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THE PROPAGATION OF TWO-DIMENSIONAL EXTREMELY SHORT OPTICAL PULSES IN SILICON NANOTUBES WITH RELAXATION AND AMPLIFICATION

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Abstract. We consider the wave equation for an electromagnetic field, which propagates in silicone nanotubes. The processes of damping and amplification are taken into account within the framework of the "phenomenological approach". Also, we study the effects observed when the initial pulse shape, in the form of either a Gaussian beam or an Airy beam, changes.

Key words: extremely short pulses, silicon nanotubes, relaxation, amplification.

1. INTRODUCTION

Since the early 90's in the twentieth century, substances with a tubular structure such as carbon [1], nitride-boron [2–3], and silicon nanotubes (SNTs) [4–7] possess a great attractiveness in terms of their unique properties. And, the great attention of researchers to silicon and the material based on it is due, first of all, to the fact that it is still the main basic element in modern microelectronics. The fact in his favor is the presence of a large spin-orbit gap [8], which makes silicon nanotube semiconductors.

On the other hand, over the past two decades, scientists have made significant progress in the field of nonlinear optics. One of the most intensively investigated research field is the study of extremely short light pulses, which are structures consisting of several oscillations of the electromagnetic field [9–15]. These physical objects are widely used in the field of light and matter interactions, generation of high order harmonics, and femto- and attosecond physics [16, 17]. Within the framework of this research direction, it is possible to single out an area that deals with the study of light bullets (LB) [18] – optical objects localized in two or three dimensions. We note only a few papers concerning the study of their evolution in nanostructures. So, in [19], the possibility of stable propagation of two-dimensional LBs in an array of carbon nanotubes (CNTs) was demonstrated. In [20], a generalization to the three-dimensional case was carried out and the inhomogeneity of the array of semiconductor CNTs was taken into account. The results of the study showed that after interaction with a region with a higher concentration, the pulse can propagate steadily without significant spreading.

All of the above circumstances stimulated us to consider the evolution of a two-dimensional light bullet in an array of SNTs, taking into account attenuation and amplification, and to answer the question of the stability of its propagation, which may have important practical applications.

2. BASIC EQUATIONS

The Hamiltonian for the SNT in the tight-binding model and the long-wave approximation can be written as [21]:

$$\begin{aligned} H_\eta &= \hbar v_f\left(k_x\tau_x - \eta k_y\tau_y\right) + \eta\tau_z h_{11} \\ h_{11} &= -\lambda_{SO}\sigma_z - a\lambda_R(k_y\sigma_x - k_x\sigma_y) \end{aligned} \tag{1}$$

where $\eta = \pm 1$, (k_x, k_y) is the electron quasi-momentum, λ_{SO} is the spin orbit interaction constant, λ_R is the Rashba interaction constant, τ_i are the Pauli matrices of the sublattice pseudospin, σ_i are the Pauli matrices for the spin, $v_f = 5.5 \cdot 10^5$ m/s is the Fermi velocity.

According to Eq. (1) the eigenvalues are:

$$\varepsilon = \pm\sqrt{\eta^2\lambda_{SO}^2 - \left(1\pm\sqrt{2}\right)\hbar^2 v_f^2\left(k_x^2+k_y^2\right) + \eta^2 a^2\lambda_R^2\left(k_x^2+k_y^2\right)} \tag{2}$$

Let us write the two-dimensional wave equation taking into account damping and amplification processes:

$$\boldsymbol{A}_{tt} - \Gamma\boldsymbol{A}_t = \frac{\partial^2\boldsymbol{A}}{\partial x^2} + \frac{\partial^2\boldsymbol{A}}{\partial z^2} + 4\pi j\left(\boldsymbol{A}\right)\cdot\exp\left(-t/t_{rel}\right) \tag{3}$$

where the parameter Γ describes the electric field pumping [22] and, accordingly its amplification, j is the electric current density, $\boldsymbol{A}$ is the vector-potential (taking into account: $\boldsymbol{E} = -\partial\boldsymbol{A}/c\partial t$). In Eq. (3) the relaxation of electrons in SNT with the time t_{rel} was phenomenologically taken into account [23]. This relaxation, in turn, leads to the attenuation of the current, which is described with the corresponding factor.

