 0.020	3.788 ± 0.190
17.3	0.480 ± 0.020	0.720 ± 0.020	3.412 ± 0.154
62.4	0.554 ± 0.018	0.720 ± 0.011	3.416 ± 0.488
130	0.567 ± 0.020	0.765 ± 0.014	4.762 ± 0.458
200	0.592 ± 0.051	0.835 ± 0.042	5.231 ± 0.849

Hadron resonance gas (HRG) turned out to be very successful in describing particle yields and their ratios produced in heavy ion collisions. In Ref. [20], the authors estimate the energy density as a function of collision energy for chemical freeze-out in central nucleus-nucleus collisions in the framework of the interacting hadron resonance gas. The energy density calculations are made including finite hadron volume corrections and leads to a suppression of particle number densities and therefore all thermodynamical functions in the hadronic gas with excluded volume corrections become much smaller than those in the ideal gas at the same temperature. The authors find that for temperatures below 120 MeV the HRG model results with and without excluded volume correction almost coincide but for larger temperatures, the HRG with interactions yields a realistic description of the hadronic phase. The energy dependence of the energy density reflect primarily the sharp increase of the temperature at chemical freeze-out (determined from fits of experimental data up to 200 GeV) followed by a saturation above $\sqrt{s_{NN}}$=10 GeV. Based on their values for energy density we calculate the time when system reaches the chemical freeze-out stage and the microscopic Hubble constant (Fig 2). It can be observed that the chemical freeze-out time is much lower than the case of ideal pionic gas and the Hubble constant is higher. The shorter time can be explained by the fact that particle density is lower due to volume correction and therefore the system reaches the freeze-out faster than in non-interacting ideal pion gas case.

Another option is presented in Ref. [21], where the authors constructed QCD equation of state based on the lattice approach in the high temperature region, while using the resonance gas equation of state in the low temperature region. They constructed several parametrizations of the equation of state which interpolate between

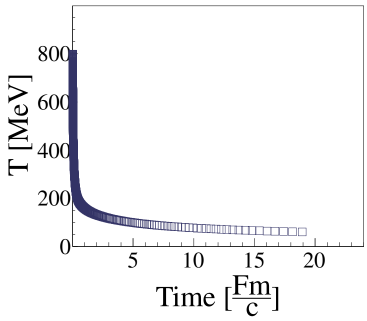

Fig. 3 – System temperature *vs.* time considering an EOS based on the lattice approach in the high temperature region, while using the resonance gas equation of state in the low temperature region.

the lattice data at high temperature and the resonance gas in the low temperature region. This corresponds to the temperature interval relevant for current experimental studies of dense matter in heavy ion collisions at RHIC as well as the experiments at the LHC. Based on their s95p-v1 parametrization for the EOS [22], we obtained the time evolution for a fireball which is in the early stages in the QGP phase and due to subsequent cooling during the expansion it reach hadronic phase. It can be seen from the Fig. 3 that the fireball is in the QGP phase only a very short time, less than 1 Fm/c and than the phase transition to the hadron gas occurs. Note that the fireball reaches thermal freeze-out temperature in a time less than 5 fm/c.

3. CONCLUSIONS

We estimated a microscopic Hubble constant around 10^{23} s^{-1} based on a Stefan-Boltzmann type equation of state. The microscopic Hubble constant is almost constant at energies for that the quark-gluon plasma formation is supported by the experimental results. Assuming a more realistic equation of state for a gas of interacting hadrons and resonances, the results are higher than those obtained in the non-interacting pionic gas.

Other interesting result is related to the estimation of the decoupling time at chemical and thermal freeze-out. The values obtained, for collisions where the quark-gluon plasma formation can be supported by the experimental results, are around a few Fm/c, in agreement with other estimations based on identical particle interferometry studies. Considering an EOS based on lattice approach in high temperature region and a hadronic gas in low temperature region, the fireball is in the QGP phase only a very short time, less than 1 Fm/c.

Acknowledgements. This work was supported by F09/30.06.2014 and PN-II-ID-PCE-IDEI 34/05.10.2011.

