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VISION OF PLASMONICS FROM MICROWAVE AND OPTICAL RANGES

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Abstract. This article analyzes the various traditions that are applied in electromagnetic field theory by experts from different branches of physics, the same results of the study may look different. Here is compared the style of presentation of dispersion characteristics and also the frequency measurement units that may not match entirely in microwave and optical ranges. We explain the causes and logic of such discrepancies. Time-dependent and position-dependent solutions of the Helmholtz equation are often with opposite signs found in literature on microwave electrodynamics and optics including plasmonics. As a result, the intermediate and final expressions may contain opposite signs. A detailed solution to the wave equation and the reasons for the appearance of opposite signs has been made here.

Key words: EM wave propagation, plane wave, light, microwaves, dispersion characteristic, Maxwell equations, wave equation, Helmholtz equation, signs of solutions, units of frequency, electromagnetics, plasmonics, optics.

1. INTRODUCTION

We will consider here non-matching approaches that appeared and became traditional in the microwave electrodynamics based on the Maxwell's equations and the electromagnetic (EM) field theory, which is used in physical optics as well as in its branch called plasmonics. The microwave electrodynamics is a part of EM field theory because the last one covers all frequency ranges.

Plasmonics is a modern field of physics that contains knowledge related to several in-touch areas such as condensed matter physics, physical optics, quantum mechanics, and atomic and molecular physics [1] – [5]. Plasmonics studies the interaction between light (photons, EM waves) and free (conduction) electrons in conductors (metals, semimetals, semiconductors). The photon is the quantum of the EM field including light. The free electrons in conductors excited by electric field components of EM wave (light) can have collective oscillations with a frequency close to the frequency of acting light. Plasma oscillations are fast oscillations of the free electron density in conducting media. Plasmonics is based on a physical phenomenon called plasmon. Plasmons by their physical nature are a composition of free electron plasma and the acting EM wave.

The surface plasmon waves are often used in the next-generation modern devices. The surface plasmon wave consists of the upper and lower parts separated by the boundaries of the two environments. Both parts of the plasmonic wave propagate at the same speed, which may be lower than the speed of an EM wave inside a dielectric (air) environment. We could imagine a wave of surface plasmons resembling a horse running through the shallow waters of a river. The lower part of the horse is in the water, thus causing concentric circles of water waves. The head of the moving horse creates an air swirl. The horse's body is in two environments at the same time. The horse, which we can compare with a plasmon wave, moves forward in the required direction as something single whole. The plasmonic wave moving along a dielectric-metal interface (surface) is also called the surface plasmon polariton (SPP) [6]. The polariton can be defined as the strong coupling of photon with another quasiparticle.

By definition, the microwave electrodynamics is the branch of physics that deals with relatively fast changing EM fields. This corresponds to ranges: SHF (super high frequency), EHF (extremely high) and THF (tremendously high) with the general frequency range $f = 3$ GHz (gigahertz, 10^9 Hz) – 3 THz (terahertz, 10^{12} Hz) and wavelength λ = 10 cm to 100 μm (micrometer, 10^{-6} m) in free space (in a vacuum).

By definition optics is the branch of physics that studies the features of light, its interactions with matter and applications created on this base. Optics is the science of light that includes the EM wave spectrum as IR (infrared, 0.3 THz – 0.43 PHz), VIS [visible, (0.43 – 0.79) PHz], UV [ultraviolet, (0.79 – 30) PHz] and the light is characterized by the frequency f=0.3 – 3000 THz and wavelength λ = 1 mm to 10 nm (nanometer, 10^{-9} m) in free space. Let us remind you that 1 petahertz (1 PHz) = 10^{15} Hz = 10^6 GHz = 10^3 THz.

The microwave electrodynamics [7], [8], [9] and physical optics [10], [11], [12] deals with EM waves and they are both based on the Maxwell's equations. It is interesting to note that the signs in the expressions describing the phenomena of plasmonics obtained by experts of microwave and optical ranges are often opposite.

The purpose of this article is to show the differences caused by the traditions of the mentioned areas of physics, and to find out at what stage of the solution occur differences in the signs.

Here we will analyze a few important differences that have to face the readers of technical literature concerning three issues: 1) dispersion characteristics; 2) units of frequency; 3) choosing opposite signs for position (spatial) terms as well for time (temporal) terms in solving the same wave equations. We are going to start parsing the differences from the simplest issue about graphs of characteristics.

2. GRAPHICS OF DISPERSION CHARACTERISTICS

Dispersion characteristics are usually turned upside down in the literature on plasmonics (optics) in comparison with the microwave electrodynamics. In the literature on plasmonics [13] – [18], is often used the dispersion characteristic in the form $f = F(k)$, where the frequency f is a function on the propagation constant (wave number) k. Instead of the frequency f can also be used the angular (cyclic) frequency $\omega = 2\pi f$ or other normalized value such as ω/ω_p, where ω_p is the plasma frequency of the conduction electrons [1], [4], [9], [12].

