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DEPENDENCE OF UNIVERSE DECELERATION PARAMETER ON COSMOLOGICAL CONSTANT: MECHANISM OF VACUUM PRESSURE EXCITATION BY MATTER

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Abstract. Based on Sakharov's idea of a 'metrical elasticity' of space, *i.e.*, of the emergence of a generalized force, preventing distortion of space, we detect the pressure of the vacuum as per the geometry of the space around the local gravity system. The gravitational defect of mass is interpreted as the transfer of energy to the vacuum, which becomes apparent from its deformation. We determine the gravitational impact of matter on the vacuum and opposite in the sign pressure of it in case of weakly gravitating static centrally symmetric distribution of matter using appropriate solution of Einstein's equations. A possibility to extend the obtained results to arbitrary gravitational systems is evaluated. A non-conservation of energy in gravitational systems is interpreted by the Extended Space Model (ESM) as the rotation of the energy-momentum vector in 5-dimensional space. A proposed approach to determining pressure as a source of gravity leads to a revision of the dependence of the deceleration parameter of the Universe on the density parameters. Under this condition we examine the ratio between the density parameters of dark energy and cosmological constant depending on the deceleration parameter.

Key words: Vacuum pressure, gravitational defect, deceleration parameter.

1. INTRODUCTION

The non-zero vacuum pressure is an element of cosmological models [1–3], resulting from the solution of Einstein's equations. He postulated that curvature of space-time is responsible for gravity. Sakharov [4] has argued that gravity emerges from quantum field theory in roughly the same sense that hydrodynamics or continuum elasticity theory emerges from molecular physics. He proposed that curvature of space "leads to a 'metrical elasticity' of space, *i.e.*, to generalized forces which oppose the curving of space." The action term of Einstein's geometrodynamics is identified with the change in the action of quantum fluctuations of the vacuum

[5–7]. From general relativity follows that the gravitational mass of bodies placed in confined volume is less than the sum of the gravitational masses of these bodies, dispersed over infinite distance. The matter, located more compactly, distorts the space in the local domain in a greater degree, however, creating smaller gravitational mass in comparison with the same amount of matter, distributed over a greater volume [1, 2]. This phenomenon is explained by transfer of energy into the gravitational field, which results in the deformation of space. Accumulation of energy during deformation demonstrates its elasticity. We will take these properties of gravity into consideration, while determining the vacuum pressure.

The Extended Space Model (ESM) [8] is a generalization of special theory of relativity in a 5-dimensional space $G(1,4)$ having an additional coordinate s, which is a extension of the concept of action in an embedded 4D space [9, 10]. In ESM, in addition to the rotations in plane (TX) relating to the Lorentz transformations, the rotations in planes (TS) and (XS) are considered [11]. Rotations of the energy-momentum vector in extended space correspond to the motion of a particle in gravity field in the embedded four-dimensional space-time [12]. Movement along additional 5-th coordinate corresponds to the presence of particles rest-mass in (1+3)D. In this paper we describe the gravitational effect of static mass by means of (TS)-rotation of energy-momentum 5-vector of matter density.

The negative pressure of the dark energy determines the dynamics of Universe extension. In standard ΛCDM cosmology the pressure of non-relativistic matter, including baryons and dark matter, does not affect the accelerated expansion of the universe, characterized by the deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$, where a is a scale factor and the overdot signifies differentiation with respect to time t. We use the units of measurement, in which a light velocity constant c and a gravitational constant G are $c = 8\pi G = 1$. The Friedmann acceleration equation can be written as

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\sum_i \rho_i(1+w_i), \tag{1}$$

where the sum i extends over the different components, matter, radiation, and dark energy, ρ_i are their equivalent mass densities, p_i are pressures, and $w_i = p_i/\rho_i$ is the equation of state for each component. The value of w_i is 0 for non-relativistic matter, $1/3$ for radiation and -1 for a cosmological constant Λ, which we denote w'_Λ. Under these conditions for density parameters Ω_i the acceleration equation gives

$$q = \frac{1}{2}\sum \Omega_i(1+3w_i) = \Omega_r + \frac{1}{2}\Omega_m + \frac{1+3w_{DE}}{2}\Omega_{DE}, \tag{2}$$

where $\Omega_r, \Omega_m, \Omega_{DE}$ relate to radiation, matter, and dark energy. At present Ω_r is negligible, and if w_{DE} corresponds to the cosmological constant [13, 14] this equation simplifies to

$$q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda. \tag{3}$$

Alternatively, it was considered a model of the Universe with the equation of state $p_v = -(1/3)\rho$, in which the expansion rate is constant [15, 16].

