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LOCALIZED STRUCTURES IN OPTICAL AND MATTER-WAVE MEDIA: A SELECTION OF RECENT STUDIES

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Abstract. A survey of some recent theoretical and experimental studies on localized structures that form and propagate in a broad class of optical and matter-wave media is presented. The article is structured as a resource paper that overviews a large series of theoretical and experimental works in diverse physical contexts: linear and nonlinear light bullets, two- and three-dimensional solitons propagating in carbon nanotubes, ultrashort few-cycle optical pulses, localized structures that form in fractional systems, rogue waves in scalar, vectorial, and multidimensional nonlinear systems, and solitons and vortices in matter-wave media.

Key words: localized structures, optical media, Bose-Einstein condensates, solitons in carbon nanotubes, few-cycle optical pulses, fractional systems, rogue waves, matter-wave localized states.

1. INTRODUCTION

The study of the existence, formation, and stability of localized structures in optical and matter-wave media is a research field of broad interest from both fundamental and applied points of view; see a series of textbooks and review papers in this area [1–21]. I also list here several influential papers published during the past few decades by many research groups in the area of localized structures that form in optical media and Bose-Einstein condensates [22–56]. I focus in this resource paper on some recent theoretical and experimental works published during the past three years, which were dedicated to linear and nonlinear localized structures in physical settings involving optical and matter-wave media.

The paper is structured in six Sections, the first one being an introduction to the topic of this overview paper, and the last one briefly presenting the conclusions. The remaing Sections are dedicated to localized structures in dispersive and/or diffractive optical media (Sec. 2), localized structures in fractional systems (Sec. 3), rogue waves (RWs) in diverse physical contexts (Sec. 4), and localized matter-wave structures (Sec. 5). The above mentioned Sections are mutually independent and can be used as resource materials in the corresponding research fields.

It should be pointed out that among the huge amount of linear and nonlinear wave structures, the two- and three-dimensional ones stand out due to their important role in many physical settings. I refer here to a viewpoint by Malomed *et al.* [12] on multidimensional solitons and their legacy in contemporary atomic, molecular, and optical physics and also to the review articles by Malomed [14] on well-established results and novel findings in the area of multidimensional solitons in nonlinear optics and atomic Bose-Einstein condensates (BECs), by Kartashov *et al.* [19] on frontiers in multidimensional self-trapping of nonlinear fields and matter, and by Malomed [20] on two- and three-dimensional vortex solitons.

In the next Section I overview the recent research activity in the area of multidimensional solitons in optical media, namely: i) theoretical and experimental studies of linear and nonlinear *light bullets*; ii) theoretical investigations of two- and three-dimensional solitons forming in carbon nanotubes, and iii) recent theoretical and experimental activities in the area of ultrashort (few-cycle) optical pulses. In Sec. 3 I present recent studies in the area of localized structures in fractional systems. I briefly review in Sec. 4 the recent theoretical and experimental works on RWs in diverse physical contexts. Matter-wave localized states, including the so-called *quantum droplets*, are discussed in Sec. 5. Section 6 concludes the article.

2. LINEAR AND NONLINEAR LOCALIZED STRUCTURES IN OPTICAL MEDIA

Space-time wave packets, which are unique one-dimensional propagation-invariant pulsed optical beams, have been studied in diverse physical settings; see, for example, an overview on diffraction-free space-time beams [57]. The propagation of optical space-time wave packets of arbitrary group velocity in free space has been demonstrated by Kondakci and Abouraddy [58]. Recently, hybrid guided space-time modes that are index-guided in one transverse dimension and are localized along the unbounded dimension have been observed by Shiri *et al.* [59] in unpatterned films. Also, anomalous refraction of optical space-time wave packets has been put forward by Bhaduri *et al.* [60] in a variety of optical materials. It should be pointed out that the space-time refraction offers new opportunities for moulding the flow of light and other wave phenomena in many physical settings; for more details see Ref. [60]. Spatiotemporally confined, multimode high-energy multidimensional solitary states have been observed in hollow-core fibers by Safaei *et al.* [61]. It is worth to mention that the numerical simulations reported in Ref. [61] have confirmed that multidimensional solitary states dynamics and spatiotemporal nonlinear enhancement can lead to the realization of a new class of compact, tunable, and high-energy spatiotemporally engineered coherent light sources that are based on picosecond ytterbium technology.

In a paper by Jia *et al.* [62], it has been demonstrated, for the first time to the best of my knowledge, that the nonlinear response of a medium can be mapped directly onto a dynamical wave profile as governed by a generalized nonlinear Schrödinger (NLS) equation, thus it has been possible to visualize a nonlinear response in a Schrödinger wave. The innovative approach put forward by Jia *et al.* [62] has been verified experimentally by directly visualizing a Kerr (saturable) nonlinearity experienced by an optical pulse (beam) in a nonlinear fiber (photorefractive crystal), thus validating its versatility for different types of optical nonlinearities. In a recent work, Kibler and Béjot [63] have studied theoretically and numerically the concept of discretized conical waves and their unique properties in structured media, and particularly, in multimode optical fibers. These space-time optical waveforms result from the linear superposition of optical fiber modes with the engineered spatiotemporal spectrum. Moreover, it was pointed out in Ref. [63] that these conical waves can also spontaneously emerge during nonlinear propagation of ultrashort optical pulses.

Zhang *et al.* [64] have studied the tightly focusing evolution of the auto-focusing linearly polarized circular Pearcey-Gaussian vortex beams. It has been demonstrated in Ref. [64] that these vortex beams are rotating during their tightly focusing evolution. Also, flat-top beams and quasi optical comb modes, which are generated by the linear polarized Pearcey-Gaussian vortex beams have been analyzed in detail, and the change of the focus depth with the beam's parameters has been also discussed [64]. Susanto and Malomed [65] have recently investigated analytically and numerically the so-called *embedded solitons*, which are exceptional modes in nonlinear-wave systems with the propagation constants falling in the system's propagation band, in second-harmonic-generating lattices. Solutions for discrete embedded solitons have been constructed by means of two analytical approximations of the governing model, which correspond to broad and narrow discrete embedded solitons, respectively [65].

