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STUDY ON THE STABILIZATION OF ATTRACTIVE BOSE-EINSTEIN CONDENSATES USING PROJECTION OPERATOR METHOD

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Abstract. We study the stability of Bose-Einstein condensates (BECs) with attractive two- and three-body interactions by analyzing the nonlinear, cubic quintic Gross-Pitaevskii equation (CQGPE). By employing a projection operator method we discuss the stabilization of an attractive BEC by varying the strength of the trapping potential. Our analysis suggests that the reduction in strength of the external trap potential increases the stability of an attractive BEC. These results were also confirmed through direct numerical simulations of the CQGPE.

Key words: Bose-Einstein condensate, projection operator method, Gross-Pitaevskii equation, split-step Crank-Nicolson method.

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1. INTRODUCTION

Since the first experimental realization of Bose-Einstein condensates (BECs) in 1995 for rubidium [1], lithium [2], and sodium [3], the stability properties of the trapped clouds of BEC atoms have been investigated intensively both experimentally and theoretically. The nonlinear nature of a BEC in the mean-field description allows investigations of many fields, such as, modulational instability [4], solitons [5], vortices [6], four-wave mixing [7], interference pattern [8] and etc. The latter soliton has been realized dark, gray and gap matter wave solitons in repulsive effective interactions, as well as bright matter-wave solitons in attractive interactions. There were also proposals to prepare experimentally solitons in a mixture of bosonic and fermionic atoms [9] and in an atomic BEC with a time-dependent scattering length [10]. In vortices, bright vortex, dark vortex and vortex lattice has been realised. Most importantly, the properties and dynamics of Bose-Einstein condensates mainly depend on the strength, geometry and type of the external trapping potentials. So in the present work, by tuning the strength of the external trapping potentials, we intent to investigate the stability properties of BEC with two- and three-body interactions [11].

At ultra low temperatures, the properties of BEC can be well described by the mean field Gross-Pitaevskii (GP) equation, a nonlinear Schrödinger (NLS) type equation for the condensate parameters. When the interatomic interactions are repulsive, the condensate is stable and its size and number have no fundamental limit. On the other hand, for attractive interactions the condensate is stable up to a critical value due to matter wave collapse. With a supply of atoms from an external source the condensate can grow again and thus a series of collapses can take place, this has been observed experimentally in BECs of ^{7}Li with attractive interaction [2]. Theoretical analysis based on the GP equation also confirms the collapse. Thus for a system of atoms with attractive two-body interaction, the condensate has no stable solution above a certain critical number of atoms N_{max} [12]. However, the addition of a repulsive potential derived from three-body interaction can stabilize the condensate against collapse beyond the critical number N_{max} [13]. Even for a very small strength of the three-body interaction, the region of stability of the condensate can be extended considerably. There were reports, by considering the possible effective interaction, that a sufficiently dilute and cold Bose gas exhibits similar three-body interaction dynamics for both signs of the s-wave scattering length [14]. It has also been suggested that, for a large number of bosons the three-body repulsion can overcome the two-body attraction, and a stable condensate will appear in the trap [15]. The attractive condensate can also be stabilized against collapse by suitably modifying the confining potential. When the confining potential is made asymmetric such that the atoms can only undergo one-dimensional motion, it can form a stable self-focusing BECs or matter-wave soliton [16].

At low temperature and density, where interatomic distances are much greater than the distance scale of atom-atom interactions, two-body interaction can be described by a single parameter (scattering length) where the effects of three-body interaction are negligible. At low enough temperatures the magnitude of the scattering length a is much less than the thermal de Broglie wavelength and the exact shape of the two-atom interaction is unimportant. On the other hand, if the atom density is considerably high the three-body interaction can start to play an important role [17, 18]. One may note that even at a very dilute limit, dominated by two-body interactions, few-body interactions are also important. At absolute zero temperature, we consider only the elastic collisions between atoms in the condensate *i.e.*, the imaginary part of the two- and three-body interactions are negligible and the real part of the two- and three-body interactions are considered [13, 18]. While at finite temperature ($\simeq$ nano (10^{-9}) K), we considered both the elastic and inelastic collisions between atoms in the condensate *i.e.*, the real and imaginary parts of the two- and three-body interactions are important [19–22].