In Fig. 1 are given the dispersion characteristics for the layered structure with parameters taken from [6]. The structure contains the following substances: the environment is air with the relative permittivity ε_1 = 1.0, the metal film with the thickness d = 30 nm is made of silver (Ag) at the relative permittivity that depends on the frequency $\varepsilon(\omega)$. The plasma frequency for silver is $\omega_p \approx 11.9989 \cdot 10^{15}$ s^{-1}. The bottom layer of the structure is made of the dielectric material with ε_2 = 2.25.

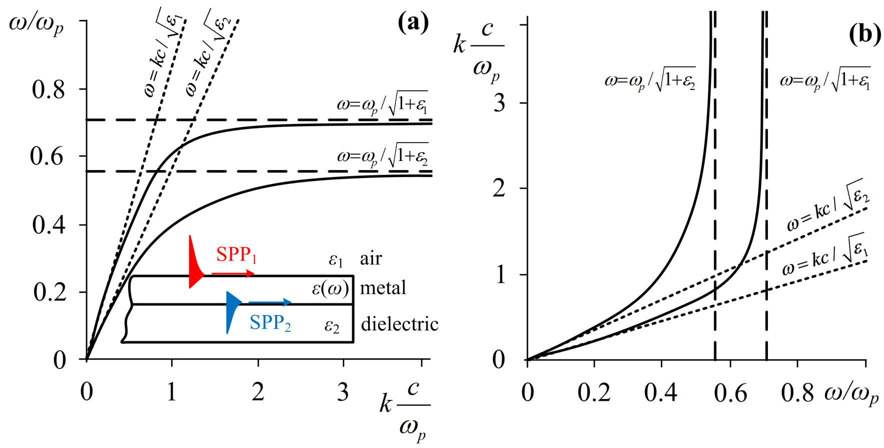

Fig. 1 – The dispersion characteristic of the SPP waves propagating on a layered structure containing metal-air and metal-dielectric interfaces on which SPPs are excited for cases: (a) in the form commonly used in the optical range and (b) in the microwave range.

Figure 1(a) corresponds to the tradition dispersion characteristic accepted in physical optics as [6] and Fig. 1(b) represents to the same characteristic but in the form usually accepted in microwave electrodynamics. In Fig. 1(a) on the ordinate (vertical) axis is placed the magnitude proportional to the normalized cyclic frequency ω/ω_p and on the abscissa (horizontal) axis is located a magnitude proportional to the propagation constant k.

In the literature on microwave electrodynamics [19] – [25] the dispersion characteristics represent the dependency $k = F(f)$, *i.e.* the propagation constant k (or other value proportional to its) is located on the ordinate axis and frequency f (or other value proportional to it) is placed on the abscissa axis (see Fig. 1(b)). Although you can see sometimes the use of two approaches in the presentation of dispersion characteristics in the same article [26].

3. VARIETY OF FREQUENCY DESIGNATION UNITS

The different units can be used for the frequency of EM waves in examinations of appropriate physical values in the microwave range and the optical range in the relevant technical literature. Throughout history, the development of microwave waveguide and antenna technologies evolved from radio and microwave frequencies with a tendency to move into higher frequencies. In the microwave range, the frequency units are only in hertz (Hz) or multiples such as kHz, GHz, THz, and PHz [7] – [9].

In the optical range for the frequency can also be used the energy unit equivalents as electronvolts (eV), kelvin (K) and the reciprocal centimeter (cm^{-1}). It is easy to find Energy Unit Conversions on the Internet [27].

Historically, the optical devices typically operate at relatively high frequencies f = 0.3 THz – 30 PHz of EM spectrum. In recent years there is a tendency in optics to develop devices that can also operate at lower frequencies. The optics was the basis for successful development of such advanced scientific research branches as plasmonics, photonics, and spectroscopy. Therefore, in optics as well as in plasmonics the measurement of frequency additionally occurs in eV, which is the amount of kinetic energy obtained (or lost) by an electron accelerating from rest through an electric potential difference of one volt in vacuum, also in K, which is the basic unit of temperature and cm^{-1}, which is a unit of wavenumber measured in cycles per unit distance or radians per unit distance [28] – [31]. Sometimes are used the normalized frequency units [32], [33].

Let us consider the reasons why additional units of measurement for frequency are used in optics.

As we know light is an EM wave that is oscillating with a frequency f in IR, VIS, and UV ranges. But light is also a particle, which is called a photon and each photon carries a packet of energy that is proportional to the frequency f. The substances consist atoms and molecules and can absorb the energy from a photon. The photon is the smallest discrete amount or quantum of EM radiation (light). The energy of a photon E can be expressed such as:

$$E = h_{pl} f = \hbar\omega = h_{pl}\frac{c}{\lambda}, \tag{1}$$

where $h_{pl} \approx 4.13567 \cdot 10^{-15}$eV·s is the Planck's constant, $\hbar = h_{pl}/2\pi$ is the reduced Planck constant, $f\lambda = c$, c is the speed of light in a vacuum (m/sec), λ is the wavelength of EM wave (light) (m).