Tail-free self-accelerating solitons and vortices have been recently investigated by Qin *et al.* [66]. Analytical and numerical methods developed in Ref. [66] show the existence of robust one- and two-dimensional self-accelerating tailless solitons and vortices in two-component BECs and in optical media with thermal nonlinearity. Zeng *et al.* [67] have investigated both one- and two-dimensional models of optical media with self-repulsive cubic nonlinearity (defocusing Kerr-type nonlinear optical media), whose local strength is subject to spatial modulation that admits the existence of flat-top solitons of various types, including fundamental ones, one-dimensional multipoles, and two-dimensional vortices.

Fedorov *et al.* [68] have proposed a simple method to control the topology of laser vortex solitons and their complexes in a wide-aperture laser with saturable absorption by means of a week coherent holding radiation. It should be pointed out that the wide variety of the stable vortex structures that have been obtained using this technique, makes the scheme put forward by Fedorov *et al.* [68] a promising one for topologically protected information processing. The existence of persistently rotating azimuthons in media with self-focusing Kerr and absorption nonlinearities has been reported by Ruiz-Jiménez *et al.* [69]. In the physical setting studied in that work, the nonlinear loss is balanced by power influx from the peripheral reservoir stored in a slowly decaying tail of the field. It has been shown by Ruiz-Jiménez *et al.* [69] that the robustness of the rotating azimuthons is enhanced in comparison to similar static dissipative patterns.

Parra-Rivas *et al.* [70] have investigated analytically and numerically the influence of the stimulated Raman scattering on the formation of bright and dark localized structures in all-fiber resonators subject to a coherent optical injection. The implications that the stimulated Raman scattering has on the bifurcation structure of localized structures in the normal group velocity dispersion regime have been investigated in detail in Ref. [70]. In a recent work, Korneev and Vysloukh [71] have applied the Wentzel-Kramers-Brillouin method to wavepackets with strong phase modulation in Kerr-type nonlinear media, and have obtained an useful information about their soliton content. Modulated solitons, soliton clusters, and vortex clusters in purely nonlinear defocusing Kerr media have been investigated both analytically and numerically by Liangwei Zeng and Jianhua Zeng [72]. The approximate analytical solutions of modulated solitons and soliton clusters in that setting have been obtained by means of the standard Thomas-Fermi approximation. The stability domains for the soliton structures studied in Ref. [72] have been identified by using both linear stability analysis and direct numerical simulations. Kochetov *et al.* [73] have proposed a mechanism to perform topological transformations that change the key features of dissipative vortices and their complexes in a controllable way. It has been shown that properly chosen potentials carry out the evolution of dissipative structures to a regime with spontaneous transformation of the topological excitations or drive the generation of vortices with control over the values of the topological charges [73].

Sazonov [74] has performed an analytical study of the propagation of spatiotemporal optical solitons in gradient fibers with Kerr nonlinearities. Two distinct approaches based on the method of the averaged Lagrangian have been used in that work. In the framework of one of the two methods developed in Ref. [74], approximate solutions have been found in the form of stationary and pulsating light bullets and the conditions for their stability have been also determined. In another work, Sazonov [75] has investigated theoretically the so-called two-color 'dancing' light bullets in graded-index waveguides. The propagation of spatiotemporal solitons in focusing quadratically-nonlinear waveguides has been studied in detail and it has been revealed that for anisotropic spatial distribution of the refractive index in the cross section of the graded-index waveguide, the soliton trajectory can be a spatial Lissajous figure. These two-color light bullets can be formed under both anomalous and normal group-velocity dispersion [75]. Sazonov and Komissarova [76] have studied analytically the possibility of the formation of two-frequency light bullets in quadratically-nonlinear media with zero group-velocity dispersion coefficient at the second harmonic frequency. It has been shown in Ref. [76] that the time duration of the light bullet component at the second harmonic frequency is two times shorter than the pulse duration at the fundamental frequency. However, the transverse dimensions of both light bullet components are identical [76].

Li and Kawanaka [77] have theoretically shown how the combination of a diffraction-free beam and an ultrashort pulse spatiotemporal coupling enables the creation of straight-line propagation light bullets with freely tunable velocity and acceleration. It has been revealed in Ref. [77] that the light bullets could propagate with a constant superluminal or subluminal velocity, and that they could also counter-propagate with a very fast superluminal velocity. Si-Liu Xu *et al.* [78] have proposed a scheme to generate stable light bullets in cold Rydberg atomic systems with parity-time-symmetric potentials, by using the electromagnetically induced transparency phenomenon. The atomic system, which has been investigated theoretically in Ref. [78], supports slow light bullets with low light intensity. Milián *et al.* [79] have introduced a new class of stable light bullets that form in twisted waveguide arrays pumped with ultrashort pulses. It has been found that, above a critical twist, three-dimensional wave packets are stabilized, with no minimum energy threshold, see Ref. [79] for more details of this study.

Peng, He, and Dong [80] have studied analytically and numerically three-dimensional chirped Airy complex-variable-function Gaussian vortex wavepackets in strongly nonlocal nonlinear media. These waveforms can rotate stably in strongly nonlocal nonlinear media and they have various shapes, including dipoles, elliptic vortices, and doughnuts [80]. Liangwei Zeng and Jianhua Zeng [81] have put forward a scheme to prevent the critical collapse of higher-order solitons by tailoring unconventional optical diffraction and nonlinearities. They have proposed and demonstrated, both theoretically and numerically, a framework of a two-dimensional nonlinear fractional Schrödinger equation (FSE) that describes the light propagation in a nonlinear periodic system with an optical lattice and competing self-focusing-self-defocusing cubic-quintic nonlinearities, a physical setting that can suppress the critical collapse of the beam [81]. Diverse families of stable solitons, including two-dimensional fundamental gap and vortical solitons as well as gap soliton clusters do exist it this model [81].

Jung *et al.* [82] have numerically studied the structural stability of vortex solitons in nematic liquid crystals with nonlocal reorientation nonlinearity response. The anisotropy-induced astigmatism can destabilize the vortex solitons and can lead to their breakup. It has been shown in Ref. [82] that for low and moderate birefringence, vortex solitons can propagate stably in nematic liquid crystals over experimentally relevant distances. In a recent work, Li *et al.* [83] have numerically investigated both the existence and stability of vector solitons in nonlocal optical media with pseudo spin-orbit coupling.

Spatiotemporal solitons in dispersion-managed multimode fibers have been investigated analytically and numerically by Mayteevarunyoo *et al.* [84] and a scheme of dispersion management for three-dimensional solitons in multimode optical fibers has been developed in that work. By means of numerical simulations, it has been found that the stability of the three-dimensional spatiotemporal solitons is determined by the usual dispersion-managed-strength parameter; see Ref. [84].

