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ANALYTICAL STUDY ON THE SLIP FLOW AND HEAT TRANSFER OF NANOFLUIDS OVER A STRETCHING SHEET USING ADOMIAN'S METHOD

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Abstract. In the present paper, an effective approach is introduced to overcome the difficulties of imposing boundary conditions at infinity, which are used to modelling the boundary layer flow of a nanofluid past a stretching sheet. The proposed scheme is mainly based on the Adomian decomposition method with an effective procedure to imposing the boundary conditions at infinity. On applying the present approach to approximate the solution of a boundary value problem in the literature, it is found that only two components of Adomian's series are sufficient to achieve the same accuracy of the homotopy analysis method using forty iterations.

Key words: Stretching sheet, boundary layer, Adomian decomposition method.

1. INTRODUCTION

The Adomian decomposition method (ADM) [1], which gives the solution in the form of an infinite series, is an effective method to solve initial/boundary value problems for ordinary, partial, and integral equations [2–12]. It has been shown that the series solution obtained by the ADM converges to the exact solution, when available. Even for a boundary value problem of no available exact solution such as the Thomas-Fermi equation, the ADM gives a sequence of approximate solutions that converge to a certain function [7]. However, the ADM counters some difficulties in solving boundary value problems with boundary conditions at infinity. This is because the conditions at infinity cannot be directly imposed in the series solution. These kinds of boundary conditions are frequently arising in many physical problems and also in the boundary layer flow of nanofluids [13–17].

To overcome the difficulties of the boundary conditions at infinity, many authors have been resorted either to the semi-analytical methods or the Padé technique [18–24]. However, the Padé technique requires a massive computational work to obtain the desired accuracy. In addition, it was the main task of many mathematicians to searching for a direct approach to effectively deal with the boundary condition at infinity. Recently, an approach has been proposed in Ref. [22] to directly implement the boundary condition at infinity by the ADM. This approach depends on obtaining a closed form solution for the governing system and then applying the ADM to approximate the physical quantities of interest. The objective of this paper is to extend the ADM proposed in [22] to solve the boundary-layer flow and heat transfer of nanofluid over a stretching sheet with partial slip [24]. Hence, we reinvestigate the problem studied very recently by Mabood *et al.* [24] in which the homotopy analysis method has been used to approximate the solutions. Moreover, it will be shown that only two components of the Adomian's series are sufficient to obtain the accuracy of the 40th-order solution of the homotopy analysis method. The current analytical results have not been reported in earlier works, to the best of our knowledge. The proposed problem is finally given by the following set of nonlinear ordinary differential equations [24]:

$$f'''(\eta) + f(\eta)f''(\eta) - \left(f'(\eta)\right)^2 = 0, \quad (1)$$

$$\frac{1}{Pr}\theta''(\eta) + f(\eta)\theta'(\eta) + Nb\,\phi'(\eta)\theta'(\eta) + Nt\left(\theta'(\eta)\right)^2 = 0, \quad (2)$$

$$\phi''(\eta) + Le f(\eta)\phi'(\eta) + \frac{Nt}{Nb}\theta''(\eta) = 0, \quad (3)$$

subject to the boundary conditions:

$$f(0) = 0, \quad f'(0) = 1 + \lambda f''(0), \quad \theta(0) = 1, \quad \phi(0) = 1, \quad (4)$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad (5)$$

where Pr, Le, Nb, Nt and λ are Prandtl number, Lewis number, Brownian motion parameter, thermophoresis parameter, and partial slip parameter, respectively. The exact solution of Eq. (1) with the boundary conditions in (4-5) is already well known and given by

$$f(\eta) = \beta(1 - e^{-\beta\eta}), \quad (6)$$

where β is a positive real root for the following cubic equation

$$\lambda\beta^3 + \beta^2 - 1 = 0. \quad (7)$$

Hence, Eqs. (2) and (3) become

$$\theta''(\eta) + Pr\left[\beta(1 - e^{-\beta\eta}) + Nb\,\phi'(\eta)\right]\theta'(\eta) + PrNt\left[\theta'(\eta)\right]^2 = 0, \quad (8)$$

$$\phi''(\eta) + Le\beta\left(1 - e^{-\beta\eta}\right)\phi'(\eta) + \frac{Nt}{Nb}\,\theta''(\eta) = 0. \tag{9}$$

