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CORRECTIONS TO STANDARD RADIATION DOMINATED UNIVERSE IN JORDAN-BRANS-DICKE COSMOLOGY

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Abstract. Brans-Dicke theory in its original form can explain the radiation and matter dominated eras with a constant Brans-Dicke scalar field and the explanation of dark energy requires a nonvanishing potential term for the scalar field in the action. Within the framework of Jordan-Brans-Dicke theory where we take the scalar field as the Jordan field with a standard kinetic term and no mass term, we find the same radiation dominated solution as given by Einstein cosmology. On the other hand, by adding a mass term for the scalar field to the action, the radiation dominated solution is modified. We make an expansion in increasing powers of the scale size. This is valid for small scale sizes. We show that this expansion describing the early radiation dominated era also gives the late time dark energy dominated era in the Friedmann equation.

Key words: Jordan-Brans-Dicke, scalar-tensor theory of gravity.

1. INTRODUCTION

Big Bang Nucleosynthesis (BBN) is a theory based on the Standard Cosmological Model and has some observational proof that a few minutes after the Big-Bang the stable low mass nuclei such as 2H, 3He, 4He, 6Li, 7Li started to form [1–6]. This BBN era is known as the radiation dominated nucleosynthesis period of the Universe. The cosmological observational data from type Ia supernovae (SNIa), the Large Scale Structure (LSS), the Cosmic Microwave Background (CMB) anisotropies, Baryon Acoustic Oscillations (BAO) in the Sloan Sky Digital Survey (SSDS) luminous galaxy sample, and Planck data show that our observable Universe is nearly spatially flat, homogeneous, and isotropic at large scale and also had a phase transition from the deceleration to the acceleration in the redshift around $0.45 \leq z \leq 0.9$. Dark energy (DE) causes the acceleration of the cosmic expansion by the negative pressure value which can cause galaxies to accelerate. Today it is accepted that DE occupies about 73% of the total energy content of the Universe and the rest has been occupied by dark matter (DM) and baryonic matter [7, 8].

General Relativity (GR) is successful in easily explaining the accelerated expansion by introducing a cosmological constant (Λ). This is achieved by using the Friedmann equations which Friedmann derived from Einstein's GR by considering the possibility of an expanding universe [9, 10]. That is the reason why in standard cosmology Friedmann-Lemaitre spacetime is used for the metric of the universe [11].

As we know, Einstein's General Relativity (GR) passes many of the observational tests but not the all of them [7]. Therefore, besides Einstein's general theory of gravity the alternative theory called the scalar-tensor theory of gravity [12, 13] which successfully explains the evolution of universe is often utilized. The important reason for this alternative theory is that it does not contradict GR but suggests that at large distances gravity does not behave as Newton and Einstein predicted [7]. As well as GR by using Mach's principle, a new approach first considered by Jordan, Brans and Dicke (JBD) [13–16] has been developed for the acceleration of the universe [17–20].

By using the property of inertia of material bodies arises because of their interaction with the matter distributed in the universe this new theory using Mach's principle, evolved. It was shown that within the framework of Brans-Dicke gravity, a constant energy density leads to a rapid power-law expansion instead of exponential. This is sufficiently rapid enough to solve the problems in standard cosmology and slow enough to make the transition from the inflationary phase to the normal state. This is known as extended inflation [21, 22]. Extended inflation limits the dimensionless Brans-Dicke parameter ω to be less than 25. If it is more than 25, there will be much more anisotropy in the Cosmic Microwave Background Radiation than what is observed today [23].

In JBD theory, the effective gravitational coupling evolves with time and asymptotically reaches the present value of the gravitational constant (G). The main idea is to use the Brans-Dicke scalar field instead of the inverse gravitational constant. Since the inverse gravitational constant is proportional to the Planck mass squared the BD scalar field is also proportional to this quantity whereas the Jordan scalar field [12] is proportional to Planck mass. These theories are described by the Jordan-Brans-Dicke (JBD) Lagrangian. There is a nonminimal coupling term used in scalar fields and this coupling term originates from a space-time dependent gravitational constant. Jordan conceived the scalar-tensor theory by embedding a four-dimensional curved manifold in five-dimensional flat spacetime. This scalar-tensor theory provides a simple generalization of GR theory and is one of the simplest alternatives to gravitational theories. By introducing a scalar field which is nonminimally coupled to the curvature tensor this theory plays an important role in explaining the accelerated expansion of the universe.