The unique properties of electromagnetic waves guided by nonlinear interfaces have been extensively studied more than three decades ago by different research groups; see Refs. [85–96]. These earlier works in the area of transverse electric (TE) and transverse magnetic (TM) surface and guided waves in layered structures containing self-focusing and self-defocusing nonlinear optical media have been reviewed in two comprehensive papers [97, 98]. Recently, Savotchenko [99–109] has revisited this research area and has obtained new results on the formation and the key features of TE and TM nonlinear surface waves in diverse physical settings. It should be pointed out here the work by Savotchenko [104] on a new model of nonlinear contacting media based on a specific form of the NLS equation that describes the modifications of the properties of the boundary regions along the interface between a Kerr-type crystal with cubic nonlinearity and a nonlinear medium characterized by an abrupt change in the dielectric constant that depends on the electromagnetic field amplitude. The short-range local interaction between the electromagnetic wave and the interface between the contacting media has been taken into account by a point potential in the NLS equation; see Ref. [104]. Savotchenko [107] has also recently put forward a new model of nonlinearity, describing a sharp change in the Kerr-type coefficients that depend on the amplitude of the electromagnetic field. Three new types of nonlinear surface waves localized near the interface separating the linear medium and the Kerr-type crystal have been found analytically. One type of such surface wave occurs in the crystal characterized by self-focusing Kerr nonlinearity and two other types of surface waves exist in the crystal characterized by self-defocusing Kerr nonlinearity; see Ref. [107]. Also, the symmetrically localized modes in a three-layered structure consisting of a linear layer between two defocusing nonlinear media, which are separated by interfaces with nonlinear optical properties, have been recently investigated analytically by Savotchenko [109].

The existence, formation, and dynamical properties of gray solitons that are described by an extended quintic NLS equation have been studied by Tsitoura *et al.* [110]. Rao *et al.* [111] have investigated the general set of nonlocal M-component NLS equations obeying the parity-time symmetry and featuring focusing, defocusing, and mixed (that is, focusing-defocusing) nonlinearities. It should be pointed out that the generic dynamical model developed in Ref. [111] has applications in nonlinear optics settings. Soliton formation and stability under the interplay between parity-time-symmetric generalized Scarf-II potentials and cubic (Kerr type) nonlinearity have been studied both analytically and numerically by Chen *et al.* [112]. In the framework of that dynamical model, the majority of fundamental nonlinear modes are stable, whereas the one-dimensional multipeak solitons and two-dimensional vortex solitons suffer from instability [112]. Li *et al.* [113] have investigated the problem of emulation of spin-orbit coupling for solitons in nonlinear optical media. The framework studied in Ref. [113] is based on the spatial-domain copropagation of two light beams with mutually orthogonal polarizations and opposite transverse components of carrier wavevectors in a nonlinear waveguide with randomly varying birefringence, the averaging with respect to which introduces an effective Manakov-type nonlinearity in the system. A a result, a two-component system of NLS equations has been derived, which is similar to the system of coupled one-dimensional Gross-Pitaevskii equations for binary spin-orbit-coupled Bose-Einstein condensates, see Ref. [113]. Stable flat-top solitons and peakons that form in parity-time-symmetric δ-signum potentials and nonlinear media have been investigated by Chen *et al.* [114]. Families of defect modes supported by parity-time-symmetric triangular optical lattices with self-defocusing cubic (Kerr-type) nonlinearity have been studied by Wang *et al.* [115] using numerical simulations. I also refer here to a work by Zhukov *et al.* [116] on the propagation of two-dimensional electroacoustic waves in silicene, namely a theoretical study of electroacoustic waves in a piezoelectric medium built upon silicene. The possibility for the amplification of the pulse amplitude through the tuning of the piezoelectric coefficient has been uncovered by Zhukov *et al.* [116].

The problem of nonconservation of the topological charge and cusps in a one-dimensional laser scheme has been investigated recently by Veretenov, Fedorov, and Rosanov [117]. Also, Rosanov [118] has recently analyzed the quasi-optical equation for a pulse of quasi-monochromatic radiation that propagates in either a homogeneous or in a one-dimensional inhomogeneous linear medium with weak dissipation. Fedorov, Veretenov, and Rosanov [119] have proposed a simple method to control the topology of laser vortex solitons and their complexes in wide-aperture lasers with saturable absorption by means of a weak coherent holding radiation. In another recent work, Rosanov *et al.* [120] have classified and analyzed in the framework of the generalized Ginzburg-Landau equation, the transformations of three-dimensional dissipative tangle solitons that form in laser media with fast saturable absorption, namely, dissipative solitons with closed and nonclosed vortex lines, under smooth variations of the system parameters.

Zhang *et al.* [121] have recently reported the observation of edge solitons at the zigzag edge of a reconfigurable photonic graphene lattice, which was created by using the electromagnetically induced transparency effect in an atomic vapor cell with a controllable nonlinearity. Kartashov and Konotop [122] have recently revealed an universal effect of gauge fields on the existence, evolution, and stability of solitons in the spinor multidimensional NLS equation. It has been found in Ref. [122] that a nonzero curvature can lead to the existence of unusual modes, namely, stable localized self-trapped fundamental and vorticity-carrying states in media with constant repulsive interactions without the need for additional external confining potentials and even in the situation of expulsive external traps. Shafeeque Ali *et al.* [123] have reported a theoretical analysis of the modulational instability of diffractionless waves in a face-centered-square lattice of waveguides featuring non-Kerr nonlinearities (namely, with cubic, quintic, and septimal nonlinearities), and with alternating signs of refractive index. Recently, Gaidoukov and Anglin [124] have used the Bogoliubov-de Gennes theory to study the snake instability of gray solitons in higher dimensions. An approximate analytical description of the snake instability within the Bogoliubov-de Gennes perturbation theory has been put forward and within that linear approximation the results obtained in two dimensions can be also applied to three dimensions, describing buckling modes of the low-density plane [124]. The interesting propagation dynamics of periodic and solitary waves (both bright and dark solitons) in inhomogeneous optical waveguides with third-order dispersion and self-steepening nonlinearity has been recently investigated by Kruglov and Triki [125]. In that work, the formation and the stability characteristics of periodic waves and envelope solitons in dispersive Kerr-type optical media under the combined influence of third-order dispersion and self-steepening effect have been studied in detail [125].

