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NONLINEAR WAVES IN A CRYSTAL CHARACTERIZED BY THE JUMP SWITCHING BETWEEN SELF-FOCUSING AND DEFOCUSING KERR NONLINEARITIES

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Abstract. In this paper, we propose a new optical nonlinearity model describing a sharp change from a self-focusing Kerr-type nonlinearity to a defocusing one and *vice versa*, depending on the electric field amplitude. Three new types of transverse electric nonlinear surface waves can propagate along the interfaces between media characterized by nonlinearities of the proposed type. The light beam propagating along the interface significantly changes the optical properties of regions near that interface. The structure of nonlinear surface waves and peculiarities of the optical domain formation near the interface are described analytically.

Key words: nonlinear surface wave; nonlinear optics; stepwise nonlinearity; Kerr-type optical medium.

1. INTRODUCTION

The numerous studies of nonlinear wave propagation [1–30] are important to design and implement various photonic devices ranging from passive splitters to active switches and light amplifiers [31–37]. The Kerr-type nonlinearity is usually weak in the most media, although some new optical materials that are developed with nanotechnologies offer promise of significantly enhanced third-order nonlinear coefficients [38]. Silicon nanocrystals can serve as a good example of such a promising material with a stronger nonlinearity.

In order to describe theoretically the optical nonlinearities of real physical systems, many authors used different models, such as the photorefractive nonlinearity [39–48], the stepwise nonlinearity [49–55], and the saturable nonlinearity [56–61]. The saturable nonlinearity coincides with the Kerr-type one at low light intensity. Such type of nonlinearity describes the smooth change of refractive index between two values in dependence on the electric field intensity.

In this paper, we want to propose a new optical nonlinearity model, which describes a sharp change in the sign of the Kerr nonlinearity, depending on the electric field. Such a model of nonlinearity may be useful in terms of potential future photonic devices. We propose a new modification of stepwise nonlinearity model in order to demonstrate the existence of exact solutions corresponding to the surface waves with specific form due to a sharp changing of dielectric constant from a self-focusing (defocusing) Kerr nonlinearity to a defocusing (self-focusing) one, depending on the electric field amplitude. We find new types of nonlinear surface waves propagating along the interface between such optical media characterized by an abrupt change of the dielectric permittivity.

This paper is organized as follows: the main equations and the modified stepwise nonlinearity model are presented in Section 2. In Section 3, we derive the solutions of the formulated nonlinear equations describing the nonlinear surface wave of the first type in a self-focusing Kerr-type medium. In Section 4, we derive the solutions describing the nonlinear surface wave of the second type in a self-focusing Kerr-type medium. In Section 5, we derive the solutions describing the nonlinear surface wave of the third type in a defocusing Kerr-type medium. Finally, in Section 6 we present our conclusions.

2. MODEL FORMULATION

We consider that the transverse electric (*TE*)-polarized (*S*-polarized) nonlinear wave propagates along the interface between Kerr-type nonlinear media, where the change between self-focusing and defocusing types of nonlinearities is possible in dependence on the electric field amplitude. The planar interface is placed in the *xy* plane, and the *z* axis is perpendicular to the interface. We suppose that all medium components are nonmagnetic.

The components of electromagnetic field of *TE*-polarized wave propagating along the *z* axis are $E_x = E_z = 0$, $H_y = 0$, and

$$E_y(x,z) = E(z)\exp\{\mathrm{i}(kx - \omega t)\}\,, \tag{1}$$

where k is the wave number, ω is the wave frequency, and c is the speed of light.

The field distribution across the interface $E(z)$ can be found from the following nonlinear equation derived from the Maxwell equations

$$\frac{\mathrm{d}^2E(z)}{\mathrm{d}z^2} + \left(\varepsilon(|E|) - n^2\right)\frac{\omega^2}{c^2}E(z) = 0\,, \tag{2}$$

where $n = ck/\omega$ is the effective refractive index, and ε is the dielectric permittivity that depends on the modulus of the electric field amplitude.