At finite temperature both the elastic and inelastic collisions are important and the corresponding GP equation includes complex coefficients in the resulting equation. Most of the complex nonlinear partial differential equations governing the nonlinear systems are in the family of NLS equation. Many important physical systems like, nonlinear fiber optics, water waves, plasma waves, BEC, etc., are governed by the NLS equation. The NLS equation is completely integrable and its N-soliton solutions can be obtained using the standard inverse scattering transform scheme. In any physical system governed by the NLS equation, a simple physical effect like dissipation, due to the inelastic collision between the atoms in BEC, is enough to destroy this integrability property of the NLS equation. To study the dynamics of such physical systems various mathematical techniques like perturbational analysis, numerical analysis, variational analysis, etc., have been developed. Lagrangian variational method (LVM) is one of the famous techniques often used in nonlinear fiber optics, fluid dynamics and in BEC [23]. Using the Lagrangian corresponding to the NLS equation and a suitable ansatz function like hyperbolic secant or Gaussian, the ordinary differential equations (ODEs) governing the parameters introduced in the ansatz can be derived with the LVM. There are numerous works related to the modification of the LVM to include various important effects neglected in the formalism of typical variational method. One of the important factors considered by various researcher to modify the LVM has been on the radiation induced solitons interactions [24]. Kuznetsov *et al.* [25], considered the nonlinear interaction of solitons and radiations, where they reported the unsuccessful application of the variational method. Nevertheless all these works were based on the fundamental LVM and its success or failure. Mikhailov has reported an interesting article about the validity of the LVM [26]. At the same time, by simplifying the linear and nonlinear loss/gain term in the GP equation the variational procedure used for the trapped BEC in Ref. [22]. Although, the variational approach works very well for conservative system, it fails for non-conservative system. However, the projection operator method (POM), a more generalized theory can be used to study the stability of the system in the presence of elastic and inelastic collisions between atoms in the condensate [27, 28]. A collective variable (CV) theory of POM can me made to be equivalent to the LVM [28]. It has been shown that from the generalized POM for complex nonlinear PDE one can drive the ODEs that could be derived either by LVM or the bare approximation of the CV theory are unique for Gaussian ansatz [28, 29]. Thus, in this work, we adopt a generalized POM to obtain the equations of motion for investigation of the stability of BEC.

The typical system under consideration in this work is the Bose gas with attractive interaction between atoms subject to different strength of the trap potentials. In the present study we analyze stabilization of the attractive BEC with attractive two- and attractive three-body interactions. In this paper, in addition to analytical studies, we also perform numerical verification for the stability of the attractive BEC in the presence of external trap. In particular, by analyzing the GP equation using POM and direct numerical integration, we address stability properties in most of the possible cases. Our present analysis strongly suggests that the reduction of the strength of the external potential can stabilize the attractive BECs. We also illustrate from numerical simulations that the attractive condensate can maintain a reasonably constant spatial profile over a sufficient interval of time.

The organization of the present paper is as follows. In Section 2, we present a brief overview of the mean-field model. Then, we discuss the stability of the condensates using a projection operator method and bring out the possible stabilization of an attractive BEC in Section 3. In Section 4, we report the numerical results of the time-dependent GP equation with two- and three-body interactions and investigate the stability of an attractive BEC for different choices of the trap strength and inter-atomic interaction. Finally, we provide the concluding remarks in Section 5.

2. CUBIC QUINTIC GROSS-PITAEVSKII EQUATION (CQGPE)

The GP equation can be used at low temperature to explore the macroscopic behavior of the Bose-Einstein condensates. In the presence of two- and three-body interactions, the time-dependent GP equation can be described by [13, 17, 18, 22]:

$$i\hbar\frac{\partial\Psi(\mathbf{r},\tau)}{\partial\tau} = \Bigg[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + \left(G_0 + \frac{i\hbar}{2}G_1\right)N|\Psi(\mathbf{r},\tau)|^2 + \left(K_0 + \frac{i\hbar}{2}K_1\right)N^2|\Psi(\mathbf{r},\tau)|^4\Bigg]\Psi(\mathbf{r},\tau), \tag{1}$$