JBD theory has received much attention and has been used to generate various cosmological phenomena such as the early inflation, Higgs inflation and dark energy [24]. There are other works done [25, 26] where the same Lagrangian was used with a spontaneous symmetry breaking potential. By using this Lagrangian some induced-gravity inflation models have been developed [27–32] and for causing accelerated expansion using only a simple potential has been shown to be sufficient [22, 33, 34]. It has also been shown that [35] the radiation dominated era starts with a closed universe expanding exponentially, and the late radiation dominated era expands linearly and the introduction of matter in the linearly expanding universe causes deceleration or acceleration. The case where inflation is driven by the quartic potential $U(\Phi)=\frac{\lambda\Phi^4}{4}$ in the framework of generalized BD theory with a scalar field dependent BD parameter $\omega_{GBD}(\Phi)$ [36] has also been investigated. The homogeneous and isotropic cosmological field equations obtained from this Lagrangian density have already been calculated for a potential $V(\phi)=\frac{1}{2}m^2\phi^2$ [37, 38] where ϕ is the Jordan field. Cosmological observations which show dark energy are often explained by a scalar field not necessarily related to JBD theory [39–46].

In [47] Friedmann equation for JBD scalar tensor theory of gravitation has been derived and it has been shown that JBD with a standard massive scalar field in Jordan canonical form can explain dark energy and make a correction to the matter density component of the Friedmann equation. The main parameter of JBD is the Brans-Dicke parameter ω which is a dimensionless constant bigger than 10^4 [48, 49].

In this paper we investigate the effect of dark energy during and before the radiation dominated era by expanding the Hubble parameter for small values of the cosmological scale size α. We show that this effect exists. It is interesting to note that although its contribution is small, this term is theoretically obtained in the Friedmann equation expansion for the square of the Hubble parameter.

2. JORDAN-BRANS-DICKE THEORY WITH A CANONICAL JORDAN KINETIC TERM AND NO MASS TERM

The Friedmann equation gives a relation between the Hubble expansion rate H of the universe and the energy density. For standard GR with a cosmological constant and flat space-like sections, the Hubble parameter can be expressed as

$$\left(\frac{H}{H_0}\right)^2=\Omega_\Lambda+\Omega_M\left(\frac{a_0}{a}\right)^3+\Omega_{rad}\left(\frac{a_0}{a}\right)^4, \tag{1}$$

where Ω_Λ is the density parameter for dark energy, Ω_M is the density parameter for matter including dark matter and Ω_{rad} is the density parameter for radiation.

These three parameters in total add up to unity. H is the Hubble parameter and α is the scale size of the universe and H_0 and a_0 denote today's values. The radiation contribution is negligible for the present era. The standard JBD action in canonical form and in units where $\hbar$ and c equals 1 is given by

$$S = \int d^4 x \sqrt{g}\left[-\frac{1}{8\omega}\phi^2 R + \frac{1}{2} g^{\mu\upsilon}\partial_\mu \phi \partial_\upsilon \phi - \frac{1}{2} m^2 \phi^2 + L_M \right] \tag{2}$$

The metric signature is (+ - - -). g is minus the determinant of the metric, ϕ is the Jordan scalar field which is related to the Brans-Dicke scalar field Φ and the two fields are related by $\Phi = \frac{1}{8\omega}\phi^2$, R is the Ricci scalar, $g^{\mu\upsilon}$ is the inverse metric tensor and L_M is the matter Lagrangian density except the scalar field ϕ. Note that the standard mass term for the scalar field can be combined with the curvature term so that this is sometimes called the cosmological term for JBD theory [50–53]. Most importantly, the term $\phi^2 R$ is the non-minimal coupling term which replaces the Newton gravitational constant G in the Einstein-Hilbert term R/G with the effective gravitational constant $G_{eff}^{-1} = \frac{2\pi\phi^2}{\omega}$.