Kevrekidis *et al.* [126] have revisited the problem of transverse instability of a two-dimensional breather stripe of the sine-Gordon equation. Using direct numerical simulations, it has been found that the instability leads to the breakup of the quasi-one-dimensional breather in a chain of interacting two-dimensional radial breathers that are fairly robust [126]. The asymptotic reductions and solitary waves of a weakly nonlocal defocusing NLS model have been studied by Koutsokostas *et al.* [127]. Recently, Ivars *et al.* [128] have uncovered the gradual self-replication of spatiotemporal Kerr cavity patterns in cylindrical microresonators.

In what follows I list a few recent experimental advances in the area of localized structures in diverse optical settings. Reyna *et al.* [129] have observed and analyzed in detail the formation, decay, and subsequent regeneration of ring-shaped clusters of (2+1)-dimensional spatial solitons in a medium with cubic-quintic (focusing-defocusing) self-interaction and strong dissipative nonlinearity. The experimental setup uses a laser beam at wavelength 800 nm, which is built of pulses with the temporal duration 150 fs, at the repetition rate of 1 kHz, propagating in a cell filled by liquid carbon disulfide (CS_2). Shen and Dierking [130] have investigated experimentally the interesting dynamics of electrically driven multidimensional solitons in nematic and cholesteric liquid crystals. They have also put forward that these solitons can be used as vehicles for two-dimensional delivery of micro-cargos. Garbin *et al.* [131] have reported the experimental realization of dissipative polarization domain walls in passive coherently driven Kerr resonators. These dissipative polarization domain walls consist of temporally localized structures where the amplitudes of the two polarization modes of the resonator interchange, thus segregating domains of orthogonal polarization states. To the best of my knowledge, the experimental results presented in Ref. [131] constitute the first evidence of isolated dissipative all-optical temporal polarization domain walls. Spatiotemporal optical vortices with controllable transverse orbital angular momentum have been recently generated experimentally [132, 133]; see also earlier theoretical and experimental works in this area [134–136].

2.1. SOLITONS IN CARBON NANOTUBES

The propagation of ultrashort optical pulses and the formation of multidimensional solitons in carbon nanotubes (CNTs) have been investigated in a series of recent papers [137–144].

A theoretical study of three-dimensional light bullets in Bragg media containing arrays of CNTs has been reported by Zhukov *et al.* [137]. The possibility of stable propagation of such spatiotemporal optical waveforms has been demonstrated in Ref. [137]. Fedorov *et al.* [138] have studied the propagation of three-dimensional bipolar ultrashort electromagnetic pulses in inhomogeneous arrays of semiconductor CNTs. The problem of stabilization of ultrashort pulses by external pumping in arrays of CNTs subject to piezoelectric effects has been investigated by Konobeeva *et al.* [139]. That work has revealed that stable propagation of ultrashort pulses can be achieved when the dissipative piezoelectric effects are properly compensated through external pumping [139]. The asymptotic dynamics of three-dimensional bipolar ultrashort electromagnetic pulses in arrays of semiconductor CNTs has been studied by Fedorov *et al.* [140]. The propagation dynamics of three-dimensional bipolar ultrashort pulses in arrays of semiconductor CNTs at times much longer than the pulse duration, yet still shorter than the relaxation time in the system, has been studied in detail both analytically and numerically [140].

Konobeeva *et al.* [141] have investigated the propagation characteristics of extremely short optical pulses in strained carbon nanotubes in the three-dimensional case. It has been shown that the mechanical stretching of carbon nanotubes significantly affects the dynamics of three-dimensional extremely short optical pulses in CNTs, namely, the larger the strain, the more stable the pulse becomes; see Ref. [141]. The problem of the propagation of three-dimensional ultrashort optical pulses in photonic crystals made of zig-zag carbon nanotubes under the condition of nonlinear absorption and amplification has been studied by Dvuzhilov *et al.* [142].

The peculiarities of propagation in CNTs of extremely short optical pulses with nonlinear absorption have been recently put forward by Konobeeva *et al.* [143]. In that work it has been demonstrated the stability of the electromagnetic pulse shape on a time scale that is significantly longer than the pulse duration but not exceeding the characteristic relaxation time; see Ref. [143]. The interesting problem of the heating of a carbon nanotubes array with few-cycle optical pulses has been investigated by Konobeeva *et al.* [144] in the framework of the hot electron model, in which an array of CNTs is irradiated by an electromagnetic wave with a high intensity.

Fedorov *et al.* [145] have recently studied both analytically and numerically the propagation of three-dimensional bipolar ultrashort electromagnetic pulses in an inhomogeneous array of semiconductor CNTs, in the presence of a control high-frequency electric field. It has been shown that as a result of the interaction of the ultrashort pulse with the barrier layer of high electron density in an array of CNTs, the pulse can either pass through the layer or can be reflected from it. The obtained results may be used for the design of soliton valves, with the transmissivity controlled by the high-frequency electric field, by adjusting the amplitude and frequency of that control field [145].

2.2. ULTRASHORT (FEW-CYCLE) OPTICAL WAVEFORMS

Many experimental and theoretical studies of optical pulses with widths ranging from tens of nanoseconds to only a few tens of femtoseconds have been reported in the course of the last three decades; see, in particular, several review papers on the fast growing research area of ultrashort high-power optical pulses [146–166]. The possibility to generate high-intensity ultrashort optical pulses relies on a revolutionary experimental achievement, namely the so-called *chirped pulse amplification* technique, which was introduced in 1985 by Strickland and Mourou [167].

Next I briefly overview recent theoretical results in the area of ultrashort optical waveforms. The excitation of molecular rotational levels by unipolar subcycle pulses has been investigated theoretically by Arkhipov *et al.* [168]. It has been shown in Ref. [168] that it is possible an efficient excitation of molecular rotational resonances by unipolar subcycle pump pulses with respect to bipolar single-cycle ones, provided that the duration of excitation pulses is much smaller than the rotational period of the molecule. The possibility of the selective ultrafast control of multi-level quantum systems by subcycle and unipolar pulses has been studied in a recent work by Arkhipov *et al.* [169]. Though the spectrum of such ultrashort pulses covers several levels at once, it has been shown that it is possible to selectively excite the levels by varying the driving pulse shape, and duration or time delay between consecutive pulses [169]. The interaction of rectangular unipolar pulses with two-level resonant media has been investigated by Arkhipov and Rosanov [170] under conditions that the pulse duration is shorter than the relaxation times of the medium. Few-cycle-pulse solitons and soliton molecules in a model governed by the fifth-order Korteweg-de Vries equation have been investigated by Chen and Jia [171].