where $\Psi(\mathbf{r},\tau)$ is the condensate wavefunction, m is the mass of a single bosonic atom, N is the number of atoms in the condensate, G_i's and K_i's, $i = 0,1$ are the nonlinear coefficients corresponding to the two- and three-body interactions, respectively and $V(\mathbf{r})$ is the external trap potential. The terms G_1 and K_1 denote two-body and three-body recombination rate coefficients. The positive and negative values of G_1 and K_1 correspond to gain and loss of atoms due to two- and three-body interactions. The normalization condition for the condensate wavefunction is $\int d\mathbf{r}|\Psi(\mathbf{r},\tau)|^2 = 1$. The two-body interaction strength is $G_0 = 4\pi\hbar^2 aN/m$ with a the inter-atomic scattering length, while the other coefficients G_1, K_0 and K_1 can be expressed in terms of G_0. In this study, we assume spherically symmetric trap of the form $V(\mathbf{r}) = \delta\frac{1}{2}m\omega^2\tilde{r}^2$ with ω is the angular frequency and $\tilde{r}$ the radial distance and δ corresponds to the strength of the external trap which can be varied between 0 and 1.

After a partialwave projection the radial part ϕ of the wave function Ψ can be written as $\Psi(\mathbf{r},\tau) = \phi(\tilde{r},t)$. The above GP equation (1) can be rewritten, using convenient dimensionless variables defined by $r = \sqrt{2}\tilde{r}/l$, $t = \tau\omega$ with $l = \sqrt{\hbar/(m\omega)}$

and $\psi(r,t) = \phi(\tilde{r},t)[l^3/(2\sqrt{2})]^{1/2}$, as

$$i\frac{\partial\psi(r,t)}{\partial t} = \left[-\frac{\partial^2}{\partial r^2} - \frac{2}{r}\frac{\partial}{\partial r} + \delta\frac{r^2}{4} + (g+ig_1)\,|\psi|^2 + (\chi+i\chi_1)|\psi|^4\right]\psi(r,t), \quad (2)$$

where, $g = 8\sqrt{2}\pi aN/l$ and $\chi = 8K_0N^2/\hbar(\omega l^6)$ are the rescaled strengths of two- and three-body interactions, respectively, the imaginary terms $g_1 = 16\pi\hbar aN/l^2$ and $\chi_1 = 4K_1N^2/(\omega l^6)$ are the rescaled two- and three-body interaction coefficients corresponding to the loss/gain of atoms. The strengths of the three-body interaction are usually very small when compared with that of the two-body interactions [13, 17, 18]. We consider the three-body interaction strengths of the order of 10% of that of two-body interaction, $(\chi \approx 0.1g)$ [18]. Since χ is a function of g, the three-body interactions can also be controlled by the tuning of s-wave scattering length [13]. The normalization condition is $4\pi\int_0^\infty r^2 dr|\psi(r,t)|^2 = 1$.

3. PROJECTION OPERATOR METHOD

The POM has been extensively used to study complex equation like the NLS equation describing the light propagation in optical fibers [28–30]. Here we perform the approach of POM to deduce the governing equation for the case of time dependent GP equation by considering an ansatz function $\psi(A,R,\alpha,\beta)$. A, R, α and β are, functions of r and t, the condensate parameters or collective variables. We have used a generalized projection operator $P_k = \exp(i\theta)\psi^*_{X_k}$, where θ is an arbitrary phase constant and $X_k \in \{A,R,\alpha,\beta\}$. To obtain the CVs equations of motion, we project (2) in the direction of P_k. By substituting the ansatz function ψ in (2), multiplying the resulting equation by P_k and then integrating, we have obtained the following equation:

$$4\pi\int_0^\infty \Bigg\{ \Im\left[\psi_t\psi^*_{X_k}e^{i\theta}\right] + \Re\left[\left(\psi_{rr}\psi^*_{X_k} + \frac{2}{r}\psi_r\psi^*_{X_k}\right)e^{i\theta}\right] - \Re\left[\delta\frac{r^2}{4}\psi\psi^*_{X_k}e^{i\theta}\right]$$
$$- \Re\left[(g+ig_1)\,\psi\psi^*_{X_k}e^{i\theta}\right]|\psi|^2 - \Re\left[(\chi+i\chi_1)\,\psi\psi^*_{X_k}e^{i\theta}\right]|\psi|^4\Bigg\} r^2 dr = 0, \quad (3)$$