The aim of this work is to study the contribution to accelerated expansion made by the vacuum pressure excited by the distributed components. We find appropriate deceleration parameter and discuss its compliance with the data of Type Ia supernovae (SNe Ia) and Plank 2018.

2. SPHERICAL SOURCE

2.1. SOLUTION OF EINSTEIN EQUATIONS

We analyze a centrally symmetrical static gravitational field. In spherical coordinates $x^i = (t, r, \theta, \varphi)$ it is described by the metric

$$ds^2 = \nu(r)\,dt^2 - \omega(r)dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\varphi^2), \tag{4}$$

where ν, ω are functions of radial coordinate. The centrally symmetric stress-energy tensor T_i^i corresponds to the source of gravitation, which creates this type of field. Solution of Einstein's equations for a spherical body with a radius of a yields [1] the required functions

$$\nu = \frac{1}{\omega} = 1 - \frac{1}{r}\int_0^a T_1^1 y^2 dy, r > a, \tag{5}$$

$$\omega = \left(1 - \frac{1}{r}\int_0^r T_1^1 y^2 dy\right)^{-1}, r \leq a, \tag{6}$$

where T_1^1 is the density of energy and variable y has the dimension of length. For the static source of gravitation it is equivalent to the density of matter: $T_1^1 = \rho$.

In the external area the obtained functions ν, w correspond to the Schwarzschild metric, and therefore the value

$$M = 4\pi \int_0^a \rho r^2 dr \tag{7}$$

is the gravitational mass of a spherical body of radius a. Integration is performed here in case of the element of volume $dV_c = 4\pi r^2 dr$, which corresponds to the coordinate frame, whereas in its proper frame the given element of space volume will be $dV_p = 4\pi r^2 \omega^{1/2} dr$. Inequality $\omega > 1$ means that the gravitational mass of body is less than the sum of individual gravitational masses of its constituent elements. This interprets as the transfer of energy, as a source of gravitational field, to the vacuum [17].

2.2. PRESSURE IN CASE OF LOW GRAVITATION

The volume of spherical body in proper frame is obtained by integration of elements dV_p with (6) and amounts to

$$V^p_{int}(a) = \int_0^a 4\pi r^2 \omega^{1/2} dr = \int_0^a 4\pi r^2 \left(1 - \frac{1}{3}\rho r^2\right)^{-1/2} dr. \quad (8)$$

For small space curvature inside the sphere, *i.e.* with $\rho a^2 << 1$, the representation of the expression under the integral into a formal power series turns out to be

$$V^p_{int}(a) = \frac{4\pi}{3} a^3 + \frac{2\pi}{15}\rho a^5. \quad (9)$$

Since the density of matter is constant, the mass of body in this frame or the proper mass will be $M^p = \rho V^p_{int}(a)$. A proper energy of static source of gravitation is defined as $E^p = M^p$.

The gravitational impact on the vacuum is determined as the relation of difference between proper energies of two spherical bodies with identical gravitational mass to the change of proper volume of space. With constant densities ρ_1,ρ_2 and radii a_1, a_2, ($a_1 < a_2$) this mass is

$$M = \frac{4}{3}\pi\rho_1 a_1^3 = \frac{4}{3}\pi\rho_2 a_2^3. \quad (10)$$

The difference of proper masses of two bodies is written as follows:

$$\Delta M^p = M^p_1 - M^p_2 = \frac{2\pi}{15} a_1^6 \rho_1^2 \left(\frac{1}{a_1} - \frac{1}{a_2}\right). \quad (11)$$