2. ANALYSIS

Suppose that $z(\eta) = 1/(\theta'(\eta))$, then Eq. (8) becomes

$$z'(\eta) - Pr\left[\beta(1 - e^{-\beta\eta}) + Nb\,\phi'(\eta)\right]z(\eta) = PrNt. \tag{10}$$

By solving this equation for $z(\eta)$ and implementing the above relation we obtain $\theta'(\eta)$ as

$$\theta'(\eta) = \frac{e^{-Pr[\beta\eta + e^{-\beta\eta} + Nb\,\phi(\eta)]}}{e^{-Pr(1+Nb)}/\theta'(0) + PrNt\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi(\sigma)]}\mathrm{d}\sigma}. \tag{11}$$

Integration of Eq. (11) yields

$$\theta(\eta) = 1 + \frac{1}{PrNt}\ln\left(1 + PrNt\,e^{Pr(1+Nb)}\theta'(0)\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi(\sigma)]}\mathrm{d}\sigma\right). \tag{12}$$

On imposing the boundary condition $\theta(\infty) = 0$, we find $\theta'(0)$ as

$$\theta'(0) = \frac{(e^{-PrNt} - 1)}{PrNt\,e^{-Pr(1+Nb)}}\left[\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi(\sigma)]}\mathrm{d}\sigma\right]^{-1}. \tag{13}$$

Inserting Eq. (13) into Eq. (12), it then follows

$$\theta(\eta) = 1 + \frac{1}{PrNt}\ln\left[1 + (e^{-PrNt} - 1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi(\sigma)]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi(\sigma)]}\mathrm{d}\sigma}\right]. \tag{14}$$

This explicit relation for $\theta(\eta)$ in terms of $\phi(\eta)$ shall be used in subsequent Sections to derive the exact solutions at particular values of the permanent parameters in which the involved integrals can be evaluated analytically. In addition, Eq. (7) is also solved for $\phi'(\eta)$ as a 1^{st}-order ordinary differential equation (ODE) and thus

$$\phi'(\eta) = \mu\,e^{-Le(\beta\eta + e^{-\beta\eta})} - \frac{Nt}{Nb}\,e^{-Le(\beta\eta + e^{-\beta\eta})}\int_0^{\eta} e^{Le(\beta\xi + e^{-\beta\xi})}\theta''(\xi)\mathrm{d}\xi, \tag{15}$$

where $\mu = e^{Le}\phi'(0)$ is a constant to be determined later. By integrating Eq. (15) once again with respect to η, where $\phi(0)=1$, we have

$$\phi(\eta) = 1 + \mu\int_0^{\eta} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma - \frac{Nt}{Nb}\int_0^{\eta} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma} e^{Le(\beta\xi+e^{-\beta\xi})}\theta''(\xi)\mathrm{d}\xi\mathrm{d}\sigma. \quad (16)$$

Now, μ can be evaluated from the condition at infinity ($\phi(\infty)=0$) as

$$\mu = \left(-1 + \frac{Nt}{Nb}\int_0^{\infty} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma} e^{Le(\beta\xi+e^{-\beta\xi})}\theta''(\xi)\mathrm{d}\xi\mathrm{d}\sigma\right) \Big/ \left(\int_0^{\infty} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma\right). \quad (17)$$

Therefore, $\phi(\eta)$ in Eq. (16) becomes

$$\phi(\eta) = 1 + \frac{\int_0^{\eta} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}\left(-1 + \frac{Nt}{Nb}\int_0^{\infty} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma} e^{Le(\beta\xi+e^{-\beta\xi})}\theta''(\xi)\mathrm{d}\xi\mathrm{d}\sigma\right)$$

$$- \frac{Nt}{Nb}\int_0^{\eta} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma} e^{Le(\beta\xi+e^{-\beta\xi})}\theta''(\xi)\mathrm{d}\xi\mathrm{d}\sigma. \quad (18)$$

This expression shall be also used in subsequent Sections to obtain the exact solutions at special cases of the physical parameters.