In solids, the atoms (molecules) vibrate about their equilibrium positions of their lattice nodes. Based on the law of equipartition of energy, the average energy of a simple harmonic oscillator is $E_k \rightarrow k_b T$, where k_b is the Boltzmann constant and T is the absolute temperature (K). The equipartition theorem relates the temperature of a system to its average energies [34], [35], see Table 1.

Table 1

Simple relations $f \propto E, k, T$.

Magnitude	Formula	Units	Equation
Energy, E	$E = h_{pl} f$	eV	(2)
Wavenumber, k	$k = 1/\lambda = f/c$	cm^{-1} (m^{-1})	(3)
Frequency, f	$f = c(1/\lambda)$	Hz	(4)
The average (kinetic) thermal energy, E_k	$E_k \rightarrow k_b T$ where $k_b = 8.6 \cdot 10^{-5}$ eV/K is the Boltzmann constant, T is the temperature	K	(5)

We can express the frequency from Eq. (2) as $f = E/h_{pl}$, which means that the frequency f is proportional ($\propto$) to the energy E, *i.e.* $f \propto E$ in units eV, from Eq. (3) $f \propto k$ in units cm^{-1}, along with this using Eq. (5) $f \propto T$ in units kelvin (K). Here we give several relationships between the most commonly used in plasmonics (optics) units: 1 THz = 0.00414 eV, 1 THz corresponds to the wavelength of 300 μm. 1 eV = 241.8 THz. 1 eV corresponds to the wavelength $\lambda = 1.2398$ μm, 2 eV corresponds to $\lambda = 0.620$ μm, 6 eV corresponds to $\lambda = 0.207$ μm etc. Magnitudes equal 1 cm^{-1} = 123.98 μeV= 0.0299795 THz. The relations between the various units that are used in the scientific literature on plasmonics are given in Table 2.

Table 2

Conversion of values eV, Hz, cm^{-1}, and K.

	eV (electron volt)	Hz (Hertz)	cm^{-1}	K (kelvin)
eV	1	$2.41804 \cdot 10^{14}$	8065.73	11604.9
cm^{-1}	$1.23981 \cdot 10^{-4}$	$2.99793 \cdot 10^{10}$	1	1.42879
Hz	$4.13558 \cdot 10^{-15}$	1	$3.33565 \cdot 10^{-11}$	$4.79930 \cdot 10^{-11}$
K	$8.61705 \cdot 10^{-4}$	$4.79930 \cdot 10^{-11}$	0.695028	1

We use definitions and quantities here that are usually applied in quantum mechanics and optics. Quantum mechanics is a branch of physics dealing with physical phenomena that refer to structures with sizes of nanoscales (1–100 nm).

4. COMMENTS ABOUT THE IMAGINARY UNIT

Reading the scientific literature we are faced with another ambiguity, related to the definition of the imaginary unit from which we can get opposite signs in expressions of EM field theory and other branches of physics.

The complex number representation gives an alternative description that simplifies the mathematical procedures. Complex exponentials are used extensively in EM field theory, physical optics, and classical and quantum mechanics. Euler's formula gives the fundamental relationship between the trigonometric functions and the complex exponential function:

$$e^{ix} = \cos x + i \sin x, \tag{6}$$

where e is the base of the natural logarithm, "i" is the imaginary unit, the argument x is a real number, and $\cos(\cdot)$ and $\sin(\cdot)$ are the trigonometric functions cosine and sine, respectively. The complex unit usually is taken as $i = \sqrt{-1}$, [10], [12], [36], [37].

First of all, Ref. [38] attracts attention to the discrepancy in the literature regarding the use of different definitions of the imaginary unit. In [38] is shown that the designation of the imaginary unit in engineering is usually "j" and in physics/science the designation "i" is applied, which differs in sign and the ratio can be $j \leftrightarrow -i$.

In [39] is taken a sign coefficient $c = \pm 1$ which is a multiplier of the imaginary unit to show a possibility to choose the desired sign from the two signs offered by the solution of the wave equation.

It is usefully to note that in some publications, *e.g.* [40], the imaginary unit marked $i = \sqrt{-1}$, *i.e.* $j \leftrightarrow i$. These definitions can be changed by simply reversing the signs of the imaginary unit, *i.e.* $\pm i \to \mp i$ [41].

The different definitions of the imaginary unit leads to the emergence of opposite signs in important expressions, such as the complex permittivity of materials, the one can be presented as $\varepsilon = \varepsilon' - i\varepsilon''$ or $\varepsilon = \varepsilon' + i\varepsilon''$, the same happened with the permeability, refractive index, propagation constant, terms for right and left-circular polarizations, Fourier series, Fourier transformation etc.