First, we consider that the crystal is characterized by a self-focusing Kerr-type nonlinearity, when the electric field is less than the threshold switching field E_s, and the nonlinearity in the crystal becomes a defocusing one, when the electric field is greater than E_s. Therefore, the dielectric permittivity ε can be written as a stepwise function

$$\varepsilon(|E|) = \begin{cases} \varepsilon_1 - \alpha_1|E|^2, |E| > E_s \\ \varepsilon_2 + \alpha_2|E|^2, |E| < E_s \end{cases} \tag{3}$$

where $\varepsilon_{1,2}$ are the unperturbed dielectric constant, and $\alpha_{1,2}$ are the positive parameters of the corresponding Kerr-type nonlinearities.

Next, we consider the opposite case of a crystal characterized initially by a defocusing Kerr-type nonlinearity that becomes a self-focusing one, when the electric field amplitude is greater than E_s.

We consider the solution of Eq. (2) satisfying the condition of extinction at infinity: $|E(z)| \to 0$ at $|z| \to \infty$. The solutions of Eq. (2) satisfying the continuity conditions describe the nonlinear surface waves of *TE* polarization propagating along the interface.

In this paper, due to the symmetry of the system, we study the field distributions of even type, for which $E(-z) = E(z)$.

We determine the total power flux as follows:

$$P = \int_{-\infty}^{+\infty} |E(z)|^2 \, \mathrm{d}z \,. \tag{4}$$

The symmetry of the system allows us to write the total power (4)

$$P = 2(P_1 + P_2), \tag{5}$$

where P_1 is the power component in the domain $0 < z < z_s$, and P_2 is the power component in the region $z > z_s$.

3. THE NONLINEAR SURFACE WAVE OF THE FIRST TYPE IN A SELF-FOCUSING KERR-TYPE MEDIUM

First, we consider the change of the self-focusing nonlinearity to a defocusing one. Let the crystal is initially characterized by a self-focusing nonlinearity with the dielectric permittivity $\varepsilon(|E|) = \varepsilon_2 + \alpha_2|E|^2$, where the nonlinearity coefficient is $\alpha_2 > 0$. The electric field strength may increase after the surface wave is excited. A symmetrically localized zone (an optical domain) of $2z_s$ width is formed after the electric field strength reaches the threshold value of the switching field. Here z_s is a point where the electric field is equal to the threshold field of switching: $|E(z_s)| = E_s$. The medium inside that optical domain is characterized by a defocusing nonlinearity with the dielectric permittivity $\varepsilon(|E|) = \varepsilon_1 - \alpha_1 |E|^2$, where the nonlinearity coefficient is $\alpha_1 > 0$, according with Eq. (3).

Two types of surface waves can occur depending on the relationship between the effective refractive index and the unperturbed dielectric constants.

In the range $\varepsilon_2 < n^2 < \varepsilon_1$ the solutions of Eq. (2) are given by

$$E(z) = \begin{cases} \mp\sqrt{\dfrac{\varepsilon_1 - n^2}{\alpha_1}} \tanh(q_t(z \mp z_1)), & |z| < z_s \\ \sqrt{\dfrac{2(n^2 - \varepsilon_2)}{\alpha_2}} \dfrac{1}{\cosh(q_2(z \mp z_2))}, & |z| > z_s \end{cases}, \quad (6)$$

where $q_t = (\varepsilon_1 - n^2)^{1/2} \omega / \sqrt{2} c$ and $q_2 = (n^2 - \varepsilon_2)^{1/2} \omega / c$.

In Eq. (6) and below, the upper sign corresponds to the region $z > 0$, and the lower one corresponds to the region $z < 0$. In addition, the value of the index $j = 1$ corresponds to the field characteristics inside the domain at $|z| < z_s$, when $|E| > E_s$, and the value of the index $j = 2$ corresponds to the field characteristics in the rest of the crystal at $|z| > z_s$, when $|E| < E_s$.