In a recent work, Bulanov *et al.* [172] have shown that the interplay between the vacuum polarization and the nonlinear effects in the interaction of counter-propagating electromagnetic waves can result in the formation of relativistic electromagnetic solitons that are described by the celebrated Kadomtsev-Petviashvili, Korteweg-de Vries, and dispersionless Kadomtsev-Petviashvili nonlinear partial differential equations. In another recent work, Zhou *et al.* [173] have developed a computational method to study the electron-positron pair creation that is induced by two sequential short pulses. In Ref. [173], the double cosine-Gaussian pulse, the alternating-sign double-Gaussian pulse, and the double-Gaussian pulse with the same sign have been used and the effects of the pulse parameters and the pulse shape on the total number of electron-positron pairs have been also discussed. Siminos *et al.* [174] have recently shown by using numerical methods (particle-in-cell simulations) that isolated carrier-envelope-phase tunable intense subcycle pulses can be created by a frequency upconversion process, which they referred to as laser wakefield driven amplification.

I overview in what follows some recent experimental works in this very broad area. An important scientific contribution towards the obtaining of intense isolated attosecond pulses, in the extreme ultraviolet and X-ray spectral ranges, from relativistic surface high harmonics has been reported by Jahn *et al.* [164]. In that work, both measurements and particle-in-cell numerical simulations to determine the optimum values for the most important parameters have been presented [164]. Recently, Aleksandrov *et al.* [165] have reported the upgrading design of a multi-TW femtosecond laser in order to increase its peak power with more than one order of magnitude.

In a comprehensive review paper, Khazanov, Mironov, and Mourou [166] have reported on the problem of nonlinear compression of high-power laser pulses, in particular, the very promising *compression after compressor approach*, when the pulse is shortened after passing a compressor. This innovative method has the following merits: its simplicity and low cost, negligible loss of pulse energy, and applicability to high-power lasers, see Ref. [166].

The current status and highlights of the Extreme Light Infrastructure Nuclear Physics (ELI-NP) research program have been reported by Tanaka *et al.* [175]. The ELI-NP pillar of the European ELI infrastructure is hosted in Magurele near Bucharest, Romania. Lureau *et al.* [176] have recently reported on a two-arm hybrid high-power laser system developed at the Bucharest-Magurele ELI-NP Facility, which is able to deliver 2×10 PW femtosecond pulses.

I also mention here the work by Söderström *et al.* [177] on the design and commissioning of a new detector array within the ELI Gamma Above Neutron Threshold (ELIGANT) set-up, with the aim of measuring weak second and third order γ-ray emitting quantum-electrodynamical processes. This new detector array is one of the key instruments in the research program *Nuclear physics and applications with high-brilliance gamma beams* at the ELI-NP pillar in Magurele, Romania.

3. LOCALIZED STRUCTURES IN FRACTIONAL SYSTEMS

In this Section I overview some recent works in the area of localized structures forming in the so-called *fractional systems*, more precisely in media with fractional dimension. Laskin has introduced two decades ago in the context of quantum mechanics, in the seminal works [178–180], the so-called fractional Schrödinger equation (FSE), The key parameter that defines the fractionality of the FSE is the Lévy index, $1 < \alpha \leq 2$. For the particular case $\alpha = 2$ the FSE becomes the standard Schrödinger equation. I also refer here to the book by Laskin on fractional quantum mechanics [181], to the textbook by Herrmann [182] on the fundamentals of the fractional calculus for physicists, and to a more specialized book by Yang, Baleanu, and Srivastava [183] on local fractional integral transforms and their applications in diverse research areas. It should be also mentioned here some recent works on fractional models in diverse physical settings and on the study of fractional differential equations with applications in physics and engineering [184–192].

In a seminal paper, Longhi [193] has introduced the FSE in optics, namely an optical realization of the FSE that is based on the transverse light dynamics in aspherical optical cavities. In that pioneering work, a laser implementation of the fractional quantum harmonic oscillator has been presented, in which dual Airy beams can be selectively generated under off-axis longitudinal pumping.

Solaimani [194] has calculated numerically the eigenvalues of one-dimensional parity-time-symmetric FSE with multiple quantum wells potential profile and has studied the effects of different parameters on the pairwise coalescence of eigenvalues. In a recent work, Lombard *et al.* [195] have investigated both analytically and numerically the observables of complex-valued parity-time-symmetric shifted potentials. The possibility of existence of stable solitons in fractional dimensions has been investigated in several works; see, for example, Refs. [196–199]. It should be pointed out here the recent work by Li and Dai [199] where double loops and pitchfork symmetry breaking bifurcations of optical solitons in nonlinear FSE with competing cubic-quintic nonlinearities have been investigated in detail. It has been revealed that stable asymmetric solitons emerge from unstable symmetric and antisymmetric ones by way of two distinct symmetry breaking scenarios [199]. Moreover, accessible solitons [200, 201] and self-trapped states of vectorial [202], gap [203], nonlocal [204], vortical [205], and multi-peak types [206] have been predicted in FSE models, as well as soliton clusters [207, 208], symmetry breaking of solitons [209, 210], and dissipative solitons in a fractional complex Ginzburg-Landau model [211]. It should be pointed out here that in the case of the ubiquitous cubic (Kerr) self-focusing media, the solitons are unstable at $\alpha \leq 1$, because the combination of such small values of the Lévy index α with the self-focusing Kerr nonlinearity gives rise to the collapse of the optical beam. Liangwei Zeng and Jianhua Zeng [196] have investigated the existence and stability of stable bright solitons in one-dimensional fractional media with spatially periodical modulated Kerr nonlinearities. It has been shown in Ref. [196] that the soliton families are stable if the Lévy index α exceeds a certain threshold value, below which the balance between the fractional-order diffraction and the spatially modulated self-focusing nonlinearity is broken. In a recent work, Li *et al.* [197] have studied analytically and numerically the $\mathcal{PT}$-symmetric optical modes and spontaneous symmetry breaking in the space-fractional Schrödinger equation. Moreover, families of vortex solitons in the fractional NLS equation with the cubic-quintic nonlinearity have been recently investigated by Li *et al.* [205]. Qiu *et al.* [206] have investigated numerically the problem of stabilization of single- and multi-peak solitons in the fractional NLS equation with a trapping potential. Metastable soliton necklaces supported by fractional diffraction and competing optical nonlinearities have been studied by Li *et al.* [208]. The unique dynamics of dissipative solitons in the framework of a one-dimensional complex Ginzburg-Landau equation of a fractional order has been investigated by Qiu *et al.* [211] by using analytical and numerical methods. The effects of the Lévy index on the soliton's dynamics have been studied and in particular, the dependence of the stability domains in the model's parameter space on the value of Lévy index has been identified in Ref. [211]. In the framework of fractional discrete NLS equation, Molina [212, 213] has recently studied the nonlinear modes, both bulk and surface states, and their stability.

