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ULTRASHORT OPTICAL PULSES IN PHOTONIC CRYSTAL WITH SUPERLATTICE AND DEFECTS

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Abstract. In this paper, we study the propagation of ultrashort optical pulses in superlattice with defects, placed in a photonic crystal. The dependence of the pulse transmission coefficient both on the parameters of the defect (depth and size) and on the parameters of the photonic crystal (depth and modulation period of the refractive index) is analyzed.

Key words: ultrashort pulses, superlattice, defect, photonic crystals.

1. INTRODUCTION

The progress of modern electronics is associated with a size reduction of electronic equipment, as well as the use of new materials with unique physical properties. From this point of view, one of the most promising candidates are materials with a controlled band spectrum, *i.e.* superlattices [1]. A superlattice is understood as a structure in which, in addition to the lattice potential, an artificially created potential acts on electrons (with a period significantly exceeding the lattice period) [2, 3].

The study of such structures is of great interest, both from the fundamental [4] and practical points of view [5]. The scientific component is due to the possibility of studying various physical effects in them (for example, the processes of localization and scattering of current carriers [6], quantum optical properties [7], and electronic energy spectrum [8]). The practical interest is due to reaching a qualitatively new level in the development of optical devices, for example, high-efficiency semiconductor lasers [9-10]. The proximity of the energy spectrum of a solitary quantum dot to atomic levels makes it possible to create one-electron transistors and memory elements on their basis [11].

Another important problem is the stable propagation of localized structures [12, 13], including ultrashort optical pulses [14-17] in a medium with a spatially variable refractive index [18, 19], and in particular, in a photonic crystal [20, 21].

In previous studies, the authors have demonstrated the propagation of such pulses while maintaining their localization in a photonic crystal based on carbon nanotubes [22], as well as the possibility of efficient generation of higher harmonics under the action of a magnetic field. In this work, we investigate the propagation of electromagnetic radiation in a medium of quantum dots with defects placed in photonic crystal.

2. MODEL AND BASIC EQUATIONS

We consider an alternating electric field propagating in a photonic crystal with a system of quantum dots in the geometry shown in Fig. 1.

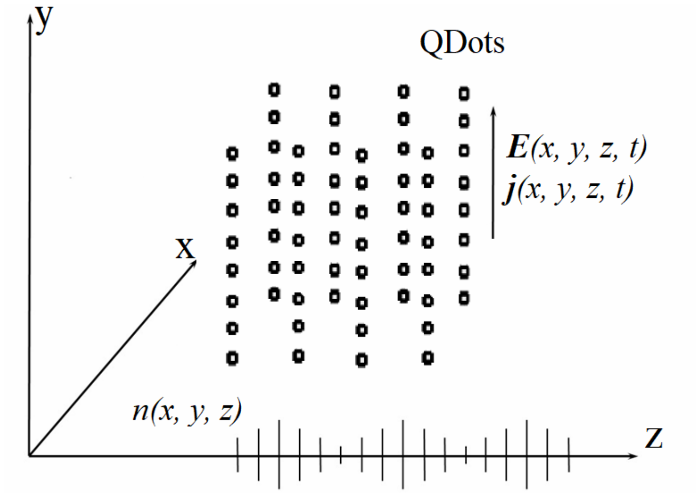

Fig. 1 – The geometry of the problem. The dashes along the z-axis conventionally show the change in the refractive index along this axis.

The dispersion law for a semiconductor superlattice has the form:

$$\varepsilon(p) = t_0 + 2t\cos(a \cdot p), \tag{1}$$

where p is the momentum along the y-axis, a is the distance between adjacent quantum wells along the y-axis, t_0 is the quantum well electron energy, and t is the transition integral determined by the overlap of electronic wave functions in neighboring quantum wells [23]. Note that we consider the system of quantum wells such that the electron tunneling between them occurs along the y-axis, while tunneling in other directions can be neglected. This determines the dispersion law of electrons (1), taking into account the fact that the quasimomentum p is directed along the y-axis.