At the symmetry center point $z = 0$, from Eq. (6), we obtain the amplitude maximum

$$E_0 = \sqrt{\frac{\varepsilon_1 - n^2}{\alpha_1}} \tanh(q_t z_1). \quad (7)$$

In order to satisfy the condition of field continuity at domain boundary $z = z_s$, from Eq. (6) it follows

$$-\sqrt{\frac{\varepsilon_1 - n^2}{\alpha_1}} \tanh(q_t(z_s - z_1)) = \sqrt{\frac{2(n^2 - \varepsilon_2)}{\alpha_2}} \frac{1}{\cosh(q_2(z_s - z_2))} = E_s. \quad (8)$$

In addition, in order to satisfy the condition of continuity of the tangential component of the field (6) at domain boundary $z = z_s$, we obtain

$$\frac{q_t^2}{\sqrt{\alpha_1} \cosh^2(q_t(z_s - z_1))} = E_s q_2 \tanh(q_2(z_s - z_2)). \quad (9)$$

Combining Eqs. (8) and (9), we can write the dispersion equation as follows

$$(\varepsilon_1 - n^2 - \alpha_1 E_s^2)^2 = \alpha_2 E_s^2 \{2(n^2 - \varepsilon_2) - \alpha_2 E_s^2\}. \quad (10)$$

The solution of the dispersion equation (10) defines the dependence of the effective refractive index on the optical characteristics of the crystal and the switching field. It can be written as

$$n^2 = \varepsilon_1 - E_s / E_n \ , \tag{11}$$

where

$$E_n = 1/\sqrt{\alpha_1(\alpha_1+\alpha_2)(E_c^2 - E_s^2)}$$

and

$$E_c^2 = 2(\varepsilon_1 - \varepsilon_2)/(\alpha_1+\alpha_2)\,.$$

From Eq. (11) it follows that the wave of the first type exists under the condition $|E_s| < E_c$. In addition, we note that the wave of the first type can propagate in the case when the dielectric constant inside the domain is greater than the dielectric constant outside it (that is, when $\varepsilon_1 > \varepsilon_2$).

From Eqs. (7)–(9) we obtain the wave parameters entering Eq. (6):

$$z_1 = z_{01}\operatorname{arctanh}\left(\frac{E_0}{E_s}K\right), \tag{12}$$

where

$$z_{01} = \frac{c}{\omega}\left(\frac{2E_n}{E_s}\right)^{1/2},$$

$$K = \left(\frac{\alpha_1}{(\alpha_1+\alpha_2)(E_c^2/E_s^2 - 1)}\right)^{1/4}$$

and

$$z_2 = z_s - z_{02}\operatorname{arccosh}\left(\frac{2\{\varepsilon_1 - \varepsilon_2 - E_s/E_n\}}{\alpha_2 E_s^2}\right)^{1/2}, \tag{13}$$

where

$$z_{02} = \frac{c}{\omega}\left(\frac{1}{\varepsilon_1 - \varepsilon_2 - E_s/E_n}\right)^{1/2},$$

and the domain width is given by

$$z_s = z_{01}\left\{\operatorname{arctanh}\left(\frac{E_0}{E_s}K\right) - \operatorname{arctanh}(K)\right\}. \tag{14}$$

This domain can exist only if $E_s < \min\{E_c, E_0\}$.

After substituting Eq. (6) into Eq. (4) we calculate the total power. The power component in the domain $0 < z < z_s$ can be written as

$$P_1 = \frac{1}{\sqrt{\alpha_1}}\left\{\frac{E_s z_s}{E_n\sqrt{\alpha_1}} - \frac{c}{\omega}\sqrt{2}(E_0 - E_s)\right\}, \tag{15}$$

where Eq. (14) must be substituted.