3. SPECIAL CASES AND EXACT SOLUTIONS

In this Section, we discuss the possibility of deriving exact solutions for the current physical problem at $Nt = 0$, $Nb = 0$. Also, comparisons with the results reported in the literature are to be introduced. Setting $Nb = 0$ into Eq. (14) and then taking the limit as $Nt \to 0$, we obtain $\theta(\eta)$ in a simpler closed form expression as

$$\theta(\eta) = 1 - \frac{\int_0^{\eta} e^{-Pr[\beta\sigma+e^{-\beta\sigma}]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma+e^{-\beta\sigma}]}\mathrm{d}\sigma}, \quad (19)$$

where β is the solution of Eq. (7). Following [16], we have

$$\int_0^{\eta} e^{-Pr[\beta\sigma+e^{-\beta\sigma}]}\mathrm{d}\sigma = \frac{1}{\beta}(Pr)^{-Pr}\,\Gamma(Pr, Pr\,e^{-\beta\eta}, Pr), \quad (20)$$

where $\Gamma(a, z_0, z_1)$ is the generalized Gamma function. On using the result of (20) as $\eta \to \infty$, we have

$$\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma}]} \mathrm{d}\sigma = \frac{1}{\beta}(Pr)^{-Pr}\, \Gamma(Pr, 0, Pr). \tag{21}$$

Accordingly, an exact solution for $\theta(\eta)$ is obtained as

$$\theta(\eta) = 1 - \frac{\Gamma(Pr, Pr\, e^{-\beta\eta}, Pr)}{\Gamma(Pr, 0, Pr)}. \tag{22}$$

Since β mainly depends on λ, then two cases for such a partial slip parameter can be addressed as following.

3.1. AT $\lambda \neq 0$

In this case, the solution given by Eq. (22) can be simplified as

$$\theta(\eta) = \frac{\Gamma(Pr, 0, Pr\, e^{-\beta\eta})}{\Gamma(Pr, 0, Pr)}, \tag{23}$$

where the relation between the generalized Gamma function and the incomplete Gamma function; $\Gamma(a, z_0, z_1) = \Gamma(a, z_0) - \Gamma(a, z_1)$ has been used.

3.2. AT $\lambda = 0$

Substituting $\lambda = 0$ into Eq. (7), we find that $\beta = \pm 1$. By considering the positive root and inserting it into Eq. (23), we have

$$\theta(\eta) = \frac{\Gamma(Pr, 0, Pr\, e^{-\eta})}{\Gamma(Pr, 0, Pr)}. \tag{24}$$

This analytical expression is valid at any Prantdl number Pr. Differentiating (24) with respect to η at $\eta = 0$, we obtain the Nusselt number as

$$-\theta'(0) = \frac{(Pr)^{Pr}\, e^{-Pr}}{\Gamma(Pr, 0, Pr)}. \tag{25}$$

This expression shall be used in Section 5 to validate the numerical results obtained from some numerical and analytical approaches in the literature.

4. SOLUTION BY ADOMIAN'S METHOD

The possibility of applying Adomian's method to obtain the approximate solutions for the governing equations is the subject of this Section. According to Adomian's method, the solutions are assumed in a decomposition series form as

$$\theta(\eta)=\sum_{n=0}^{\infty}\theta_n(\eta),\quad \phi(\eta)=\sum_{n=0}^{\infty}\phi_n(\eta). \tag{26}$$