In a paper by Huang and Dong [214], the existence and stability of one- and two-dimensional dissipative surface solitons supported by the nonlinear FSE with an interface between a semi-infinite chirped lattice and a uniform Kerr medium have been studied in detail. It has been shown in Ref. [214] that stable dissipative surface solitons feature low energy and small propagation constants and adapt well to a wide range of two-photon absorption. Robust nonlinear dissipative surface solitons can be excited by Gaussian input beams. Vortex solitons in fractional systems with partially parity-time-symmetric azimuthal potentials have been studied by Dong and Huang [215]. The domain of variation of the Lévy index α greatly impacts the properties of nonlinear vortices, including the existence domain, power, and stability of such vortex solitons; see Ref. [215] for a detailed study of these problems.

Recently, Zhu *et al.* [216] have investigated the existence and stability of in-phase three-pole and four-pole gap solitons in the FSE supported by one-dimensional parity-time-symmetric periodic potentials with defocusing Kerr nonlinearity. These multipole gap solitons in FSE with parity-time-symmetric optical lattices exist in the first finite gap and are stable for moderate power values [216]. However, when the Lévy index α decreases, the domains of stability of the in-phase multipole gap solitons shrink, see Ref. [216]. In a subsequent work by the same group [217], the existence and stability of mixed-gap vector surface solitons at the interface between a uniform medium and an optical lattice with fractional-order diffraction have been reported. The mixed-gap vector surface solitons can be stable in the nonlinear FSE. The two components of these vector surface solitons arise from the semi-infinite and the first finite gaps of the optical lattices, respectively, and for some propagation constants of the first component, the stability domain of these vector surface solitons can be widened by decreasing the Lévy index α, see Ref. [217].

The interesting propagation dynamics of abruptly autofocusing circular Airy-Gaussian vortex beams in the (2+1)-dimensional FSE has been studied by He *et al.* [218]. By means of specific numerical techniques, the propagation dynamics of the beams with vorticities from 0 to 4 has been explored in Ref. [218]. It has been put forward that the propagation leads to abrupt autofocusing, followed by its reversal (rebound from the center). The Lévy index α ($1 < \alpha \leq 2$), the relative width of the Airy and Gaussian factors, and the vorticity determine the autofocusing dynamics, including the focusing distance, radius of the focal light spot, and peak intensity at the focus; see Ref. [218] for more details of this study. In a recent paper by Xin, Song, and Li [219] the interesting evolution dynamics of Gaussian beams governed by the two-dimensional FSE with variable coefficients has been studied analytically and numerically. The impact of the longitudinal modulation, the Lévy index, and the chirp parameters on the evolution of Gaussian beams has been analyzed in detail [219]. Li *et al.* [220] have investigated analytically and numerically the existence, symmetry breaking bifurcation, and stability of two-dimensional optical solitons supported by fractional diffraction, in the framework of nonlinear FSE with either self-focusing or self-defocusing saturable nonlinearities. The physical setting supports two-dimensional symmetric, antisymmetric, and asymmetric solitons, and it has been revealed that the asymmetric solitons emerge by way of a symmetry breaking bifurcation, see Ref. [220]. In a recent work, Zeng *et al.* [221] have constructed families of fundamental and multipole solitons (dipole and tripole solitons) in a cubic-quintic nonlinear lattice in fractional dimension. The multipole complexes exist only in the presence of the nonlinear lattice and the shapes and stability of all solitons strongly depend on the value of the Lévy index. The stability domains are broadest for the fundamental solitons and narrowest for the tripoles, see Ref. [221].

4. ROGUE WAVES

In this Section I overview some recent theoretical and experimental advances in the area of rogue waves (RWs), also termed as freak waves. These special wave structures emerge in a broad class of physical settings, for example in hydrodynamics, optics and photonics, Bose-Einstein condensates; see the comprehensive reviews [222–225]. In a recent work, Akhmediev [226] has overviewed the intense research activity in the area of rogue waves or extreme events, which are known as waves that appear from nowhere. Actually, that overview paper [226] is concentrated on the Peregrine wave, its analogs, and its higher-order combinations.

The huge amount of theoretical and experimental papers that have been published in the past decade in the area of RWs were strongly influenced by the seminal works of Peregrine [227], Kuznetsov and Ma [228, 229], and Akhmediev *et al.* [230]. In these pioneering works, exact rational solutions of the integrable NLS equation were reported. It should be also mentioned here the papers published in 1993 by Mihalache and Panoiu [231] and by Mihalache *et al.* [232] in which the above-mentioned exact rational solutions of the NLS equation and other types of the so-called first-order exact solutions of this nonlinear partial differential equation were obtained in the context of the pulse propagation in optical fibers, for both the anomalous and the normal dispersion regime. I point out here that the multiparameter families of first-order exact solutions of the NLS equation in the normal-dispersion regime have been independently reported in 1993 by Akhmediev and Ankiewicz [233] and by Gagnon [234].

In what follows I will overview recent theoretical and experimental works in this fast growing research area. Ward and Kevrekidis [235] have revisited the problem of rogue wave structures in the context of a few dispersive nonlinear dynamical models: the NLS, Hirota, and Zakharov equations. The rogue wave patterns have been found as self-similar solutions that are associated with a (potentially complex) time-dependent prefactor and a self-similar profile (in the NLS equation case, a Lorentzian profile), arising against the backdrop of a constant, nonvanishing background; see Ref. [235]. A general n-fold Darboux transformation matrix for the integrable Sasa-Satsuma equation has been obtained by Guo *et al.* [236]. Different kinds of solitons on a continuous wave background have been obtained by using the Darboux transformation, including W-shaped first-order and second-order rational solutions, co-existing W-shaped second-order rational solutions, first-order semi-rational solutions, first-order and second-order periodic solutions, first-order half-periodic solutions, and second-order breather-positon solutions; see Ref. [236]. The unique dynamics of dust-acoustic and dust-cyclotron freak waves in a magnetized dusty plasma has been recently studied analytically and numerically by Akhtar *et al.* [237].