Due to equality of gravitational masses of both bodies, the space distortion in the area $r > a_2$, created by them, will be identical. Let's find the difference between the volumes in the proper frame, which are set in the coordinate frame by the condition $r \leq a_2$. This volume for the first body is the sum of this body's own volume and the peripheral area $a_1 < r \leq a_2$, namely,

$$V^p_1 = V^p_{int}(a_1) + V^p_{ext}(a_1, a_2), \quad (12)$$

where the second term is given by

$$V^p_{ext}(a_1, a_2) = \int_{a_1}^{a_2} 4\pi r^2 \omega^{1/2} dr. \quad (13)$$

Breaking the expression under integral into the formal power series, in case of $M/r << 1$ we obtain

$$V^p_{ext}(a_1, a_2) = \frac{4}{3}\pi(a_2^3 - a_1^3) + 2\pi M(a_2^2 - a_1^2). \quad (14)$$

As a result, the volume (12) will amount to

$$V_1^p = \frac{4}{3}\pi a_2^3 + \frac{1}{15}\pi\rho_1 a_1^3(5a_2^2 - 3a_1^2). \tag{15}$$

The area $r \le a_2$ restricts the second body, whose proper volume for the weak gravitational field according to (9) is

$$V_2^p = \frac{4}{3}\pi a_2^3 + \frac{2}{15}\pi\rho_2 a_2^5. \tag{16}$$

The difference between the proper volumes, confined within the radius a_2 in the coordinate frame, will be

$$\Delta V^p = V_1^p - V_2^p = \frac{1}{5}\pi\rho_1 a_1^3(a_2^2 - a_1^2). \tag{17}$$

The ratio of change in the energy of the spherical body $\Delta E^p = \Delta M^p$ to the change of its volume for small $\Delta a = a_2 - a_1$ retaining its gravitational mass taking (11) into consideration yields

$$\wp = \frac{\Delta E^p}{\Delta V^p} = \frac{1}{3}\rho. \tag{18}$$

With increasing masses defect, the difference between the proper volume of the spheres and their identical volumes in a remote frame

$$\Delta V = (V_1^p - V_c) - (V_2^p - V_c) = \Delta V^p \tag{19}$$

increases. In the theory of elasticity $\wp$ corresponds to the pressure of an perfect liquid. Positive pressure of gravity field characterizes the gravitational impact of matter on the vacuum, which lies in its constraint. The field pressure upon vacuum is compensated by pressure of the vacuum itself:

$$p_v = -\wp. \tag{20}$$

This is the mean vacuum pressure in case of weak gravitation inside the static sphere. The relationship between density and pressure in expression (18) coincides with the state equation of photon gas [18].

3. ENERGY TRANSFORMATION IN GSM

In the extended space $G(1,4)$ a 4-vector of energy and momentum is completed to a 5-vector

$$\overline{p} = \left(\frac{E}{c},\ p_x\ ,\ p_y\ ,\ p_z\ ,\ mc\right)\ , \tag{21}$$

where m is the rest mass of the particle. For simplicity we have recorded this vector in (1 + 2)-dimensional space:

$$\overline{p}_m = \left(\frac{E}{c}\ ,\ \ P\ ,\ \ p_s\right). \tag{22}$$

Its hyperbolic rotations on an angle ϕ_{TS} in the plane (TS) [8] yields

$$\frac{E'}{c} = \frac{E}{c}\cosh\phi_{TS} + p_s\sinh\phi_{TS}\ , \tag{23}$$

$$P' = P\ , \tag{24}$$

$$p'_s = p_s\cosh\phi_{TS} + \frac{E}{c}\sinh\phi_{TS}\ . \tag{25}$$

In terms of ESM in the static case the energy-momentum vector of the unit volume of matter with total density of matter $\varepsilon_p = \varepsilon\omega^{1/2}(r)$ can be represented as the 5-vector

$$\overline{p}_{mt} = \left(c\varepsilon\omega^{1/2}(r)\ ,\ \ 0\ ,\ \ c\varepsilon\omega^{1/2}(r)\right).$$

Its hyperbolic rotation in the plane (TS) (23)-(25) on an angle $\phi_{TS} = -\frac{1}{2}\ln\left(\omega\left(r\right)\right)$ yields

$$\overline{p}_{mg} = (c\varepsilon\ ,\ \ 0\ ,\ \ c\varepsilon)\,.$$

This rotation corresponds to transition from the total density of matter to the density as a source of gravity.