REFERENCES

1. E.W. Kolb, M.S. Turner, *The Early Universe*, (Addison Wesley. Redwood City, C.A. 1990).
2. S.Weinberg, *The first three minutes*, (Basic Books Publishing, New York, 1993).
3. S. W. Hawking, *A Brief History of Time. From the Big Bang to Black Holes*, Romanian translation (Humanitas Publishing Company, Bucharest, 2005).
4. I. Arsene et al., (BRAHMS Coll), Nucl. Phys. A **757**, 1 (2005); B.B. Back et al., (PHOBOS Coll), Nucl. Phys. A **757**, 28 (2005); J. Adams et al., (STAR Coll), Nucl. Phys. A **757**, 102 (2005); K. Adcox et al., (PHENIX Coll), Nucl. Phys. A **757**, 184 (2005)
5. C. Besliu, A. Jipa et al., Nucl.Phys. A **820**, 235C (2009); A. Jipa, C. Besliu et al., Int.J.Mod.Phys. E **16**, 1790 (2007); A. Jipa, V. Covlea, C. Besliu et al., Indian J.Phys. **85**, 167 (2011), C. Ristea et al., Rom. Rep. Phys. **65**, 1321 (2013)
6. M. Chojnacki, W. Florkowski, T. Csorgo, Phys.Rev. C **71**, 044902 (2005)
7. D. Boyanovsky, H. J. de Vega, D. J. Schwarz, arXiv:hep-ph/0602002
8. B. Tomasik, arXiv:nucl-th/0610042
9. N. Xu and M. Kaneta, Nucl. Phys. A **698**, 306 (2002); F. Becattini and G. Pettini, Phys. Rev. C **67**, 015205 (2003); W. Schmitz et al., J. Phys. G **28**, 1861 (2002); J. Cleymans, H. Oeschler and K. Redlich, J. Phys. G **25**, 281 (1999); J. Cleymans et al. Phys. Rev. C **57**, 3319 (1998); F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, Phys. Rev. C **64**, 024901 (2001)
10. W. Reisdorf et al., Nucl. Phys. A **612**, 493 (1997); M.A. Lisa et al., Phys. Rev. Lett. **75**, 2662 (1995); C. Muntz, arXiv:nucl-ex/9806002.; H. Appelshauser et al., Eur. Phys. J. **C2**, 661 (1998); J. Adams et al., STAR Collaboration, Phys. Rev. Lett. **92**, 112301 (2004)
11. B. Abelev et al., (ALICE Collaboration), Phys.Rev.Lett. **109**, 252301 (2012)
12. C. Grupen, *Astroparticle Physics*, (Springer Verlag, 2005); K.A. Olive, arXiv:hep/ph/1005.3955.
13. E. Recami, arXiv:physics/0505149; arXiv:physics/0105080; arXiv:gr-qc/9509005.
14. B. Muller, J. Schukraft, B. Wyslouch, arXiv:hep-ex/1202.3233; K. Aamodt, et al. (ALICE Collaboration), Phys.Lett.B **696**, 328 (2011)
15. B. Abelev et al., STAR Collaboration, Phys. Rev. C **79**, 34909 (2009), arXiv:nucl-ex/0808.2041
16. P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, Phys. Lett. B **344**, 43 (1995); P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, Phys. Lett. B **365**, 1 (1996); P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B **465**, 15 (1999)
17. A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, arXiv:nucl-th/1210.7724.
18. E. Schnedermann, J. Sollfrank, and U. Heinz, Phys. Rev. C **48**, 2462 (1993); D. Teaney, J. Lauret, and E.V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001); U. Heinz and P. Kolb, Nucl. Phys. A **702**, 269 (2002); F. Retiere and M.A. Lisa, Phys. Rev. C **70**, 044907 (2004)
19. J. Adams et al., STAR Collaboration, Phys.Rev.Lett. **92**, 112301 (2004), arXiv:nucl-ex/0310004.
20. A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, arXiv:nucl-th/1201.0693.
21. P. Huovinen, P. Petreczky, arXiv:hep-ph/0912.2541.
22. `http://th.physik.uni-frankfurt.de/∼huovinen/eos/s95p-v1.tar.gz` and `https://wiki.bnl.gov/hhic/index.php/Lattice calculatons of Equation of State.`