Liu and Wazwaz [238] have investigated the problem of inelastic collisions of lumps and line solitons in the Maccari nonlinear system. By using the Hirota bilinear method combined with the Kadomtsev-Petviashvili hierarchy reduction method, determinant semi-rational solutions consisting of lumps and line solitons have been constructed. The inelastic interaction scenarios have been investigated by Liu and Wazwaz [238], namely, line solitons fissioning into line solitons and lumps, lumps and line solitons fusing into line solitons, or a mixture of fusion and fission of lump and line soliton.

The rogue wave structure and the formation mechanism in the framework of coupled nonlinear Schrödinger equations have been studied by Li *et al.* [239]. In that work, by adjusting the parameters of the nonlinear dynamical model, different types of rogue wave structures, namely, bright, dark, and eye-shaped rogue waves have been obtained. Breather solutions for a six-component Ablowitz-Kaup-Newell-Segur (AKNS) system with a negative flow have been studied by Shen *et al.* [240]. Darboux transformation for the AKNS system has been constructed by setting a restrictive condition on two spectral functions. Three types of breather solutions for the AKNS system have been obtained, including the space-time periodic solution, Kuznetsov-Ma breather solution, and Akhmediev breather solution. A special kind of Akhmediev breather that is composed of bright-dark lump waves has been put forward in Ref. [240]. Kaur and Wazwaz [241] have investigated bright-dark lump wave solutions for a new form of the (3+1)-dimensional BKP-Boussinesq equation. By employing the Hirota bilinear form, they have explored a variety of lump solutions, generated from quadratic functions [241].

In a recent work, Crabb and Akhmediev [242] have studied the rogue wave multiplets in the complex Korteweg-de Vries equation. A multi-parameter family of rational solutions to the complex Korteweg-de Vries equation has been obtained. The family of solutions obtained by Crabb and Akhmediev [242] includes some particular cases with high-amplitude peaks at the centre, as well as a multitude of cases in which higher-order rogue waves are partially split into lower-order fundamental components. Hou *et al.* [243] have investigated the sine-Gordon breathers and formation of extreme waves in self-induced transparency media, within the framework of the sine-Gordon equation. It has been revealed that the extreme waves in that physical settings occur as a result of an intense interaction between individual breather components, by an optimal choice of the breathing periods and the propagation directions for each component [243]. In a recent paper, Ankiewicz *et al.* [244] have revisited the Gardner equation in order to understand the general rogue wave solutions of this nonlinear partial differential equation. Interesting structures revealed by the rational solutions of the Gardner equation have been revealed by Ankiewicz *et al.* [244]. The obtained solutions have applications in the studies of internal ocean waves and dusty-type plasmas; see Ref. [244].

The dynamics of pole trajectories in the complex plane of the Peregrine solitons for higher-order NLS equations has been investigated by Peng *et al.* [245]. It was shown in Ref. [245] that the poles of the Peregrine soliton travel down and up the imaginary axis in the complex plane, and at the turning point of the pole trajectory, the real part of the complex variable coincides with the location of maximum height of the rogue wave in physical space. Ye *et al.* [246] have obtained exact Peregrine soliton solutions on a periodic-wave background caused by the interference in the vector cubic-quintic NLS equation involving the self-steepening effect.

The interesting problem of rogue waves arising on the double-periodic background in the focusing NLS equation has been investigated independently by Chen *et al.* [247] and by Conforti *et al.* [248]. Yang *et al.* [249] have analyzed the temporal and spectral characteristics of the Akhmediev breather and have revealed the amplification and transmission of attenuated multi-solitons in nonlinear optical fibers. The analytical and numerical calculations have been based on the specific spectral characteristics of the Akhmediev breather; see Ref. [249]. The generation and propagation of hyperbolic secant solitons, Peregrine solitons, and various breathers in a coherently prepared three-level atomic system have been investigated by Guan *et al.* [250]. The high-order rogue waves excited from multi-Gaussian perturbations on a continuous wave have been studied numerically by Gao *et al.* [251]. It has been revealed that the maximal intensity approaches 63.8 times that of the power of the initial background wave, and it also retains a large value under the influence of fiber loss and noise [251].

Recently, Sullivan *et al.* [252] have studied the Kuznetsov-Ma breather-like solutions in the Salerno model, which is a discrete variant of the celebrated NLS equation interpolating between the discrete NLS equation and the completely integrable Ablowitz-Ladik model. The main scope of the work by Sullivan *et al.* [252] has been to investigate in detail the existence and stability of time-periodic solutions of the Salerno model. Panajotov *et al.* [253] have shown that optical rogue waves are controllable by means of time-delayed feedback and optical injection, thus the control of dissipative rogue waves in nonlinear cavity optics is feasible. In addition it has been revealed by Panajotov *et al.* [253] that the optical injection may suppress the rogue wave formation in semiconductor lasers with saturable absorbers. Two-dimensional rogue waves on zero background in a Benney-Roskes model have been studied by Guo *et al.* [254]. The analytically obtained solutions, which are localized in both space and time, can be viewed as a two-dimensional analog of the Peregrine soliton; see Ref. [254]. I also refer here to several relevant works in this broad research area, reporting different types of rogue-wave structures and other localized waveforms in many physical contexts [255–269].

A lot of experiments on rogue (freak) waves have been reported during recent years. It should be mentioned here the work by Chabchoub *et al.* [270] on the observation of slanted solitons and breathers propagating at an angle with respect to the direction of propagation of the wave field. The experimental results of Chabchoub *et al.* [270] are based on the theoretical predictions obtained from the (2D+1) hyperbolic NLS equation for deep-water waves. The unstable waveforms in this physical setting explain the formation of directional large-amplitude rogue waves with a finite crest length within a wide range of nonlinear dispersive media. The recent research progress on optical rogue waves in fiber lasers has been reviewed by Song *et al.* [271]. It should be pointed out here that fiber lasers that have unique nonlinear dynamics provide an ideal platform to observe the formation of rogue waves, that is, high amplitude localized wave packets, in optical settings. Xu *et al.* [272] have recently reported the observation of a novel type of breather interaction in telecommunication optical fibers, in which two identical breathers propagate with opposite group velocities. The special interaction of breathers, namely the so-called ghost-like breather interaction has been fully described by an N-breather solution of the NLS equation [272]. Recently, Finot [273] has reported experimental results on Peregrine soliton-like structures in normally dispersive fibers. The emergence of a highly peaked structure on a continuous background was not linked to any Peregrine soliton dynamics and was mainly ascribed to the impact of the Brillouin backscattering; see a detailed study of this issue in Ref. [273].