where $\psi^*_{X_k} = \partial\psi^*/\partial X_k$, $X_k \in \{A,R,\alpha,\beta\}$, $\Im[\ldots]$ represents imaginary part of $[\ldots]$ and $\Re[\ldots]$ corresponds to real part of $[\ldots]$. When the phase constant θ in the above equation (3) is chosen as $\pi/2$, the corresponding projection operator scheme is equivalent to the LVM [29]. Substituting $\theta = \pi/2$ in (3), we get the equivalent Lagrangian variations as

$$4\pi\int_0^\infty\left\{\Re\left[\psi_t\psi^*_{X_k}\right]+\Im\left[\psi_{rr}\psi^*_{X_k}+\frac{2}{r}\psi_r\psi^*_{X_k}\right]-\Im\left[\delta\frac{r^2}{4}\psi\psi^*_{X_k}\right]\right.$$
$$\left.-\Im\left[(g+ig_1)\,\psi\psi^*_{X_k}\right]|\psi|^2-\Im\left[(\chi+i\chi_1)\,\psi\psi^*_{X_k}\right]|\psi|^4\right\}r^2dr=0 \qquad (4)$$

In other words equation (4) is equivalent to the variations of the form [29]

$$\frac{\partial L_{eff}}{\partial X_k}-\frac{d}{dt}\left(\frac{\partial L_{eff}}{\partial \dot{X}_k}\right)=0, \qquad (5)$$

The effective Lagrangian L_{eff} is calculated by integrating the average Lagrangian density as $L_{eff}=4\pi\int_0^\infty\mathcal{L}(t)\,r^2dr$. The average Lagrangian density of (2)

$$\mathcal{L}(t)=\frac{i}{2}\left(\psi_t\psi^*-\psi^*_t\psi\right)-|\nabla\psi|^2-\delta\frac{r^2}{4}|\psi|^2-\frac{1}{2}(g+ig_1)|\psi|^4-\frac{1}{3}(\chi+i\chi_1)|\psi|^6, \qquad (6)$$

when $g_1=0$ and $\chi_1=0$, the results of equations (4) and (5) are equivalent.

In the present work, we essentially look for stabilization of attractive BEC. Normally one may expect solitons in BEC using GP equation when the system should be conservative. It means that the GP equation should not have any dissipative term like gain/loss etc. If we include the effect of gain/loss of atoms then the corresponding GP equation will be a non-conservative system and hence there is no soliton in the conventional sense. However, one can still look for non-autonomous solitons by suitably tailoring the gain/loss of atoms. For example, such non-autonomous solitons have been studied by Rajendran *et al.* [19], Serkin *et al.* [20]. The nature of such solitons in the case with both two- and three-body interactions and gain/loss of atoms have been considered to some extent by Roy *et al.* [21]. However, in the present study, we mainly focus on the stabilization of attractive BEC for conservative system.

In order to obtain the governing equation of motions of the condensate parameters, we use the following Gaussian ansatz [30].

$$\psi(r,t)=A(t)\exp\left[-\frac{r^2}{2R(t)^2}+\frac{i}{2}\beta(t)r^2+i\alpha(t)\right], \qquad (7)$$

where $A(t)$, $R(t)$, $\alpha(t)$ and $\beta(t)$ are the time dependent amplitude, width, chirp and phase, respectively. By applying the POM one may obtain a set of four coupled ordinary differential equations, which describe the dynamics of BEC parameters,

$$\dot{A} = -3A(t)\beta(t), \quad \dot{R} = 2\beta(t)R(t), \tag{8a}$$

$$\dot{\alpha} = -\frac{7g}{8\sqrt{2}}A(t)^2 - \frac{2\chi}{3\sqrt{3}}A(t)^4 - \frac{3}{R(t)^2}, \tag{8b}$$

$$\dot{\beta} = \frac{\delta}{2} - 2\beta(t)^2 + \frac{2}{R(t)^4} + \frac{g}{2\sqrt{2}}\frac{A(t)^2}{R(t)^2} + \frac{4\chi}{9\sqrt{3}}\frac{A(t)^4}{R(t)^2}. \tag{8c}$$

At this juncture, it may be noted that there is no quantitative (or qualitative) difference between the above equations (8a)-(8c) and those obtained through LVM. On the other hand, if one uses other ansatz like hyperbolic secant or raised cosine function in POM, then it may generate different sets of ODEs [28, 29].