It is important to note that the second term on the right hand side of Eq. (14) is a nonlinear term in ϕ, which can be decomposed into a series of Adomian's polynomials A_n as

$$\ln\left(1+(e^{-PrNt}-1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma+e^{-\beta\sigma}+Nb\phi(\sigma)]}d\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma+e^{-\beta\sigma}+Nb\phi(\sigma)]}d\sigma}\right)=\sum_{n=0}^{\infty}A_n, \tag{27}$$

where these A_n are defined by

$$A_n=\frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[\ln\left(1+(e^{-PrNt}-1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma+e^{-\beta\sigma}+Nb\sum_{i=0}^{\infty}\lambda^i\phi_i(\sigma)]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma+e^{-\beta\sigma}+Nb\sum_{i=0}^{\infty}\lambda^i\phi_i(\sigma)]}\mathrm{d}\sigma}\right)\right]_{\lambda=0}. \tag{28}$$

Inserting (27) and (28) into (14) and (18) and applying Adomian's method, we can then set the following system of two-coupled recurrence schemes for $\theta(\eta)$ and $\phi(\eta)$ as

$$\theta_0(\eta)=1, \tag{29}$$

$$\theta_{n+1}(\eta)=\frac{1}{NtPr}A_n, \tag{30}$$

$$\phi_0(\eta)=1-\frac{\int_0^{\eta} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}, \tag{31}$$

$$\phi_{n+1}(\eta)=\frac{Nt}{Nb}\left(\frac{\int_0^{\eta}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}{\int_0^{\infty}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\mathrm{d}\sigma}\right)\int_0^{\infty}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma}e^{Le(\beta\xi+e^{-\beta\xi})}\theta_{n''}(\xi)\mathrm{d}\xi\mathrm{d}\sigma-$$

$$-\frac{Nt}{Nb}\int_0^{\eta}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma}e^{Le(\beta\xi+e^{-\beta\xi})}\theta_{n''}(\xi)\mathrm{d}\xi\mathrm{d}\sigma,\ n\geq 0. \tag{32}$$

The recurrence scheme for $\phi(\eta)$ can be also simplified and written as

$$\phi_0(\eta)=\frac{\Gamma(Le,0,Le\,e^{-\beta\eta})}{\Gamma(Le,0,Le)}, \tag{33}$$

$$\phi_{n+1}(\eta)=\frac{Nt}{Nb}\left[(1-\phi_0)I_n(\infty)-I_n(\eta)\right], \tag{34}$$

where $I_n(\eta)$ and $I_n(\infty)$ are given by

$$I_n(\eta)=\int_0^{\eta}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma}e^{Le(\beta\xi+e^{-\beta\xi})}\theta_{n''}(\xi)\mathrm{d}\xi\,\mathrm{d}\sigma, \tag{35}$$

and

$$I_n(\infty)=\int_0^{\infty}e^{-Le(\beta\sigma+e^{-\beta\sigma})}\int_0^{\sigma}e^{Le(\beta\xi+e^{-\beta\xi})}\theta_{n''}(\xi)\mathrm{d}\xi\,\mathrm{d}\sigma. \tag{36}$$

The double integrations in (35) and (36) can be further simplified to single integrations by changing the order of integration as

$$\int_0^{\eta}f(\sigma)\int_0^{\sigma}g(\xi)\mathrm{d}\xi\,\mathrm{d}\sigma=\int_0^{\eta}g(\xi)\int_{\xi}^{\eta}f(\sigma)\mathrm{d}\sigma\,\mathrm{d}\xi. \tag{37}$$

Accordingly, we have

$$I_n(\eta)=\frac{1}{\beta}(Le)^{-Le}\int_0^{\eta}e^{Le(\beta\xi+e^{-\beta\xi})}\,\Gamma(Le,Le\,e^{-\beta\eta},Le\,e^{-\beta\xi})\,\theta_{n''}(\xi)\mathrm{d}\xi, \tag{38}$$

and hence

$$I_n(\infty)=\frac{1}{\beta}(Le)^{-Le}\int_0^{\infty}e^{Le(\beta\xi+e^{-\beta\xi})}\,\Gamma(Le,0,Le\,e^{-\beta\xi})\,\theta_{n''}(\xi)\mathrm{d}\xi. \tag{39}$$