The power component in the region $z > z_s$ is given by

$$P_2 = \frac{2c}{\alpha_2\omega}\left\{\sqrt{\varepsilon_1 - \varepsilon_2 - E_s / E_n} + \sqrt{\varepsilon_1 - \varepsilon_2 - E_s / E_n - E_s^2\alpha_2 / 2}\right\}. \tag{16}$$

The total power (4) of the wave of the first type depends on the threshold value of the switching field and the amplitude (7) at the interface: $P = P(E_s, E_0)$.

4. THE NONLINEAR SURFACE WAVE OF THE SECOND TYPE IN A SELF-FOCUSING KERR-TYPE MEDIUM

In the range $n^2 > \max\{\varepsilon_{1,2}\}$ the solutions of Eq. (2) are given by

$$E(z) = \begin{cases} \pm\sqrt{\dfrac{2(n^2-\varepsilon_1)}{\alpha_1}}\dfrac{1}{\sinh(q_1(z \mp z_1))}, & |z| < z_s, \\ \sqrt{\dfrac{2(n^2-\varepsilon_2)}{\alpha_2}}\dfrac{1}{\cosh(q_2(z \mp z_2))}, & |z| > z_s, \end{cases}, \tag{17}$$

where $q_1 = (n^2 - \varepsilon_1)^{1/2}\omega / c$.

Equation (17) corresponds to a nonlinear surface wave of the second type existing for values of the effective refractive index $n^2 > \max\{\varepsilon_{1,2}\}$.

At the symmetry center point $z = 0$, from Eq. (17), we obtain the amplitude maximum

$$E_0 = -\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\frac{1}{\sinh(q_1 z_1)}. \tag{18}$$

Note that the value of z_1 must be negative.

In order to satisfy the condition of field continuity at the domain boundary $z = z_s$, from Eq. (17) it follows that

$$\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\frac{1}{\sinh(q_1(z_s-z_1))} = \sqrt{\frac{2(n^2-\varepsilon_2)}{\alpha_2}}\frac{1}{\cosh(q_2(z_s-z_2))} = E_s\,. \tag{19}$$

In addition, in order to satisfy the condition of continuity of the tangential component of the field (17) at the domain boundary $z = z_s$, we obtain

$$q_1\coth(q_1(z_s-z_1)) = q_2\tanh(q_2(z_s-z_2))\,. \tag{20}$$

Combining Eqs. (19) and (20), we obtain that the threshold value of the switching field is not an arbitrary parameter, but it is completely determined by the properties of the medium, as follows:

$$E_s^2 = E_c^2 = 2\frac{\varepsilon_1-\varepsilon_2}{\alpha_1+\alpha_2}\,. \tag{21}$$

From Eqs. (18)–(21) we obtain the wave parameters entering Eq. (17):

$$z_1 = -\frac{c}{\omega\sqrt{(n^2-\varepsilon_1)}}\operatorname{arcsinh}\left(\frac{1}{E_0}\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\right), \tag{22}$$

and

$$z_2 = z_s - \frac{c}{\omega\sqrt{(n^2-\varepsilon_2)}}\operatorname{arcsinh}\left(\sqrt{\frac{\alpha_1+\alpha_2}{\alpha_2}\frac{n^2-\varepsilon_2}{\varepsilon_1-\varepsilon_2}}\right), \tag{23}$$

where the domain width is given by

$$z_s = \frac{c}{\omega\sqrt{(n^2-\varepsilon_1)}}\left\{\operatorname{arcsinh}\left(\sqrt{\frac{\alpha_1+\alpha_2}{\alpha_1}\frac{n^2-\varepsilon_2}{\varepsilon_1-\varepsilon_2}}\right) - \operatorname{arcsinh}\left(\frac{1}{E_0}\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\right)\right\}. \tag{24}$$

The wave (17) of the second type exists under conditions $E_s < E_0$ and $\varepsilon_1 > \varepsilon_2$. Note that the threshold value of the switching field is determined by Eq. (21) in order to excite a wave of the second type. Therefore, the condition of existence of wave of the second type can be written as $E_0 > E_c$.