For this reason, we have to pay attention to the value of the imaginary unit in the publications.

5. COMMON REASONING

Since solutions to wave equations derived from Maxwell's equations may contain expressions with opposite signs, we will consider the stages of their obtaining. The Faraday's and Ampère's law equations in microwave and optics ranges are the same [7], [12], [19]:

- Maxwell-Faraday equation, in microwave and optical ranges of frequency:

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}. \tag{7}$$

- Maxwell-Ampère equation, in microwave and optical ranges of frequency:

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}. \tag{8}$$

where $\mathbf{E}(\mathbf{r},t)$ is the electric field strength of an EM wave, $\mathbf{H}(\mathbf{r},t)$ is the magnetic field strength, $\mathbf{D}(\mathbf{r},t)$ is the electric flux density, $\mathbf{B}(\mathbf{r},t)$ is magnetic flux density, and $\mathbf{J}(\mathbf{r},t)$ is total electric current density. These values depend on the coordinate $\mathbf{r}$ and time t. The equations are interesting to us because they are the base for obtaining the wave equations, the solutions of which we are going to study.

We will consider here the propagation of monochromatic plane EM waves in a homogeneous isotropic environment. The monochromatic wave is characterized by oscillations that occur at only a single frequency at each spatial point. The simplest solution of the wave equations is for a plane wave, *i.e.* a wave whose surfaces of constant phase are infinite planes, perpendicular to the direction of propagation. There is a reason why the solution in the form of plane monochromatic waves plays a big role in electromagnetics. The fact is that using the Fourier transform by time and coordinates, any function of these variables can be decomposed into plane monochromatic waves, if only the function quickly descends on time (temporal) and space (spatial) at infinity [42], [43]. We are going to explore expressions for the time-periodic case that are usually used in all branches of physics. In this case we can use simplifications for the differentiation and integration with respect to time t.

We can write time-harmonic Maxwell's equations in terms of vector field phasors $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$. Vector phasors contain the information on magnitude, direction, and phase, where the radius vector is $\mathbf{r} = \sqrt{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z}$. The wave equations for the time-periodic case are $\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$ (and $\nabla^2\mathbf{H} + k^2\mathbf{H} = 0$), where $k = \omega/c$ is the wavenumber (propagation constant) in a medium. The phase speed of light in a vacuum is $c = k/\omega = 1/\sqrt{\varepsilon_0\mu_0}$, the phase speed of light in an isotropic medium is $v = 1\big/\sqrt{\varepsilon\mu} = c\big/\sqrt{\varepsilon_r\mu_r}$, $\varepsilon = \varepsilon_r\varepsilon_0$ is the absolute permittivity, $\mu = \mu_r\mu_0$ is the absolute permeability, $\varepsilon_0 \approx 8.85 \cdot 10^{-12}$ (F/m) and $\mu_0 \approx 1.26 \cdot 10^{-6}$ (H/m) are the vacuum permittivity and permeability, ε_r and μ_r are the relative permittivity and permeability of an isotropic medium, respectively, $n = c/v$ is the index of refraction.

Because of the linearity of Maxwell's equations the solutions can be decomposed into a superposition of sinusoids or cosinusoids for the EM waves propagating in a vacuum. The convenience of complex notation has its origins in the Euler identity (formula) according to Eq. (6) such: $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ and $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$. By inverting Euler's expression we obtain the following representation of the cosine and sine functions: $\cos(\omega t) = \mathrm{Re}\left(e^{i\omega t}\right) = \left(e^{i\omega t} + e^{-i\omega t}\right)/2$ as well as $\sin(\omega t) = \mathrm{Im}\left(e^{i\omega t}\right) = \left(e^{i\omega t} - e^{-i\omega t}\right)/2i$. This is the basis for the Fourier transform method for the solution of the wave equations, which are second-order linear partial differential equations [44] – [46].

Reading the technical literature in electromagnetics we notice that the solution of wave equations may contain dependencies on time and coordinates with the plus or minus signs for every term, for example as

$$E(\mathbf{r},t) \sim e^{-i\,(\omega t-(kr))}, \tag{9}$$

and

$$E(\mathbf{r},t) \sim e^{i\,(\omega t-(\mathbf{kr}))} = e^{-i\,((\mathbf{kr})-\omega t)}. \tag{10}$$