The equation for the evolution of the width R can be obtained by combining the equations (8a)-(8c) as,

$$\ddot{R}(t) = -R(t)\delta + \frac{4}{R(t)^3} + \frac{g(t)}{\sqrt{2\pi^3}R(t)^4} + \frac{8\chi(t)}{9\sqrt{3}\pi^3 R(t)^7} \equiv -\frac{\partial U(R)}{\partial R}, \tag{9}$$

and the effective potential $U(R)$ corresponding to the above equation of motion can be written as,

$$U(R) = \delta\frac{R^2}{2} + \frac{2}{R^2} + \frac{g}{3\sqrt{2\pi^3}R(t)^3} + \frac{4\chi}{27\sqrt{3}\pi^3 R^6}. \tag{10}$$

We analyse the nature of the effective potential for the different possible interactions between atoms in the condensate with and without the external trap. First we graphically explore the stability of an attractive and a repulsive BEC in the presence of full external trap ($\delta = 1$). Figs. 1(a) and 1(b) show the effective potential as a function of R for different g values in both two-body repulsive and attractive regimes ($|g| = 10, 20, 30$ and $\chi = 0.1g$). The minima in potential indicates that stabilization is possible. In the case of repulsive interactions (when $g > 0$ and $\chi > 0$) one expects stabilization. On the other hand, when the interactions are attractive, stable condensate may occur for inter-atomic interaction strengths $g > g_{min}$. We also consider the case with attractive two-body and repulsive three-body interactions. Figure 2 shows the effective potentials for the case with attractive two-body ($g < 0$) and repulsive three-body ($\chi > 0$) interactions [13]. In the repulsive case, BEC is stable even when the strength of the interaction is very high in the system. However, in the attractive interaction case, BEC tend to collapse due to inter-atomic attraction. In general, in the presence of trap, the stability of BEC is mainly depends on two important forces. First one is the confining trap potential and the other is the attractive inter-atomic interactions. This fact is clearly seen in Figures 1 and 2, where the effective potential has a minimum while increasing the strength of the attractive interactions in the system.

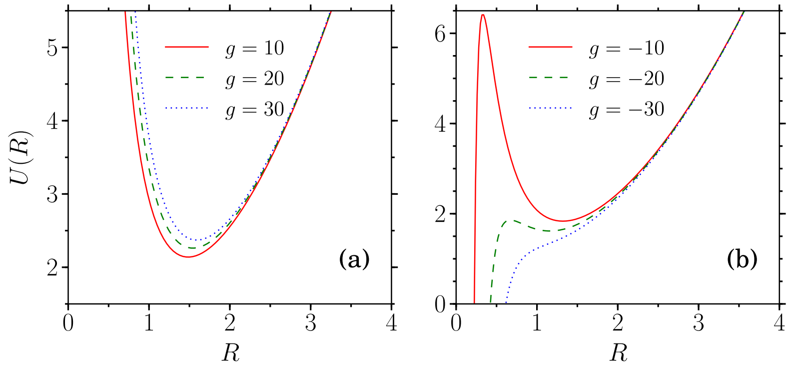

Fig. 1 – (a) Potential energy curves for both two- and three-body interactions are repulsive $g > 0$ and (b) Potential energy curves for both two- and three-body interactions are attractive $g < 0$. The parameters are $\chi = 0.1g$, $|g| = 10, 20, 30$ and $\delta = 1$ in (10)

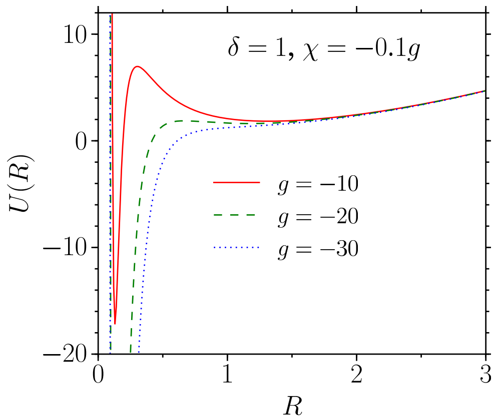

Fig. 2 – Potential curves for attractive two-body ($g = -10$, -20, and 30) and repulsive three-body ($\chi = -0.1g$) interactions.