The electromagnetic field in our model (Fig. 1) is described classically, taking into account Maxwell's equations. Choosing $\boldsymbol{E} = -c^{-1}\partial \boldsymbol{A}/\partial t$ and taking into account the dielectric and magnetic properties of the medium, we write the Maxwell's equations in the form:

$$\frac{\partial^2 \boldsymbol{A}}{\partial x^2} + \frac{\partial^2 \boldsymbol{A}}{\partial y^2} + \frac{\partial^2 \boldsymbol{A}}{\partial z^2} - \frac{n^2(x,y,z)}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \frac{4\pi}{c}\boldsymbol{j} = 0, \tag{2}$$

where $n(x, y, z)$ is the spatially modulated refractive index, *i.e.* a photonic crystal with modulation of the refractive index, and $\boldsymbol{j}$ is the current due to the action of the electric field of the pulse on electrons in the superlattice. The vector potential has the form: $\boldsymbol{A} = (0, A_y(x, y, z, t), 0)$. The electric field is assumed to be directed along the y-axis, and c is the light speed in a semiconductor matrix containing quantum dots.

The electric field of the substrate is not taken into account here. In this model, we do not take into account interband transitions. Thus, we limit the frequency of laser pulses, which lies in the near infrared region. Since the typical size of the superlattice and the distance between quantum dots are much smaller than the typical size of the spatial region in which the ultrashort pulse is localized, we can use the continuous medium approximation and we assume that the current is distributed over the volume. The typical length at which the refractive index for a photonic crystal changes significantly is larger and does not introduce additional restrictions.

Since the typical relaxation time for electrons in the superlattice τ can be estimated as $10^{-12} – 10^{-13}$ s [24], the ensemble of electrons at times typical for the dynamics of ultrashort optical pulse (of the order 10^{-14} s) can be described using the collisionless kinetic Boltzmann equation.

Let us write the current density $\boldsymbol{j} = (0, j_y, 0)$:

$$j_y = \frac{q}{\pi}\sum_s \int \mathrm{d}p_y v_y f\,, \tag{3}$$

where we introduce the group velocity of electrons: $v_y = \partial\varepsilon(p)/\partial p$, and f is the electron distribution function. We solve Eq. (3) by the method of characteristics:

$$j_y = \frac{q}{\pi}\sum_s \int_{-q_0}^{q_0} \mathrm{d}p_y v_y\left(p_x, p_y - \frac{q}{c}A(t), p_z\right)F_0 \tag{4}$$

Integration in (4) is carried out in the first Brillouin zone and $q_0 = 2\pi/a$.

According to the calculations presented in [25], the final effective equation can be represented in the form:

$$\frac{\partial^2 A_y}{\partial x^2}+\frac{\partial^2 A_y}{\partial y^2}+\frac{\partial^2 A_y}{\partial z^2}-\frac{n^2\left(x,y,z\right)}{c^2}\frac{\partial^2 A_y}{\partial t^2}+\frac{qb}{\pi\hbar\tau}\sin\left(\frac{aq}{c}A_y\right)=0,$$
$$b=\int_{-q_0}^{q_0}\mathrm{d}p_y\cos\left(ap_y\right)\cdot F_0 \tag{5}$$

Passing to a cylindrical coordinate system and taking into account that charge accumulation can be neglected [26], we can write the effective equation in the following form:

$$\frac{\partial^2 A}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)-\frac{n^2\left(r,z\right)}{c^2}\frac{\partial^2 A}{\partial t^2}+\frac{qb}{\pi\hbar\tau}\sin\left(\frac{aq}{c}A\right)=0,$$
$$r=\sqrt{x^2+y^2} \tag{6}$$

Equation (6) is solved numerically using a direct finite-difference scheme of the cross type. The time and coordinate steps are determined from the standard stability conditions, and are decreased until the solution changed in the eighth significant digit. Here we assume that $\partial / \partial\varphi \rightarrow 0$ due to cylindrical symmetry because the charge accumulation can be neglected [26].

3. NUMERICAL SIMULATION RESULTS

The initial conditions for the vector potential are:

$$A\left(r,z,t=0\right)=Q\cdot\exp\left(-\frac{r^2}{\gamma_r^2}\right)\exp\left\{-\left(\frac{z-z_0}{\gamma_z}\right)^2\right\},$$
$$\frac{\mathrm{d}A\left(r,z,t=0\right)}{\mathrm{d}t}=\frac{2\cdot Q\cdot u\cdot\left(z-z_0\right)}{\gamma_z^2}\cdot\exp\left(-\frac{r^2}{\gamma_r^2}\right)\exp\left\{-\left(\frac{z-z_0}{\gamma_z}\right)^2\right\}, \tag{7}$$

where Q is the pulse amplitude; parameters γ_z, γ_r determine the pulse width along the z and r axes, u is the initial velocity of the pulse, and the refractive index is modeled as:

$$n\left(r,z\right)=n_0\left(1+\alpha\cos\left(2\pi z/\chi\right)\right)\cdot\left(1-d\cdot\exp\left(-\left(\frac{z-z_0}{g}\right)^2\right)\right), \tag{8}$$

where n_0 is the average refractive index, α is the refractive index modulation depth, χ is the refractive index modulation period, d defines the depth of the defect, *i.e.*

how much the periodic structure of the refractive index is violated, and g determines the defect size, *i.e.* how strongly the region with violation of the refractive index is limited in space.