The wave of the first type (6) differs from the wave of the second type (17) not only in the range of existence, but also in that, the effective refractive index of wave (17) is a free parameter. It can be chosen as a control parameter for the wave of the second type, as well as the amplitude at the interface; see Eq. (18).

After substituting Eq. (17) into Eq. (4) we calculate the total power of the second type wave. The power component in the domain $0 < z < z_s$ can be written now as

$$P_1 = -\frac{2c}{\alpha_1\omega}\left\{\sqrt{n^2 - \varepsilon_1 + \frac{\alpha_1 E_0^2}{2}} + \sqrt{n^2 - \varepsilon_1 + (\varepsilon_1 - \varepsilon_2)\frac{\alpha_1}{\alpha_1 + \alpha_2}}\right\}. \tag{25}$$

The power component in the region $z > z_s$ is given by

$$P_2 = \frac{2c}{\alpha_2\omega}\left\{\sqrt{n^2 - \varepsilon_2} + \sqrt{n^2 - \varepsilon_2 - (\varepsilon_1 - \varepsilon_2)\frac{\alpha_2}{\alpha_1 + \alpha_2}}\right\}. \tag{26}$$

It follows from Eqs. (25) and (26) that the range of admissible values of the effective refractive index changes, as follows: $n > n_{\min}$, where $n^2_{\min} = \max\{\varepsilon_1, \varepsilon_{02}\}$, $\varepsilon_{02} = \varepsilon_2 + \alpha_2(\varepsilon_1 - \varepsilon_2)/(\alpha_1 + \alpha_2)$.

The power (4) of the wave of the second type depends now on the effective refractive index and the amplitude at the interface $P = P(n, E_0)$. We can consider such dependence as the inverse function $n = n(P, E_0)$ to choose the total power as a control parameter.

5. THE NONLINEAR SURFACE WAVE IN A DEFOCUSING KERR-TYPE MEDIUM

Now we consider the crystal characterized by a defocusing Kerr-type nonlinearity, when the electric field is less than the threshold field of switching E_s, and the nonlinearity in the crystal becomes a self-focusing one, when the electric field is greater than E_s. Therefore, we use instead Eq. (3) the following expression of the dielectric permittivity

$$\varepsilon(|E|) = \begin{cases} \varepsilon_1 + \alpha_1|E|^2, |E| > E_s \\ \varepsilon_2 - \alpha_2|E|^2, |E| < E_s \end{cases} \tag{27}$$

where $\alpha_{1,2}$ are positive parameters of the nonlinearity.

In the range $n^2 > \max\{\varepsilon_{1,2}\}$ the solutions of Eq. (2) with dielectric permittivity (27) are given by

$$E(z)=\begin{cases}\sqrt{\dfrac{2(n^2-\varepsilon_1)}{\alpha_1}}\dfrac{1}{\cosh(q_1(z\mp z_1))}, & |z|<z_s\\ \pm\sqrt{\dfrac{2(n^2-\varepsilon_2)}{\alpha_2}}\dfrac{1}{\sinh(q_2(z\mp z_2))}, & |z|>z_s\end{cases}, \tag{28}$$

where $q_{1,2}$ are determined by the same equations.