Let us consider the reason for the inconsistency of the signs in solving the same problems by specialists of different branch of physics. About agreements of the signs (plus or minus) in solutions of the same EM problems, see the discussion in Refs. [36], [38], [47], [48]. Reference [36] provides the reasoning and research on agreements and definitions regarding signs in the literature on ellipsometry. We remind here that ellipsometry is known as an optical technique for investigating the dielectric properties of films and layers. The ellipsometry let us to measure changes of an EM wave polarization when the wave reflects or transmits through a material structure. As noted in that paper many parameters that appear in the theory of ellipsometry crucially depend on the choice of arbitrary conventions and definitions. Here are analyzed two alternatives for the complex formulation of sinusoidal oscillations. The paper highlights that the negative temporal (time) exponent is preferred here because of its use in most of modern physics, particularly quantum mechanics, although oppositive alternative is firmly established in electrical engineering. A direct consequence of the choice in sign for the exponent in the time-factor is the sign of the imaginary part of the complex refractive index n. In Ref. [36] it was noted that the people studying of ellipsometry are confronted with a confusing multiplicity of conventions and definitions employed. In the articles on the subject, this problem persists in the present literature. In an effort to untangle this situation, nine conventions and definitions have been singled out, where arbitrary choices between two alternatives have to be made in the theory of ellipsometry.

In Ref. [38] specialists from Massachusetts Institute of Technology (MIT) identified that opposed signs are used in Engineering, *i.e.* "the Negative Sign Convention", and physics (probably, in optics), *i.e.* "the Positive Sign Convention" for

EM wave propagating in $+z$-direction. MIT presented a table with expressions for the Maxwell's equations, wavenumber, propagation constant, complex permittivity ε, permeability μ, the index of refraction n, Fourier transform etc. Article [48] addresses the issues of signs in exponential terms containing time and coordinates. In Ref. [40] is is shown that plane forward waves moving in $+k$ direction can be described as

$$E = E_0\, e^{i(-\omega t + kz)}, \tag{11}$$

or

$$E = E_0\, e^{i(\omega t - kz)}. \tag{12}$$

We can write for the first case of EM forward wave:

$$E = \mathrm{Re}\{E_0\, e^{i(-\omega t + kz)}\} = E_0 \cos\left(-\omega t + kz\right), \tag{13}$$

and the peak of EM wave has moved to a positive location:

$$\Delta z = (\omega/k)\,\Delta t, \tag{14}$$

where Δz is a positive shift for the plane forward waves moving in $+k$ direction.

On other hand, we can write for the backward waves also two variants:

$$E = E_0\, e^{i(\omega t + kz)}, \tag{15}$$

or

$$E = E_0\; e^{i(-\omega t - kz)}. \tag{16}$$

We can write for the first case of the backward waves:

$$E = \mathrm{Re}\{E_0\, e^{i(\omega t + kz)}\} = E_0 \cos\left(\omega t + kz\right). \tag{17}$$

The peak of EM wave moves toward the negative direction:

$$\Delta z = -\,(\omega/k)\,\Delta t, \tag{18}$$

where Δz is a negative shift for EM plane waves moving in $-k$ direction, *i.e.* this represents the backward wave.

In [12], [37], [49] the electric field is taken in the form $E\left(\mathbf{r},t\right) \propto e^{i((\mathbf{kr})-\,\omega t)}$. It fits the meaning that $e^{-i\omega t}$ corresponds to the positive frequency and the exponent with the opposite sign, *i.e.* with the minus sign, such that:

$$e^{i\omega t} = e^{-i(-\omega)t}, \tag{19}$$

corresponding to the negative frequency $(-\omega)$.

The physical field can be presented as the sum of the positive and negative frequency parts [37], [49]: $E\left(\mathbf{r},t\right) = \left(E_0 e^{i((\mathbf{kr})-\,\omega t)} + E_0^{*} e^{-\,i((\mathbf{kr})-\,\omega t)}\right)/2$. We can obtain the real parts of the field by adding their respective complex conjugates E_0^{*} and dividing the result by 2.

We remind here that the values of the wavenumber (propagation constant) k and angular frequency ω are not independent values, because the product of wavenumber k on the speed of EM wave in a matter v was marked by us as $\omega = kv$ and we can re-write $\omega - kv = 0$. So we see that in expressions ω and k have the opposite signs.

As an example we take and show the dependencies on time and coordinates for the electric field $\mathbf{E}$, of course, we can do the same for all of other fields $\mathbf{H}$, $\mathbf{B}$, $\mathbf{D}$ etc.

6. SOLUTIONS OF WAVE EQUATIONS AND SIGNS

Now we examine the reason why opposite signs occurred in the solutions of the wave equation. For this reason, we will analyze these solutions of the wave equation [37] – [40], [47] – [54]. The wave equation is a second-order linear partial differential equation for the characterization of EM waves. In [7], [55] – [58] is given the EM wave equation and its solutions. The homogeneous form of the wave equation, written in terms of the electric field $\mathbf{E}$ can be expressed as:

$$\left(v^2\,\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0, \tag{20}$$

where the Laplace operator (Laplacian) $\nabla^2 = \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is a second-order differential operator in a Cartesian coordinate system, t is time, $\partial^2/\partial t^2$ is a second order derivative of time.