The current density can be determined according to the following formula:

$$j = e\sum_p \mathrm{v}_y\left(p - \frac{e}{c}A\left(x,z,t\right)\right)\left\langle a_p^+ a_p\right\rangle, \tag{4}$$

where $\mathrm{v}_y\left(p\right) = \dfrac{\partial\varepsilon\left(p\right)}{\partial p_y}$, <…> denote the averaging with a non-equilibrium density matrix $\rho\left(t\right){:}Sp\left(B\left(0\right)\rho\left(t\right)\right)$ and a_p^+, a_p are the operators of creation and annihilation of electrons in SNT. In the expression for the current density, one can use the number of particles that follows from the Fermi-Dirac distribution. In the case of low temperatures, only a small region in the momentum space near the Fermi level contributes to the sum (4):

$$j = e\int_{-\Delta}^{\Delta}\int_{-\Delta}^{\Delta} \mathrm{d}p_z \mathrm{d}p_y \mathrm{v}_y\left(p - \frac{e}{c}A(x,z,t)\right). \tag{5}$$

The domain of integration over the momenta in (5) is determined from the equality of the number of particles:

$$\int_{-\Delta}^{\Delta}\int_{-\Delta}^{\Delta} \mathrm{d}p_z \mathrm{d}p_y = \int_{ZB} \mathrm{d}p_z \mathrm{d}p_y \left\langle a^{+}_{pz,py} a_{pz,py}\right\rangle,$$

where the integration on the right hand side is carried out over the first Brillouin zone. Thus, we can replace the integral of the Fermi function over the Brillouin zone by an integral over a rectangular region with coordinates [(–Δ; Δ), (–Δ; Δ)].

3. MAIN RESULTS

Equation (3) was solved numerically after performing its dimensionlessness [24]. The initial condition was chosen in two forms: the Gaussian form (6a) and the Airy beam (6b):

$$A(x,z,0) = Q \cdot \exp\left(-\frac{z^2}{\gamma_z^2} - \frac{x^2}{\gamma_x^2}\right),$$
$$\frac{\mathrm{d}A(x,z,0)}{\mathrm{d}t} = \frac{2Qz\nu_z}{\gamma_z^2}\exp\left(-\frac{z^2}{\gamma_z^2} - \frac{x^2}{\gamma_x^2}\right), \tag{6a}$$

$$F(x) = \int_x^{\infty} Ai(y)\mathrm{d}y,$$
$$A(x,z,0) = Q \cdot F\left(\frac{z}{\gamma_z} + \kappa\left(\frac{z}{\gamma_z}\right)^2\right) J_0\left(\frac{x}{\gamma_x}\right)\exp\left(-\frac{x}{\gamma_x}\right), \tag{6b}$$
$$\frac{dA(x,z,0)}{\mathrm{d}t} = Q\left.\frac{\mathrm{d}F\left(\frac{z-\nu_z t}{\gamma_z} + \kappa\left(\frac{z-\nu_z t}{\gamma_z}\right)^2\right)}{\mathrm{d}t}\right|_{t=0} \cdot J_0\left(\frac{x}{\gamma_x}\right)\exp\left(-\frac{x}{\gamma_x}\right)$$

where r is the radius, Q is the amplitude, v_z is the initial pulse velocity, γ_z, γ_x determine the pulse width, $Ai(y)$ is the Airy function, κ is the parameter that determines the form of the Airy beam and is related to the dispersion coefficient in a linear medium [25]. This initial condition corresponds to the fact that an extremely short pulse has an Airy profile along the direction of propagation and a Gaussian profile in the transverse direction.

The evolution of the extremely short pulse is shown in Fig. 1 (for a Gaussian beam) and Fig. 2 (for an Airy beam). The sections are taken at the maximum pulse amplitude (in the longitudinal direction).