At the symmetry center point $z = 0$, from Eq. (17), we obtain the amplitude maximum

$$E_0=\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\frac{1}{\cosh(q_1 z_1)}. \tag{29}$$

In order to satisfy the condition of the field continuity at the domain boundary $z = z_s$, from Eq. (28) it follows that

$$\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\frac{1}{\cosh(q_1(z_s-z_1))}=\sqrt{\frac{2(n^2-\varepsilon_2)}{\alpha_2}}\frac{1}{\sinh(q_2(z_s-z_2))}=E_s. \tag{30}$$

In addition, in order to satisfy the condition of continuity of the tangential component of the field (28) at the domain boundary $z = z_s$, we obtain

$$q_1\tanh(q_1(z_s-z_1))=q_2\coth(q_2(z_s-z_2)). \tag{31}$$

Combining Eqs. (30) and (31), we obtain that the threshold value of the switching field is not an arbitrary parameter, but it is completely determined by the properties of the medium, as follows:

$$E_s^2=E_{sc}^2=2\frac{\varepsilon_2-\varepsilon_1}{\alpha_1+\alpha_2}. \tag{32}$$

From Eqs. (29)–(32) we obtain the wave parameters entering Eq. (28):

$$z_1=\frac{c}{\omega\sqrt{(n^2-\varepsilon_1)}}\operatorname{arccosh}\left(\frac{1}{E_0}\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\right), \tag{33}$$

and

$$z_2=z_s-\frac{c}{\omega\sqrt{(n^2-\varepsilon_2)}}\operatorname{arcsinh}\left(\sqrt{\frac{\alpha_1+\alpha_2}{\alpha_2}\frac{n^2-\varepsilon_2}{\varepsilon_2-\varepsilon_1}}\right), \tag{34}$$

where the domain width is given by

$$z_s = \frac{c}{\omega\sqrt{(n^2-\varepsilon_1)}}\left\{\operatorname{arccosh}\left(\sqrt{\frac{\alpha_1+\alpha_2}{\alpha_1}\frac{n^2-\varepsilon_2}{\varepsilon_2-\varepsilon_1}}\right)+\arccos\left(\frac{1}{E_0}\sqrt{\frac{2(n^2-\varepsilon_1)}{\alpha_1}}\right)\right\}. \quad (35)$$

The wave (28) in a defocusing medium exists under conditions $E_0 > E_{sc}$ and $\varepsilon_2 > \varepsilon_1$.

After substituting Eq. (28) into Eq. (4) we calculate the total power. The power component in the domain $0 < z < z_s$ can be written now as

$$P_1 = \frac{2c}{\alpha_1\omega}\left\{\sqrt{n^2-\varepsilon_1-\frac{\alpha_1 E_0^2}{2}}+\sqrt{n^2-\varepsilon_1-(\varepsilon_2-\varepsilon_1)\frac{\alpha_1}{\alpha_1+\alpha_2}}\right\}. \quad (36)$$

The power component in the region $z > z_s$ is given by

$$P_2 = \frac{2c}{\alpha_2\omega}\left\{\sqrt{n^2-\varepsilon_2+(\varepsilon_2-\varepsilon_1)\frac{\alpha_2}{\alpha_1+\alpha_2}}-\sqrt{n^2-\varepsilon_2}\right\}. \quad (37)$$

It follows from Eqs. (36) and (37) that the range of admissible values of the effective refractive index changes, as follows $n > n_{\min}$, where $n^2_{\min} = \max\{\varepsilon_{01}, \varepsilon_2\}$, $\varepsilon_{01} = \varepsilon_1 + \alpha_1 E_0^2/2$.

6. CONCLUSION

In order to investigate theoretically the effects of controlling surface wave propagation by input electric field amplitude, we proposed a new model of optical nonlinearity. Such a model describes a sharp change of Kerr-type nonlinearity from a self-focusing one to a defocusing one and *vice versa* in dependence on electric field amplitude.

We found three new types of nonlinear surface waves propagating along the interface between Kerr-type media with dielectric permittivity described by the proposed model of nonlinearity. We calculated all the wave parameters and the power flux of the found three types of surface waves. The conditions of the wave existence were analyzed.

We hope that results of this paper may be useful in designing of optical switching systems of control and semiconductor optical amplifiers based on crystalline structures with changing types of optical nonlinearity, in which the electric field controls the optical input for switching the corresponding output values [62–66].

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