In the present study, our main focus is to study the stabilization of BEC with attractive two- and three-body interactions and by tuning the strength of the external trap. For this purpose, we consider the cases with different strengths of the trap, δ. In Figure 3 we plot the effective potential for different δ values ($\delta = 1$, 0.75 and 0.5). The depth of the minimum in the effective potential found to increase while lowering the trap strength. This means the attractive force acting on the system is reduced and the condensate stability is increased. Thus lowering the trap strength results in the increase of the stability of an attractive condensate with three-body interactions.

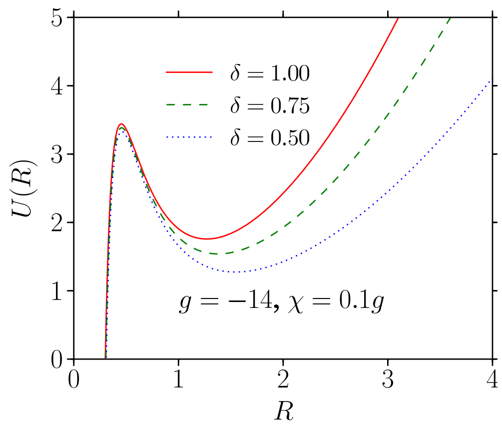

Fig. 3 – Potential energy curves for different trap strength ($\delta = 1$, 0.75 and 0.5) with both two-and three-body interactions are attractive ($g = -14$ and $\chi = 0.1\,g$).

4. NUMERICAL STUDY

Next we study the stability of BEC with different possible interactions considered above, by solving the time-dependent GP equation (2) numerically. The numerical solutions were obtained using a split-step Crank-Nicolson method (SSCN) [31, 32]. The harmonic oscillator solution is used as initial condition and the space and time steps for solving SSCN method are $dx = 0.01$ and $dt = 0.0001$, respectively.

We verify the analytically studied stability of BEC (Figure 1) with both two- and three-body interactions through numerical simulation. Figures 4 and 5 depict the evolution of the density (Figure 4) and the rms radius (Figure 5), respectively, for (a) both two- and three-body interactions are repulsive and (b) both two- and three-body interactions are attractive. In the case of repulsive interactions between

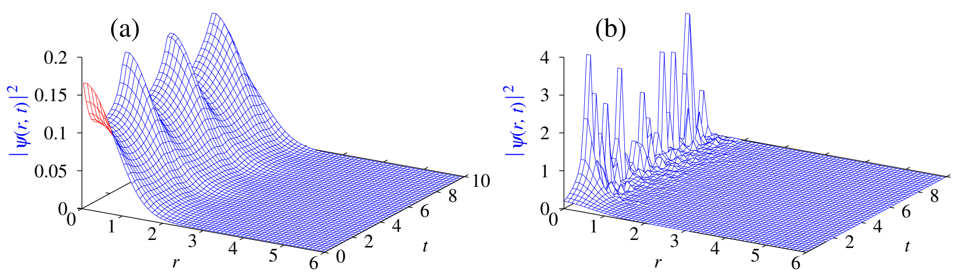

Fig. 4 – Plot of the density $|\psi(r,t)|^2$ illustrating the stable and unstable dynamics corresponding to Figure 1: (a) both two- and three-body interactions are repulsive ($g = 14$, and $\chi = 0.1g$) and (b) both two- and three-body interactions are attractive ($g = -14$, and $\chi = 0.1g$).

atoms in the condensate the trapped BEC is known to be stable and the rms radius, $\langle r \rangle_{\text{rms}}$ shows steady oscillations as illustrated in Figure 5(a). The spacetime plot of the density $|\psi(r,t)|^2$ also exhibits similar steady oscillations as shown in Figure 5(a). On the other hand, when the interactions are attractive, collapse occurs for strength of interaction g is less than certain critical value. This is illustrated in Figure 5(b)