To compute $\theta_0(\eta)$, the Adomian polynomial A_0 should be first calculated by using formula (28) for $n=0$, hence

$$A_0 = \ln\left(1 + (e^{-PrNt} - 1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi_0(\sigma)]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + Nb\,\phi_0(\sigma)]}\mathrm{d}\sigma}\right) =$$

$$= \ln\left(1 + (e^{-PrNt} - 1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}\right), \quad (40)$$

where we have assigned

$$\Lambda = \frac{Nb}{\Gamma(Le,0,Le)}. \quad (41)$$

Therefore, the Adomian solution components $\theta_1(\eta)$ and $\phi_1(\eta)$ are given as

$$\theta_1(\eta) = \frac{1}{NtPr}\ln\left(1 + (e^{-PrNt} - 1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}\right), \quad (42)$$

$$\phi_1(\eta) = 0. \quad (43)$$

It is clear from (43) that $\phi_1(\eta)$ does not contribute to the approximate solution for $\phi(\eta)$. Thus we have to evaluate $\phi_2(\eta)$ from (34) that yields

$$\phi_2(\eta) = \frac{Nt}{Nb}\left[(1-\phi_0)I_1(\infty) - I_1(\eta)\right], \quad (44)$$

where $I_1(\eta)$ and $I_1(\infty)$ are evaluated from (38) and (39) for $n = 1$. In view of the above analysis, we can write the two-term approximate solution for the temperature distribution $\theta(\eta)$ and the three-term approximate solution for the nano-particle concentration $\phi(\eta)$ as

$$\theta(\eta) = 1 + \frac{1}{NtPr}\ln\left(1 + (e^{-PrNt} - 1)\frac{\int_0^{\eta} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}{\int_0^{\infty} e^{-Pr[\beta\sigma + e^{-\beta\sigma} + \Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma}\right), \quad (45)$$

$$\phi(\eta) = \phi_0 + \frac{Nt}{Nb}\left[(1-\phi_0)I_1(\infty) - I_1(\eta)\right]. \quad (46)$$

Here, it is important to mention that the integrals involved in (45) reduce to the following exact expression, when $Pr = Le = \nu$, say

$$\int_0^{\eta} e^{-Pr[\beta\sigma+e^{-\beta\sigma}+\Lambda\Gamma(Le,0,Le\,e^{-\beta\sigma})]}\mathrm{d}\sigma = \frac{\nu^{-\nu-1}}{\beta\Lambda}\left[e^{-\nu\Lambda\Gamma(\nu,0,\nu\,e^{-\beta\eta})} - e^{-\nu\Lambda\Gamma(\nu,0,\nu)}\right]$$

$$= \frac{\nu^{-\nu-1}}{\beta\Lambda}\left[e^{-\nu\Lambda\Gamma(\nu,0,\nu\,e^{-\beta\eta})} - e^{-\nu Nb}\right]. \qquad (47)$$

Therefore, the closed-form solution given by (45) reduces to the following exact expression, when $Pr = Le = \nu$,

$$\theta(\eta) = 1 + \frac{1}{\nu Nt}\ln\left[1 + (e^{-\nu Nt} - 1)\left(\frac{e^{-\nu\Lambda\Gamma(\nu,0,\nu\,e^{-\beta\eta})} - e^{-\nu Nb}}{1 - e^{-\nu Nb}}\right)\right], \qquad (48)$$

where $\Gamma(\nu,0,\nu) = 0$ was used to obtain the last expression. It is obvious that the obtained approximate solutions satisfy the given boundary conditions.