We restrict our analysis to the Robertson-Walker metric to emphasize that space-time is necessarily spatially homogeneous and isotropic:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\frac{\mathrm{d}r^2}{\left[1 + \left(\frac{k}{4}\right)^2\right]^2} \tag{3}$$

where k is the curvature parameter, $a(t)$ is the scale factor of the universe has the dimension of length and r is dimensionless. $k = -1, 0, 1$ corresponding to open, flat, closed universes respectively.

When the field equations from the variation of the action (2) are derived, the Robertson-Walker metric and the cosmological background value of the scalar field $\phi = \phi(t)$ are inserted together with the energy-momentum tensor of a perfect fluid with energy density ρ_M and pressure p_M, so we obtain

$$\frac{3}{4\omega}\phi^2\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 + \frac{2}{2\omega}\frac{\dot{a}}{a}\dot{\phi}\phi = \rho_M \tag{4}$$

$$\frac{1}{4\omega}\phi^2\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{\omega}\frac{\dot{a}}{a}\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi\left(\frac{1}{2} + \frac{1}{2\omega}\right)\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 = p_M \tag{5}$$

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}+\left[m^2-\frac{3}{2\omega}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)\right]\phi=0 \tag{6}$$

where dot denotes d/dt and the subscript M denotes matter. For $k = 0$, $m = 0$, $a \sim t^{\alpha}$, $\phi \sim t^{\beta}$, $\rho \sim a^{-4}$ and $p=\frac{\rho}{3}$ in (4–6) give

$$a \sim t^{\frac{1}{2}} \quad \text{and} \quad \phi = \text{constant} \tag{7}$$

where this is the standard radiation dominated solution. The general solutions of the field equations for $m = 0$ and general equation of state $p = \upsilon\rho$ have been discussed in [54]. The case where ω is taken to be a function of ϕ have been compared with the model of Brans-Dicke for $p = \rho = 0$, for $p = \rho$ and for $p=\frac{1}{3}\rho$ [55]. The importance of equation (7) is that it agrees with standard cosmology which seems to agree with the radiation dominated era of standard cosmology [56]. Since m is different than zero for a JBD theory which explains dark energy, the solution given by (7) must be generalized. Instead of working with the field equations (4–6) which are given in terms of $\phi(t)$, $a(t)$ and their derivatives with respect to cosmological time t, we adopt an approach where we choose the fractional rate of change of ϕ

$$F(a)=\frac{\dot{\phi}}{\phi} \tag{8}$$

and the Hubble parameter

$$H(a)=\frac{\dot{a}}{a}, \tag{9}$$

where the scale size a as the independent variable and express functions of a. The left-hand side of the field equations are rewritten in terms of $H(a)$, F(a) and their derivatives with respect to a.

For $k = 0$ the equations become in the following form:

$$H^2-\frac{2\omega}{3}F^2+2HF-\frac{2\omega}{3}m^2=\left(\frac{4\omega}{3}\right)\frac{\rho}{\phi^2} \tag{10}$$

$$H^2+\left(\frac{2\omega}{3}+\frac{4}{3}\right)F^2+\frac{4}{3}HF+\frac{2a}{3}(HH'+HF')-\frac{2\omega}{3}m^2=\left(-\frac{4\omega}{3}\right)\frac{p}{\phi^2} \tag{11}$$

$$H^2-\frac{\omega}{3}F^2-\omega HF+a\left(\frac{HH'}{2}-\frac{\omega}{3}HF'\right)-\frac{\omega}{3}m^2=0. \tag{12}$$

From these three equations, it may be shown that the continuity equation for the matter-energy excluding the JBD scalar field follows as

$$\dot{\rho}+3\left(\frac{\dot{a}}{a}\right)(p+\rho)=0. \tag{13}$$

The continuity equation may be used in place of (11) because (10, 12 and 13) can be used to derive (11) where provided $\rho \neq 0$. Using (13), it is seen that the energy density ρ evolves with a in the same manner as in standard Einstein cosmology when the universe is solely governed by radiation,

$$\rho=\frac{C}{a^4}, \tag{14}$$

where C is an integration constant.