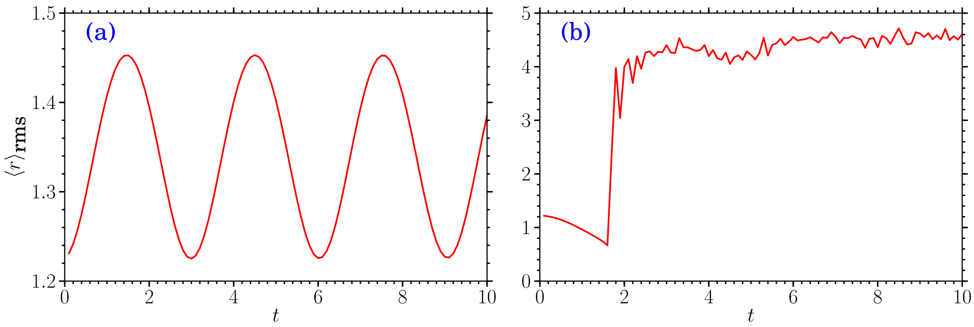

Fig. 5 – Plot of the $\langle r \rangle_{\text{rms}}$ versus t for the cases illustrated in Figure 4. The parameters are the same as in Figure 4.

(rms radius) and Figure 4(b) (density). In the presence of attractive interactions, the BEC collapses due to the high inter-atomic attraction below a certain critical value. While in the presence of repulsive interactions, the attraction due to the external trap is compensated by the repulsive inter-atomic interaction between atoms. However, the addition of a repulsive three-body interaction with attractive two-body interaction is consistent with a number of atoms larger than critical value [13].

This fact is illustrated with Figure 6, corresponding to the case shown in Figure 2, where we have used an attractive two-body ($g = -14 < 0$) and a repulsive three-body ($\chi = 1.4 > 0$) interactions. Figure 6(a) shows the evolution of the rms radius and Figure 6(b) is the contour plot of the density in the $t - r$ plane. It is evident from Figure 6 that the BEC is stable with attractive two-body and repulsive three-body interactions.

Now we consider the stabilization of BEC with both attractive two- and three-body interactions by tuning the strength of the external trap potential from numerical simulations. In Figure 7, we show the evolution the density $|\psi(r,t)|^2$ for different trap strengths, $\delta = 1.0$, 0.75 and 0.748 with $g = -14$ and $\chi = -1.4$ in (2). It is clear from the figures that the initial Gaussian profile tend to collapse when $\delta = 1$ and 0.75, while the collapse is prevented for $\delta = 0.748$. This means that the attractive forces acting on the system is reduced by tuning the strength of the external trap and the size of condensate. Figure 8 depicts the evolution of the rms radius $\langle r \rangle_{\text{rms}}$ for different δ values considered above, which shows steady oscillations for $\delta = 0.748$.

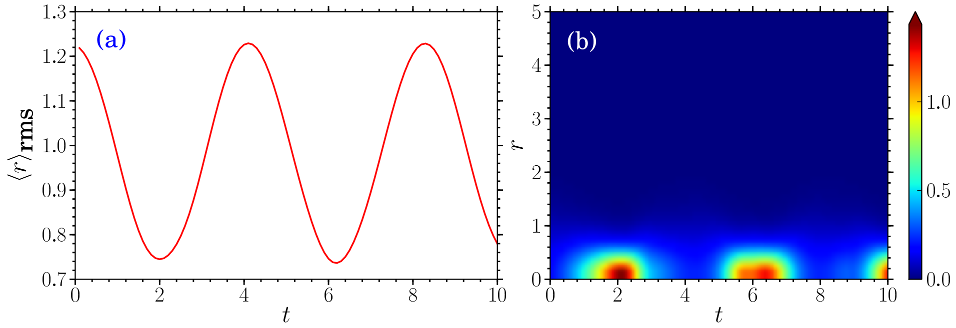

Fig. 6 – Plot of (a) the rms radius, $\langle r \rangle_{\rm rms}$ as a function of t and (b) the contour of $|\psi(r,t)|^2$ in $t-r$ plane for attractive two-body and repulsive three-body interactions discussed in Figure 2. The parameters are $g = -14$, $\chi = 1.4$ and $\delta = 1$ in (2).