4. ISOTROPIC SOURCE

Let us examine an arbitrary space-time, containing a source of gravitation with density ρ, which is described by the metric $ds^2 = g_{ij}dx^i dx^j$. We allocate a small area, in which metrical coefficients and density can be considered as constant in the first approximation, pressure is isotropic, and whose boundary is a sphere in the proper frame. The gravity, created by this ball, is described by metric (4).

The metrical coefficients of the space-time without a source of gravitation in this sphere will be slightly different from g_{ij}. The transition to a locally inertial system [1] with the beginning in the point x_0^k is made for the changed metrics using the transformation

$$x'^k = x^k + \frac{1}{2}\left(\Gamma^{\mathrm{k}}_{\mathrm{ij}}\right)_{x^i = x_0^i} x^i x^j \tag{26}$$

with Christoffel's symbols Γ^k_{ij}. In this locally flat space we place the absent source of gravitation in the empty sphere. This one will comply to conditions under which pressure of gravitational field was obtained (18). In case of static space-time, the proper pressure of vacuum is determined according to (20) and will be

$$p_v = -\frac{1}{3}\rho. \tag{27}$$

5. COSMOLOGICAL SOLUTION

A model is considered in which the source of gravity is the vacuum pressure excited in it by the gravitational field of matter and radiation. In FLRW cosmological model the source of gravitation is static in comoving coordinates, which is a locally geodesic system. Equation of state (27) corresponds to static space-time but if expansion of the universe is accelerating, then the inequality $-p_v > \wp$ holds. However, the relative velocity of the universe expansion is equal to Hubble parameter H, which is small at present period, and this equation of state is suitable. The vacuum pressure induced by the energy density of the distributed components will be

$$p_e = \sum (w_i^e)\rho_i. \tag{28}$$

Tolman [18] has applied solutions of Einstein's equation for the electromagnetic field in the case of weak gravity to analyze the gravitational interaction of a light packet and beam with a material particle. This resulted in a double active gravitational mass of the directed electromagnetic radiation compared to a material particle, equivalent to its energy. This is also true for the photon gas and is consistent with the Lagrangian mechanics analysis for the passive gravitational mass of the photon [12, 19].

Matter and gas of relativity particles are considered to induce vacuum pressure (27), which corresponds to the equation of state parameters $w_m^e = w_r^e = -1/3$. If the vacuum energy associated with the cosmological constant creates additional vacuum pressure, it is reflected in the additional term of the equation of state: $w_\Lambda^e = -1/3$. Its summation with w'_Λ yields $w_\Lambda = -4/3$ and the deceleration parameter (3) provided $w_m = w_m^e$ at the present becomes $q_0 = -1.5\Omega_\Lambda$ (i). Otherwise, an additional member of the equation of state for the cosmological constant is $w_\Lambda^e = 0$ and the deceleration parameter will be $q_0 = -\Omega_\Lambda$ (ii).

Initial estimate of the accelerating expansion of the universe from supernovae observation [13] resulted in a current deceleration parameter $q_0 = -1 \pm 0.4$. The Planck measurements of the CMB anisotropies, combining information from the temperature and polarization maps and the lensing reconstruction [20] gives matter density parameter $\Omega_m = 0.315 \pm 0.007$. This result is consistent with analysis of the the anisotropic galaxy clustering measurement from the Baryon Oscillation Spectroscopic Survey [21]. Under the assumption $\Omega_m + \Omega_{DE} = 1$ the dark energy density is $\Omega_{DE} = 0.685 \pm 0.007$ and in the case $\Omega_{DE} = \Omega_\Lambda$ this yields the deceleration parameter $q_0 = -1.0275 \pm 0.011$ (i) and $q_0 = -0.685 \pm 0.007$ (ii) depending on whether the cosmological constant creates additional vacuum pressure or not. Wider SNe Ia data sampling and advanced statistical analysis, taking into account series expansion of luminosity distance as a function of redshift [14] yields for ΛCDM cosmologies $q_0 = -0.474^{+0.112}_{-0.109}$ with curvature and $q_0 = -0.552^{+0.049}_{-0.047}$ with a flat universe. Estimates of deceleration parameter are highly dependent on the data sample, the cosmological model and the analysis method [14, 22–24]. Dependence of q_0 on the parameter $\Omega_\Delta = \Omega_{DE} - \Omega_\Lambda$ for models (i) and (ii) is shown in Fig. 1.