The results of numerical simulations of the dynamics of three-dimensional ultrashort pulse in a photonic crystal with a superlattice and defects are presented in Fig. 2.

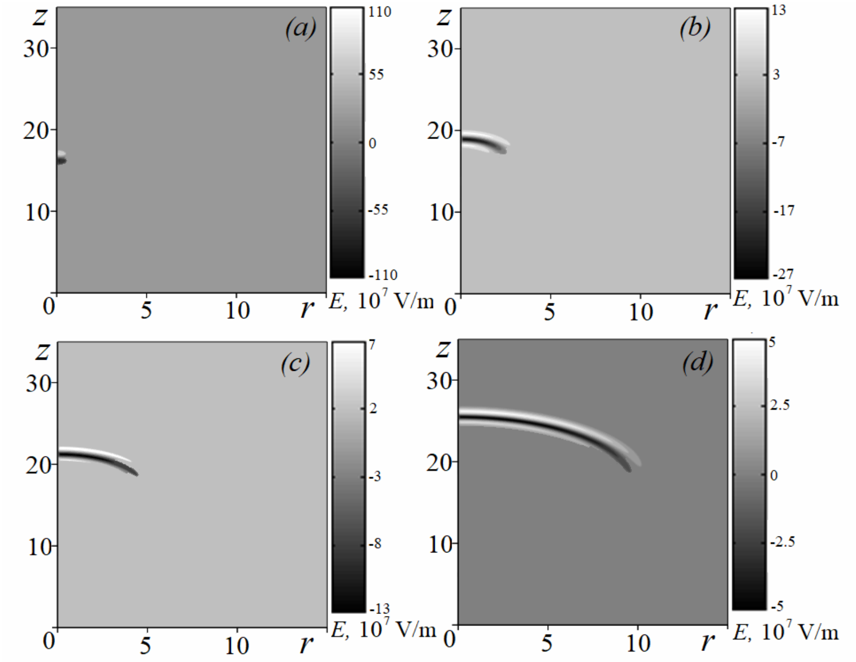

Fig. 2 – The evolution of a three-dimensional extremely short optical pulse at different instants of time: a) 0.0002 ps; b) 1 ps; c) 5 ps; d) 10 ps. The dimensionless unit along the r and z axes corresponds to $2 \cdot 10^{-5}$ m.

It can be seen that the pulse propagates rather stably in a superlattice placed in a nonideal photonic crystal, experiencing diffraction spreading over time. However, in spite of the presence of the defect in the photonic crystal, the energy of the three-dimensional ultrashort optical pulse remains localized in a limited spatial region. The dispersion spreading along the pulse propagation axis is compensated by the nonlinearity of the medium (superlattice). It should be noted that the effect of the modulation depth of the refractive index of the photonic crystal manifests itself in a change in the pulse shape. An increase in the modulation period of the refractive index leads to an increase in the group velocity of the pulse wavepacket. This is due to the fact that the processes of interference in the nodes of the photonic crystal occur less frequently. It is obvious that with an infinite period, the group velocity of the wavepacket of the pulse will be maximum.

These results are repeatedly confirmed in works devoted to the propagation of ultrashort pulses in the photonic crystal based on carbon nanotubes, which also have a nonparabolic electron dispersion law [19, 27].

In Figs. 3 and 4 we show the effect of the depth and size of the defect on the ultrashort pulse, namely, on the transmission coefficient K_{tr}:

$$K_{tr} = \frac{I_{pass}}{I_0}, \tag{9}$$

where I_{pass} is the intensity of the electric field of the transmitted pulse and I_0 is the the intensity of the electric field of the pulse at the initial moment of time.

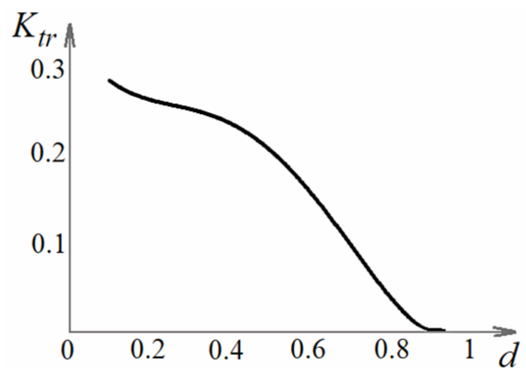

Fig. 3 – The dependence of the pulse transmission coefficient on the defect depth (to determine d, see (9)).