3. PERTURBATIVE RADIATION DOMINATED SOLUTION OR $a \to 0$

In this section we discuss solutions of (10–12) for a radiation dominated universe with $p=\frac{\rho}{3}$ and flat spacelike sections $k = 0$. In this case (10–12) become

$$\frac{4H^2}{3}+\left(\frac{4\omega}{9}+\frac{4}{3}\right)F^2+2HF+\frac{2a}{3}(HH'+HF')-\frac{8\omega}{9}m^2=0 \tag{15}$$

$$H^2-\frac{\omega}{3}F^2-\omega HF+a\left(\frac{HH'}{2}-\frac{\omega}{3}HF'\right)-\frac{\omega}{3}m^2=0. \tag{16}$$

As we discussed in the previous section the solution with $m = 0$ gives

$$H=H_1\left(\frac{a_1}{a}\right)^2 \tag{17}$$

$$F = 0 \tag{18}$$

where H_1 is constant. We now take $m \neq 0$ and consider (17) as the lowest order term and do the expansions for small a, in increasing powers of $\left(\frac{a}{a_1}\right)^4$ where $\frac{a}{a_1} < 1$ and we obtain $H(a)$ and $F(a)$ as given below:

$$H = H_1\left(\frac{a_1}{a}\right)^2 + H_2\left(\frac{a}{a_1}\right)^2 + H_3\left(\frac{a}{a_1}\right)^6 + ... \tag{19}$$

and

$$F = 0 + F_2\left(\frac{a}{a_1}\right)^2 + F_3\left(\frac{a}{a_1}\right)^6 + ... \tag{20}$$

By neglecting a^6 terms we find

$$H_2 = \frac{\omega m^2}{3H_1} \tag{21}$$

and

$$F_2 = -\frac{8\omega m^2}{5H_1}. \tag{22}$$

By using these values in (19), (20) and keeping a^6 terms, (15) and (16) give two solutions for H_3

$$H_3 = -\frac{\omega^2 m^4}{(H_1)^3}\left(\frac{32\omega}{75} + \frac{1327}{900}\right) - \frac{9F_3}{4} \tag{23}$$

$$H_3 = -\frac{\omega^2 m^4}{(H_1)^3}\left(\frac{368\omega}{150} + \frac{1}{18}\right) + \frac{3\omega F_3}{4}, \tag{24}$$

where

$$F_3 = \frac{(1824\omega - 1277)\omega^2 m^2}{(675\omega + 2025)(H_1)^3}. \tag{25}$$

Using this full expression of F_3 in (23) and keeping the only leading terms in ω for both solutions we have

$$H_3 \cong -\frac{32\omega^3 m^4}{75(H_1)^3} \tag{26}$$

$$F_3 \cong \frac{608\omega^2 m^4}{225(H_1)^3}. \tag{27}$$

Summarizing, up to order a^6 we find H and F as

$$H = H_1\left(\frac{a_1}{a}\right)^2 + \frac{\omega m^2}{3H_1}\left(\frac{a}{a_1}\right)^2 - \frac{32\omega^3 m^4}{75(H_1)^3}\left(\frac{a}{a_1}\right)^6 + \ldots \tag{28}$$

and

$$F = -\frac{8\omega m^2}{5H_1}\left(\frac{a}{a_1}\right)^2 + \frac{608\omega^2 m^4}{225(H_1)^3}\left(\frac{a}{a_1}\right)^6 + \ldots \tag{29}$$

Finally, we get

$$H^2 = H_1^2\left(\frac{a_1}{a}\right)^4 + \frac{2\omega m^2}{3} - \frac{64\omega^3 m^4}{75H_1^2}\left(\frac{a}{a_1}\right)^4 + \ldots \tag{30}$$

where the first term confirms the standard radiation dominated universe term. From late time fit to dark energy and matter [47] today's Hubble parameter H_0 is of the order of $\sqrt{\omega}m$ and for a less than a_1 for which our expansion is valid, the higher order terms are negligible.

4. CONCLUSION

The present work starts with the JBD theory with a canonical Jordan kinetic term and no mass term where it is seen that the energy density ρ evolves with a in the same manner as in standard Einstein cosmology when the universe is solely governed by radiation. We calculated the corrections to the radiation dominated solution in massive JBD theory. We showed that JBD theory with a standard mass term describing the early radiation dominated era also gives the late time dark energy dominated era in the Friedmann equation provided that an expansion in increasing powers of the scale size a is made. This is what one expects in a universe with only radiation but no matter.

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