Converting Eq. (20) gives:

$$\nabla^2 E(\mathbf{r},t) - \frac{1}{v^2}\frac{\partial^2 E(\mathbf{r},t)}{\partial t^2} = 0. \tag{21}$$

The wave equation (21) describes the propagation of EM waves through a homogeneous isotropic medium. The three-dimensional form of the wave equation often arises in the study of physical problems involving partial differential equations in both space and time. We use for a solution the method of the separation of variables [55] – [59]:

$$E(\mathbf{r},t) = T(t)\,R(\mathbf{r}). \tag{22}$$

We put expression (22) to Eq. (21) and get:

$$\Delta\left(T(t)\,R(\mathbf{r})\right) - \frac{1}{v^2}\frac{\partial^2 T(t)}{\partial t^2}R(\mathbf{r}) = 0. \tag{23}$$

We divide the right and left side of the equation by the product $T(t)\,R(\mathbf{r})$:

$$\frac{\Delta R(\mathbf{r})}{R(\mathbf{r})} - \frac{1}{v^2}\frac{\partial^2 T(t)/\partial t^2}{T(t)} = 0. \tag{24}$$

The first term of Eq. (24) depends only on the radius-vector $\mathbf{r}$, and the second term depends only on the time of t. This is only possible if both components are equal to the same constant.

Let us define this constant as $-k^2$, then we can write:

$$\frac{\Delta R(\mathbf{r})}{R(\mathbf{r})} = -k^2, \tag{25}$$

if the constant $-k^2$ turns out to be positive, then let us assume that k is a purely imaginary number.

After converting Eq. (25), we get:

$$\Delta R(\mathbf{r}) + k^2 R(\mathbf{r}) = 0. \tag{26}$$

In Eq. (26) the function $R(\mathbf{r})$ can be the electric field $E(\mathbf{r})$ or the magnetic field $H(\mathbf{r})$. In mathematics, the eigenvalue problem for the Laplace operator Δ is called Helmholtz equation. For the spatial (space) part of the solution of the linear partial differential wave equation (21), we will use Eq. (26) which is the Helmholtz equation. The term k is the eigenvalue and in Eq. (26) also means the wave number.

For the time (temporal) part of the solution of the wave equation (21), we will get:

$$\frac{1}{v^2}\frac{\partial^2 T(t)/\partial t^2}{T(t)} = -k^2. \tag{27}$$

After transformation of Eq. (27) we get:

$$\frac{\partial^2 T(t)}{\partial t^2} + (k\,v)^2\, T(t) = 0. \tag{28}$$

We introduce here for brevity the designation $\omega = k\,v$.

$$\frac{\partial^2 T(t)}{\partial t^2} + \omega^2 T(t) = 0, \quad \text{(Equation of harmonic oscillations)}. \tag{29}$$

The solutions of Eqs. (26) and (29) in the complex form are simpler comparing with the solutions in the real form. Therefore, it is advisable to look for solutions in a complex form, and then consider the physical part of a complex solution. We use the complex algebra but we recognize that only the real part of the final answer has physical significance [37], [50]. For a linear differential equation with physical coefficients, the real part of the general complex solution is a final physical solution.

It has been applied the underlining of the symbol or letter to designate the complex value $\underline{T}(t)$ and real value $T(t)$. The general complex solution to the harmonic oscillation equation is the linear combination:

$$\underline{T}(t) = \underline{T}_{01}\, e^{i\omega t} + \underline{T}_{02}\, e^{-i\omega t}, \tag{30}$$

where $\underline{T}_{01}$ and $\underline{T}_{02}$ are arbitrary complex integration constants. The question of which of the two terms of Eq. (30) with the plus or minus sign to choose as a solution is your personal choice. You can choose $\underline{T}_{01}\, e^{i\omega t}$ or $\underline{T}_{02}\, e^{-i\omega t}$. The general solution is just the superposition of all possible solutions of a certain differential equation. At this stage, these decisions are tantamount to each other. It is a matter of agreement.

A common solution to the harmonic oscillation equation can be obtained as a linear combination of species solutions: $\underline{T}(t) = \underline{T}_0\, e^{i\omega t}$, where ω can have two possible values with the same modulus but different signs. $\underline{T}_0$ is an arbitrary integration constant, which is different for positive and negative values of ω.