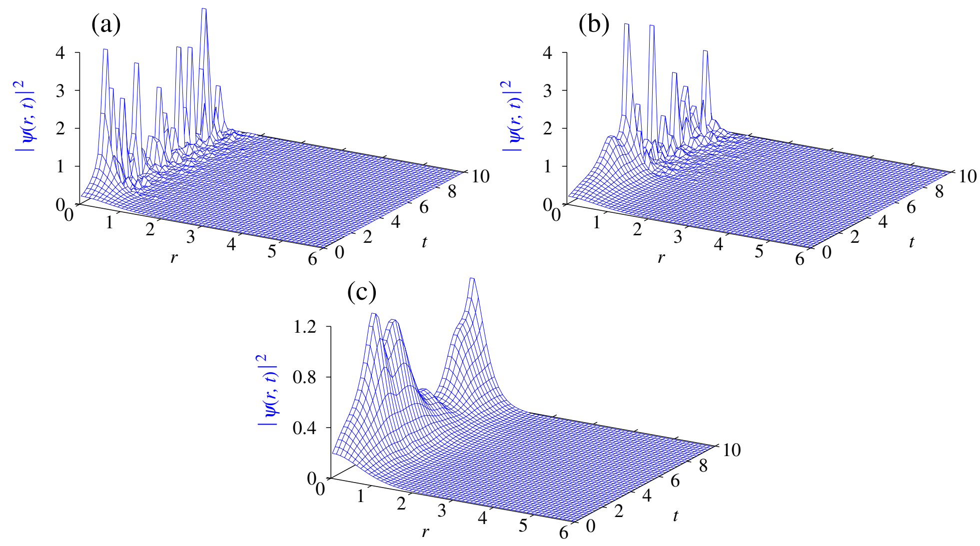

Fig. 7 – Plots of the numerically computed density $|\psi(r,t)|^2$ as a function of time (evolution initial Gaussian profile) for different trap strengths (δ) with attractive two- and three-body interactions. (a) $\delta = 1.0$, (b) $\delta = 0.75$, and (c) $\delta = 0.748$. The other parameters are fixed as $g = -14$ and $\chi = -1.4$ in (2).

As predicted in Figure 3, lowering the trap strength increases the stability of BEC with attractive two- and three-body interactions.

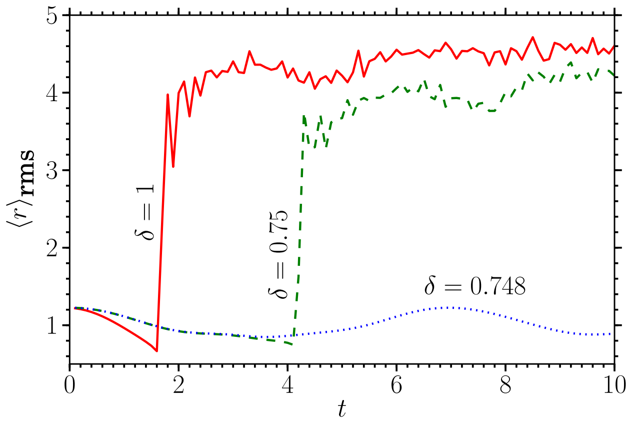

Fig. 8 – Plot of the root means squared distance $\langle r \rangle_{\text{rms}}$ as a function of time t corresponding to the three different choices of trap strengths considered in Figure 7. The other parameters as the same as in Figure 7.

5. CONCLUSIONS

In conclusion, we have theoretically analysed the stability of BECs with two- and three-body interactions through both analytical and numerical methods. We have studied analytically, the stability of both attractive as well as repulsive BEC with two- and three-body interactions using the projection operator method in the presence of harmonic trap. We have shown that there is an enhancement of the stability of an attractive BEC when the strength of external trap is reduced from above. We have also confirmed our analytical results through direct numerical simulations and results are agreement well with the analytical predictions.

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