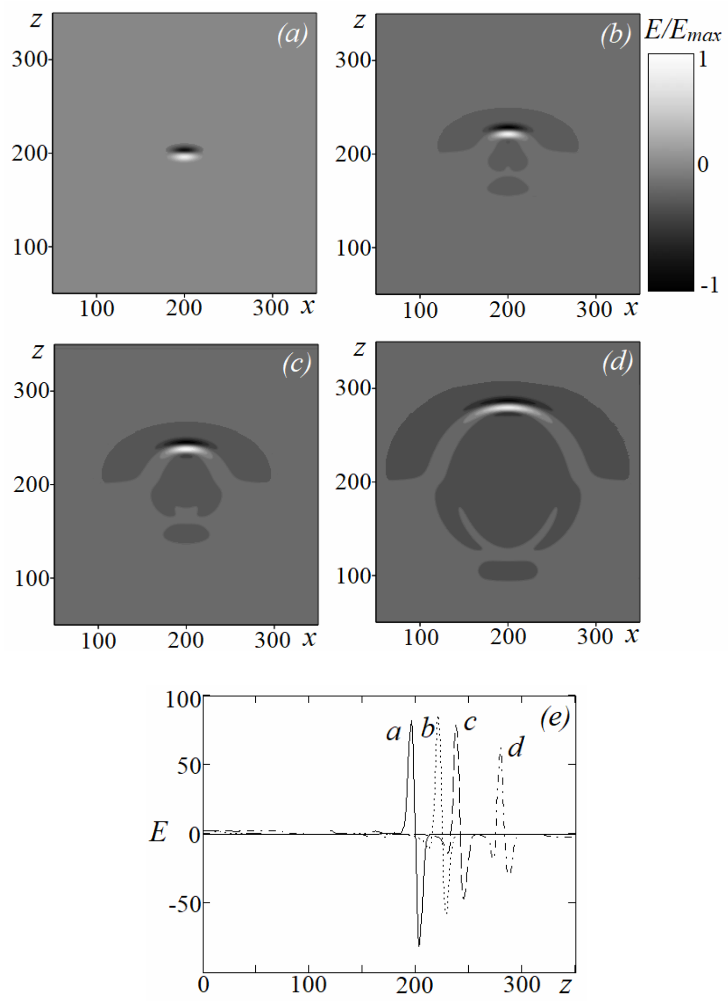

Fig. 1 – The dependence of the electric field on coordinates at different instances of time for initial condition (6a): a) initial pulse; b) $t = 3 \cdot 10^{-13}$ s; c) $t = 5 \cdot 10^{-13}$ s; d) $t = 10 \cdot 10^{-13}$ s; e) the cross-section along the z-axis.

It can be seen from Fig. 1 that in the course of time the pulse amplitude decreases but this decrease is insignificant, due to the balance between the dispersion and the nonlinearity of the medium, and to the establishment of a balance of current dissipation and electric field pumping.

It follows from Fig. 2 that the extremely short optical Airy pulse with a Gaussian cross section propagates in the medium of silicon nanotubes with a considerable spreading. In the course of time, we observe an insignificant decrease in its amplitude.