5. MATTER-WAVE LOCALIZED STRUCTURES

In this Section I briefly overview some recent theoretical and experimental studies of physical properties of matter-wave localized structures in atomic BECs; see, for example, a short review paper by Malomed [274] on the creation of matter-wave solitons by means of spin-orbit coupling, an overview paper by Pang *et al.* [275] on 2D vortex solitons in spin-orbit-coupled dipolar BECs, and a recent comprehensive review on spatiotemporal engineering of matter-wave solitons in BECs [276].

In the past few years there have been published a series of theoretical and experimental works in the area of the so-called *quantum droplets*, with the aim of creation of stable multidimensional soliton-like structures in BECs. The quantum droplets that have been predicted in a seminal paper by Petrov [49], are self-bound states in mixtures of two BECs; see also the influential earlier work by Lee, Huang, and Yang [277]. More precisely, the quantum droplets are robust two- and three-dimensional self-trapped states in BECs that are stabilized by effective self-repulsion induced by quantum fluctuations around the mean-field states.

It should be pointed out here that the three-dimensional droplets with embedded vorticity have been investigated by Kartashov *et al.* [278]; for two recent overviews of this topic, see Kartashov *et al.* [19] and Luo *et al.* [279]. The interesting problem of spontaneous symmetry breaking of quantum droplets in dual-core traps has been recently investigated by Liu *et al.* [280]. Recently, Zin *et al.* [281] have revisited the problm of stability of two-component quantum droplets beyond approximations that rely on single-component approximation. It has been shown in Ref. [281] that the densities of the two components of a stable droplet are limited to a range depending on the interaction strength. Kartashov *et al.* [282] have predicted that the Lee-Huang-Yang effect [277] makes it possible to create stable quantum droplets in binary BECs with a heterosymmetric or heteromultipole structure, that is, for different vorticities or multipolarities in their components. In a recent work, Lin *et al.* [283] have studied two-dimensional vortex quantum droplets that are trapped by a thicker transverse confinement. It has been revealed in Ref. [283] that stable two-dimensional vortex quantum droplets with topological charge number up to at least four can be supported in that physical setting. Self-bound droplet clusters in laser-driven BECs have been recently investigated by Zhang *et al.* [284].

In the experimental area, I mention here that nearly two-dimensional [285, 286] and fully three-dimensional [287, 288] quantum droplets have been created in BECs containing ^{39}K atoms.

The problem of Faraday and resonant waves in dipolar cigar-shaped BECs has been investigated by Vudragović and Balaž [289] using a mean-field variational method and a full numerical approach; see also earlier relevant contributions in the area of Faraday waves in BECs [290–297].

The formation and stability properties of gap-type dark localized modes in a BEC trapped in an optical lattice have been studied analytically and numerically by Liangwei Zeng and Jianhua Zeng [298]. Two types of stable dark localized modes, namely gap solitons and soliton clusters, have been predicted in both one- and two-dimensional settings; see Ref. [298]. Liangwei Zeng and Jianhua Zeng [299] have also introduced a model of nonlinear Kerr (cubic) media of ultracold atoms with spatially modulated repulsive interactions, which is supporting a vast variety of stable flat-top matter-wave solitons, including one-dimensional flat-top fundamental and multipole solitons, and two-dimensional flat-top fundamental and vortex solitons. This dynamical model can be implemented in both nonlinear optics and BEC settings. In a recent work, a rigorous mathematical study of vortex solitons in attractive two-dimensional BECs has been reported by Jianfu Yang and Jinge Yang [300]. Kartashov *et al.* [56] have shown that attractive two-dimensional spinor BECs with helicoidal spatially periodic spin-orbit coupling support a rich variety of stable fundamental solitons and bound soliton complexes. The problem of the reduction of the three-dimensional dynamics of BECs, under the action of strong confinement in one direction has been recently revisited [301].

Harko *et al.* [302] have studied the dynamics of vortices with arbitrary topological charges in weakly interacting BECs, using the Adomian decomposition method to solve the Gross-Pitaevskii equation in polar coordinates. In a recent work, Ravisankar *et al.* [303] have published the OpenMP versions of FORTRAN programs for solving the Gross-Pitaevskii equation for harmonically trapped three-component spin-1 spinor BECs in both one and two spatial dimensions and with or without spin-orbit and Rabi couplings.

On the experiment arena, I mention here the work by Kono *et al.* [304] on emergent critical phenomenon in spin-$1/2$ ferromagnetic-leg ladders and its relationship with the quasi-one-dimensional BEC, and the work by Luo *et al.* [305] on the creation and characterization of matter-wave breathers. The creation of quasi-one-dimensional excited matter-wave solitons (breathers) has been demonstrated to be feasible by quenching the strength of the interactions in a BEC with attractive interactions; see Ref. [305] for details. The observation of multiple dark-antidark solitons in two-component repulsively interacting $^{87}\mathrm{Rb}$ BEC has been reported by Katsimiga *et al.* [306]. Also, the existence, stability, and dynamics of different multisoliton dark-antidark states obtained in the framework of the theoretical model developed in Ref. [306] have been analyzed in detail.

6. CONCLUSION

The objective of this article has been to provide an overview of recently obtained theoretical and experimental results in the study of localized structures in optical and matter-wave media.

This paper has been structured, essentially, as a resource text, which provides a large number of references to relevant publications reporting recent advances in the broad research field of localized structures that form in diverse optical media and Bose-Einstein condensates. In particular, I have included in this overview new findings concerning light bullets, the creation and diverse applications of few-cycle (ultra-narrow) optical pulses, the emergence of rogue waves in various media, and the creation of quantum droplets in Bose-Einstein condensates.

The survey of the recently reported theoretical and experimental results clearly demonstrates that there remains a vast room for further studies of solitons and related self-trapped states in optics and photonics, Bose-Einstein condensates, and other quickly developing areas of pure and applied physics.

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