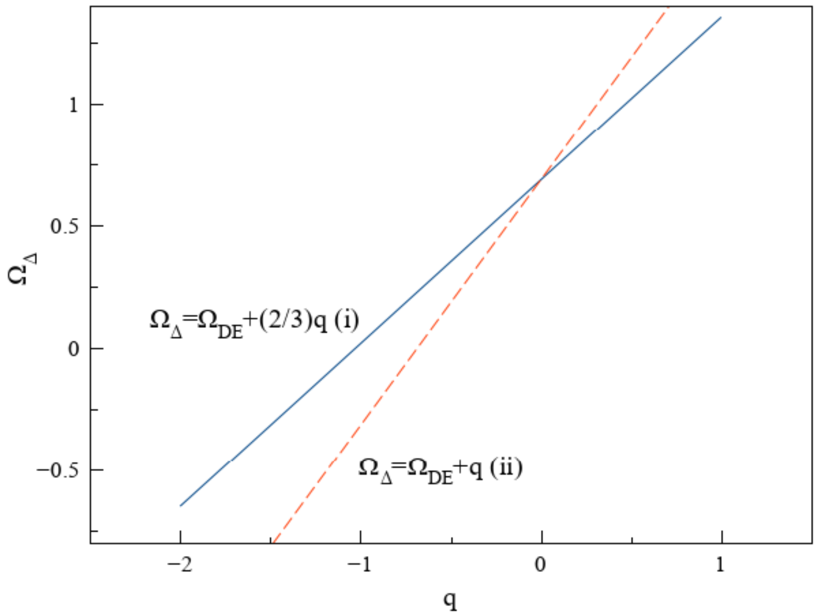

Fig. 1 – The deceleration parameter dependence of $\Omega_\Delta = \Omega_{DE} - \Omega_\Lambda$ for models, in which the cosmological constant creates additional vacuum pressure (i) and don't induce it (ii). The parameter Ω_{DE} corresponds to the Planck data under condition $\Omega_m + \Omega_{DE} = 1$.

6. CONCLUSIONS

Matter, even static, warps space, and it is natural to assume that the result is a vacuum pressure, which does not take into account the standard ΛCDM model. We have examined a possible mechanism for the occurrence of this pressure, based on the gravitational effect of the masses and the assumption of elasticity of space in compliance with the law of energy conservation. The component of the equation of state for a vacuum containing a distributed locally isotropic static gravity source is $w = -(1/3)$. Originally the spherical sources of gravity with constant densities and identical gravitational masses are considered in the spheres with the same volume in the remote frame. With the increase in mass defect, the difference between the proper volume of the spheres and their volume in the remote frame increases, which gives a positive $\wp$ and a negative vacuum pressure. We have shown that the equation of state, obtained for a weakly gravitating sphere, can be extended to a more general case. In statics, according to the theory of elasticity, determined on the basis of the vacuum model as a perfect liquid, the vacuum pressure balances the impact of gravity on vacuum. In ESM the gravitational defect of static mass is described by hyperbolic rotation of energy-momentum 5-vector of matter density in plane (TS).

Application of the considered mechanism of excitation of vacuum pressure by matter to the FLRW cosmological model yields a deceleration parameter that depends only on cosmological constant. If the vacuum energy associated with Λ creates additional vacuum pressure, the deceleration parameter at the present time is $q_0 = -1.5\Omega_\Lambda$, otherwise it is reduced to $q_0 = -\Omega_\Lambda$. The second result is closer to the present estimates under condition $\Omega_{DE} = \Omega_\Lambda$, but the difference is still considerable. Without this assumption, both options remain relevant.

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