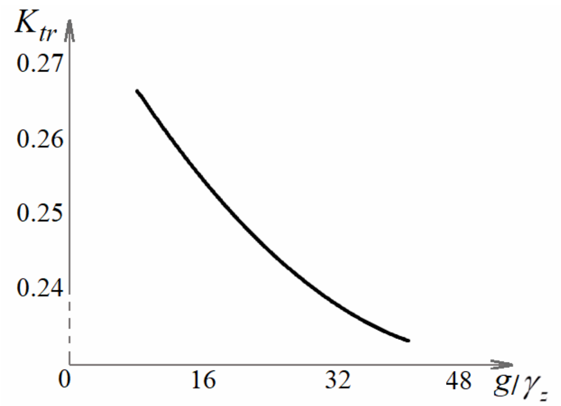

Fig. 4 – The dependence of the pulse transmission coefficient on the defect size.

From Figs. 3 and 4 we can conclude that there is a monotonic dependence of the transmission coefficient of the electromagnetic wave on the defect parameters.

Moreover, in both cases, an increase in these parameters (depth and size of the defect) leads to a decrease in the amplitude of ultrashort pulse during its propagation in a superlattice placed in the photonic crystal.

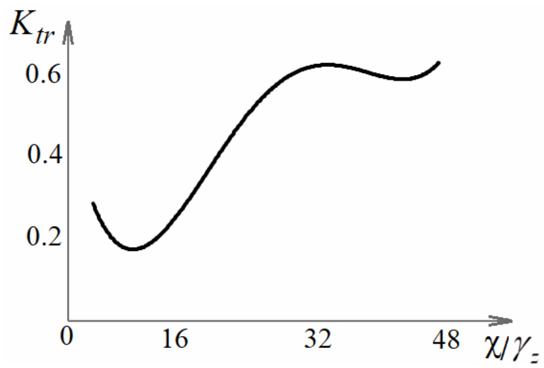

Fig. 5 – The dependence of the pulse transmission coefficient on the modulation period of the refractive index. For the defect parameters: d = 0.1, g =1.5 μm.

The dependence of the transmission coefficient of the electromagnetic wave on the modulation period of the refractive index is nontrivial, which is associated with the processes of the wave reflection at the boundaries of the defect. In this case, additional reflected waves are formed, and their interference with the incident pulses in antiphase leads to a decrease in the transmission index.

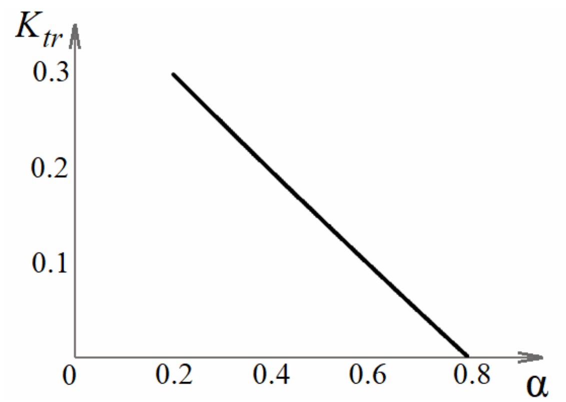

Fig. 6 – The dependence of the pulse transmission coefficient on the modulation depth of the refractive index (t = 10 ps). For the defect parameters: d = 0.1, g = 1.5 μm.

Figure 6 shows that with an increase in the modulation depth, a decrease in the pulse amplitude is observed. This dependence is close to a linear one.

The results obtained are very promising, since they make it possible to control the value of the pulse energy passing through the defect. This plays an important role for practical applications, for example, in information transmission devices, as well as in photonics, optoelectronics, and nanoelectronics.

4. CONCLUSIONS

The following main conclusions of this work can be drawn:

1. A three-dimensional ultrashort optical pulse propagates stably in the medium of a photonic crystal with superlattice and defect.
2. The parameters of the refractive index of a photonic crystal (period and depth of modulation) with superlattice have a significant effect on the shape and amplitude of the ultrashort pulse and can lead to its complete reflection.
3. Along with the refractive index, the control parameters of the pulse include the depth and size of the defect, which also make it possible to control the pulse transmission coefficient.

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