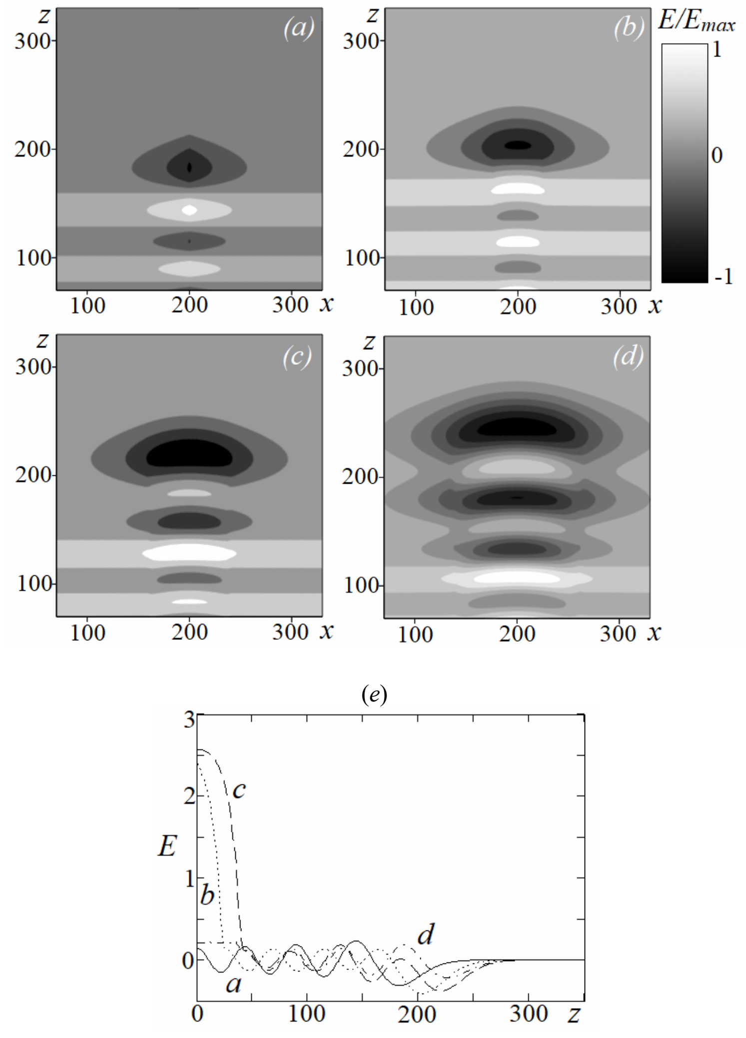

Fig. 2 – The dependence of the electric field on coordinates at different instances of time for initial condition (6b): a) initial pulse; b) $t = 1 \cdot 10^{-13}$ s; c) $t = 5 \cdot 10^{-13}$ s; d) $t = 8 \cdot 10^{-13}$ s; e) the cross-section along the z-axis.

We also carried out numerical experiments on the effect of the dissipation and pumping parameters on the shape of the electromagnetic pulse propagating along the sample. It was shown that in the case of a Gaussian pulse, these parameters do not have a significant effect on the evolution of the pulse, in contrast to the case with the Airy beam. The corresponding dependences for the initial conditions (6b) are shown in Fig. 3.

These dependences show that the dissipation parameter exerts a greater influence on the shape of the Airy pulse. Moreover, its increase causes not only a decrease in the amplitude at the very beginning of the pulse propagation, but also stabilizes it leading to complete attenuation. According to Fig. 3B, the change in the gain parameter does not have a significant effect on the Airy momentum.

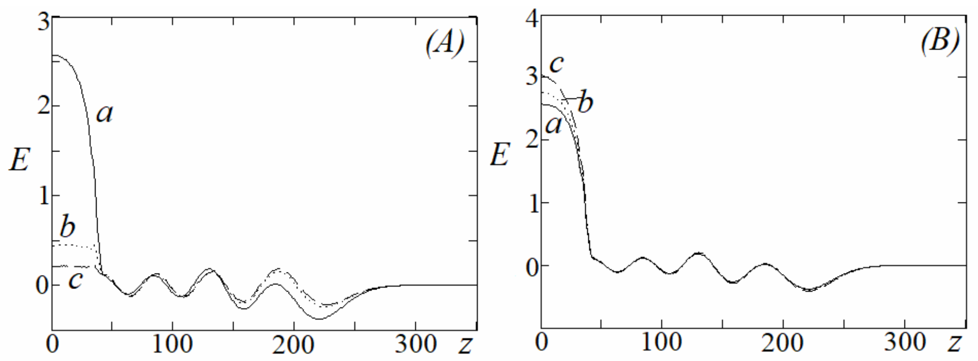

Fig. 3 – The dependence of the electric field of the Airy pulse (longitudinal section) on the coordinate: (A) for different values t_{rel}: a) $t_{rel}/t = 5$; b) $t_{rel}/t = 25$; c) $t_{rel}/t = 50$; (B) for different values of the gain parameter: a) $\Gamma = 0.01$ s^{-1}; b) $\Gamma = 0.05$ s^{-1}; c) $\Gamma = 0.1$ s^{-1}.

4. CONCLUSIONS

1. Two-dimensional extremely short optical pulse of the Gaussian shape propagates stably with a slight damping of the amplitude, which is a consequence of the balance of its energy losses and the energy scattered by dissipation.

2. There is a strong spreading of the two-dimensional Airy pulse as it propagates in the array of silicon nanotubes but with the conservation of the input pulse amplitude.

3. The dissipation parameters most strongly affect the shape of the Airy pulse and do not affect the pulse of a Gaussian form.

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