Let us go back now to the spatial part of the solution of the wave equation to the Helmholtz equation (26). We will continue to search for particular solutions to the wave equation by separating variables. We are looking for a solution to the Helmholtz equation for the spatial part in the form of three functions, each of which depends only on one spatial coordinate:

$$R(\mathbf{r}) = X(x)\, Y(y)\, Z(z)\,. \tag{31}$$

We will put Eq. (31) into the Helmholtz equation (26) and get:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(X(x)\, Y(y)\, Z(z)) + k^2 (X(x)\, Y(y)\, Z(z)) = 0. \tag{32}$$

We denote $X = X(x)\,,\ Y = Y(y)\,,\ Z = Z(z)$:

$$\left(Y\, Z \frac{\partial^2 X}{\partial x^2} + X\, Z \frac{\partial^2 Y}{\partial y^2} + X\, Y \frac{\partial^2 Z}{\partial z^2}\right) + k^2 X\, Y\, Z = 0. \tag{33}$$

We denote the derivatives with respect to the appropriate coordinates as follows: $X'' = \partial^2 X/\partial x^2$, $Y'' = \partial^2 Y/\partial y^2$, and $Z'' = \partial^2 Z/\partial z^2$, thus

$$\left(X'' Y\, Z + Y'' X\, Z + Z'' X\, Y\right) + k^2 X\, Y\, Z = 0. \tag{34}$$

The two strokes indicate the second derivative in each case with respect to the corresponding variable.

We will divide Eq. (34) by the product X, Y, Z and get:

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2, \tag{35}$$

the first term of Eq. (35) depends only on the x coordinate, the second term depends only on the y coordinate and the third term depends only on the z coordinate. This is only possible if each of these components is a constant. We identify these constants as $\left(-k_x^2\right),\ \left(-k_y^2\right),\ \left(-k_z^2\right)$. Then:

$$k^2 = k_x^2 + k_y^2 + k_z^2, \tag{36}$$

where $k_x,\ k_y,\ k_z$ can be seen as the corresponding projections of the wavenumber vector $\mathbf{k}$.

$$\frac{X''}{X} = -k_x^2, \tag{37}$$

$$\frac{Y''}{Y} = -k_y^2, \tag{38}$$

$$\frac{Z''}{Z} = -k_z^2, \tag{39}$$

This equation of harmonic oscillations $\partial^2 X / \partial x^2 = -k_x^2 X$ is dependent from the coordinate but not from time (see Eq. (37)). A complex solution to Eq. (37) can be taken as a linear combination of particular solutions:

$$\underline{X} = \underline{X}_{01}\, e^{ik_x x} + \underline{X}_{02}\, e^{-ik_x x}. \tag{40}$$

We can choose solution $\underline{X} = \underline{X}_{01}\, e^{ik_x x}$ or $\underline{X} = \underline{X}_{02}\, e^{-ik_x x}$. We will now make a choice as in the microwave electrodynamics and engineering $\underline{X} = \underline{X}_0\, e^{-ik_x x}$, where the projection k_x of the wavenumber vector $\mathbf{k}$ can take two possible values with the same modulus, but different signs. The term with the minus component is a matter of agreement.

Similarly for the coordinates y and z:

$$\underline{Y} = \underline{Y}_0\, e^{-ik_y y}, \tag{41}$$

$$\underline{Z} = \underline{Z}_0\, e^{-ik_z z}. \tag{42}$$

We will put the terms (40)–(42) in Eq. (31) and get:

$$\underline{R}(\mathbf{r}) = \underline{X}_0\, \underline{Y}_0\, \underline{Z}_0\, e^{-ik_x x}\, e^{-ik_y y} e^{-ik_z z} = \underline{R}_0(\mathbf{r})\, e^{-i\,(k_x x + k_y y + k_z z)} \tag{43}$$

Equation (43) is a particular solution to the Helmholtz equation (26), which can be rewriten as:

$$\underline{R}(\mathbf{r}) = \underline{R}_0(\mathbf{r})\, e^{-i\,(\mathbf{kr})}. \tag{44}$$

The complete solution of the wave equation Eq. (21) on the basis of Eq. (22) can be written as:

$$\underline{E}(\mathbf{r},t) = \underline{T}(t)\, \underline{R}(\mathbf{r}) = \underline{T}_0\, e^{i\omega t}\, \underline{R}_0(\mathbf{r})\, e^{-i\,(\mathbf{kr})}. \tag{45}$$

A particular solution to the wave equation (21) in the form of plane monochromatic waves is:

$$\underline{E}(\mathbf{r},t) = \underline{E}_0 e^{i\,(\omega t - (\mathbf{kr}))}, \tag{46}$$

where $\underline{E}_0$ is an arbitrary integration constant. This solution is usually used in EM problems in the microwave range.

Analyzing the technical literature we see that in [9], [10], [57] the dependence on coordinates and time is used in the form $e^{i\,(\omega t - (\mathbf{kr}))}$ and in [12], [18], [37] is used in form $e^{i\,((\mathbf{kr}) - \omega t)}$.

The signs in expressions that are used in different branches of physics are as follows:

- Microwave electrodynamics, Engineering:
  - The electric and magnetic field strengths of the time-periodic plane wave: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\, e^{i\omega t}$, $\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\, e^{i\omega t}$.
  - We can replace: $\frac{\partial e^{i\omega t}}{\partial t} = i\omega\, e^{i\omega t}, \frac{\partial}{\partial t} \to i\omega$
  - We can replace: $\frac{\partial^2 e^{i\omega t}}{\partial t^2} = (i\omega)^2\ e^{i\omega t}, \quad \frac{\partial^2}{\partial t^2} \to -\omega^2$.
  - For differentiation with respect to time t: $\partial\mathbf{E}(x,y,z,t)/\partial t = i\omega\mathbf{E}(x,y,z)$, the same for the magnetic field strengths.
  - Faraday's law for the Time-Periodic Case: $\nabla \times \left(\mathbf{E}(\mathbf{r})\, e^{i\omega t}\right) = -\frac{\partial \mathbf{B}(\mathbf{r}) e^{i\omega t}}{\partial t}$, $\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}$.
  - Ampère's law for the Time-Periodic Case: $\nabla \times \left(\mathbf{H}(\mathbf{r})\, e^{i\omega t}\right) = \mathbf{J}(\mathbf{r})e^{i\omega t} + \frac{\partial \mathbf{D}(\mathbf{r}) e^{i\omega t}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} + i\omega\varepsilon\mathbf{E}$.
- Optics, Plasmonics:
  - The electric and magnetic field strengths of the time-periodic plane wave: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\, e^{-i\omega t}$, $\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\, e^{-i\omega t}$.
  - We can replace: $\frac{\partial e^{-i\omega t}}{\partial t} = -i\omega\, e^{-i\omega t}, \frac{\partial}{\partial t} \to -i\omega$
  - We can replace: $\frac{\partial^2 e^{-i\omega t}}{\partial t^2} = (-i\omega)^2\ e^{-i\omega t}, \quad \frac{\partial^2}{\partial t^2} \to -\omega^2$.
  - For differentiation with respect to time t: $\partial\mathbf{E}(x,y,z,t)/\partial t = -i\omega\mathbf{E}(x,y,z)$, the same for the magnetic field strengths.
  - Faraday's law for the Time-Periodic Case: $\nabla \times \left(\mathbf{E}(\mathbf{r})\, e^{-i\omega t}\right) = -\frac{\partial \mathbf{B}(\mathbf{r}) e^{-i\omega t}}{\partial t}$, $\nabla \times \mathbf{E} = +i\omega\mu\mathbf{H}$.
  - Ampère's law for the Time-Periodic Case: $\nabla \times \left(\mathbf{H}(\mathbf{r})\, e^{-i\omega t}\right) = \mathbf{J}(\mathbf{r})e^{-i\omega t} + \frac{\partial \mathbf{D}(\mathbf{r}) e^{-i\omega t}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} - i\omega\varepsilon\mathbf{E}$.

The signs between the relative and imaginary parts in the complex relative permittivity $\underline{\varepsilon}_r = \underline{\varepsilon}/\varepsilon_0$ and index of refraction $\underline{n}$:

- Microwave electrodynamics, Engineering:
  - Relative permittivity: $\underline{\varepsilon}_r = \varepsilon_r' - i\varepsilon_r''$.
  - Refractive index: $\underline{n} = n' - in'' = \sqrt{\underline{\varepsilon}_r} = \sqrt{(\varepsilon_r' - i\varepsilon_r'')}$; at $\underline{\mu}_r = 1$, where $\varepsilon_r' = (n')^2 - (n'')^2$; $\varepsilon_r'' = 2n'n''$.
- Optics, Plasmonics:
  - Relative permittivity: $\underline{\varepsilon}_r = \varepsilon_r' + i\varepsilon_r''$.

– Refractive index: $\underline{n} = n' + in'' = \sqrt{\underline{\varepsilon}_r} = \sqrt{(\varepsilon_r' + i\varepsilon_r'')}$; at $\underline{\mu}_r = 1$, where $\varepsilon_r' = (n')^2 - (n'')^2$; $\varepsilon_r'' = 2n'n''$.

In the scientific literature on the engineering and microwave electrodynamics the time dependency is usually expressed by the complex exponential function $e^{i\omega t}$ while in the physics literature and especially in optics and plasmonics this dependence is usually taken as $e^{-i\omega t}$. It should be noted that in some works, *e.g.* in *Electromagnetic Wave Theory* [7] is taken as $E(\mathbf{r},t) \propto e^{i((\mathbf{kr})-\omega t)}$, and in *The plane wave spectrum representation of electromagnetic fields* is presented as $E(\mathbf{r},t) \propto e^{i(\omega t \pm (\mathbf{kr}))}$.

7. CONCLUSIONS

The approach to the processes of EM wave propagation in the area of plasmonics from the point of view of experts in the microwave and optical range specialists is often different.

In fact, the choice of signs in expressions is more related to the traditions of scientific school, which include experts who write articles and books in different branches of physics.

It is important to draw readers attention to different approaches to certain issues in the technical literature in the microwave and optical ranges.

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