5. NUMERICAL VALIDATION

This Section is devoted to compare between the present results and the corresponding results in the literature at the same values of the selected parameters. In Table 1, comparisons of the present exact numerical results for $(-\theta'(0))$ at different values of Pr, when $Nt = 0, Nb = 0, \lambda = 0$, with those obtained in [12, 25–27] are presented. It is clear from Table 1 that the results reported in the literature coincide with the present numerical values in most cases. However, the value of $(-\theta'(0))$ obtained in [12] by using the homotopy analysis method when $Pr = 0.7$ may be not accurate enough, where it agrees the current exact value up to only two decimal places. In addition, the calculated value obtained in [27] may need some revisions because it was completely different than the current exact value and also than those in references [12], [25], and [26]. Moreover, the values of the skin friction $(-f''(0))$ are calculated by using the exact solution given by Eqs. (6–7) and compared in Table 2 with those obtained in [14] and [24] by using the homotopy analysis method at different values of λ.

Figure 1 displays the variation of β against λ. It seems from Fig. 1 that β is always positive for all values of λ. However, before commenting on the obtained results it should be noted that the present analytical solution (6–7) is valid only when $\lambda \geq \frac{2}{3\sqrt{3}}$. In view of the current exact solution for the f-equation, it can be concluded from Table 2 that the present results of the skin friction $(-f''(0))$ agree with those obtained by Noghrehabadi *et al*. [14] and by Mabood *et al*. [24] up to five or six decimal places. On the other hand, to stand on the accuracy of Adomian's method, the approximate solutions given by Eqs. (45–46) and Eq. (48) are used to conduct several numerical results for $(-\theta'(0))$ and $(-\phi'(0))$, which are then compared with the available results in the literature at various values of λ, Nt, and Nb when $Pr = Le = \nu = 10$.

It is seen from Table 3 that the current numerical results obtained by applying the ADM are in full agreement with the corresponding ones in [14] and [24] by using the homotopy analysis method. Therefore, the 2-term approximate solution of Adomian's method has achieved similar accuracy of the 40-term approximate solution of the homotopy analysis method.

Table 1

Comparison of results for $(-\theta'(0))$ at $\lambda = 0$, $Nt = 0$, $Nb = 0$

Pr	Khan and Pop [25]	Wang [26]	Gorla and Sidawi [27]	Hassani *et al*. [12]	Present results
0.07	0.0663	0.0656	0.0656	–	0.0655625
0.20	0.1691	0.1691	0.1691	0.1692	0.169089
0.70	0.4539	0.4539	0.5349	0.4582	0.453916
2.00	0.9113	0.9114	0.9114	0.9114	0.911358
7.00	1.8954	1.8954	1.8905	1.8956	1.895403
20.00	3.3539	3.3539	3.3539	3.3539	3.353904
70.00	6.4621	6.4622	6.4622	6.4623	6.462199

Table 2

Comparison of results for $(-f''(0))$ at different values for λ

λ	Noghrehabadi *et al*. [14]	Mabood *et al*. [24]	Present results
0.1	0.872082	0.872082	0.87208247
0.3	0.701548	0.701548	0.70154821
0.5	0.591195	0.591195	0.59119548
1.0	0.430160	0.430160	0.43015971
2.0	0.283980	0.283981	0.28397959
5.0	0.144841	0.144843	0.14484019
10.	0.081243	0.081246	0.08124198

Table 3

Comparison of results for $(-\theta'(0))$ and $(-\phi'(0))$ at $\nu = Le = \Pr = 10$

λ	Nb	Nt	$(-\theta'(0))$			$(-\phi'(0))$		
			[14]	HAM [24]	Present (ADM)	[14]	HAM [24]	Present (ADM)
0.5	0.2	0.1	0.424328	0.424328	0.42432776	1.999070	1.999072	1.99907252
		0.2	0.306640	0.306640	0.30664021	2.110990	2.110993	2.11099348
		0.3	0.229206	0.229207	0.22920659	2.228691	2.228691	2.22869197
1.0	0.3	0.1	0.190347	0.190346	0.19034699	1.819268	1.819267	1.81926938
		0.2	0.137084	0.137084	0.13708403	1.898513	1.898513	1.89851463
		0.3	0.102297	0.102297	0.10229664	1.969337	1.969337	1.96933898

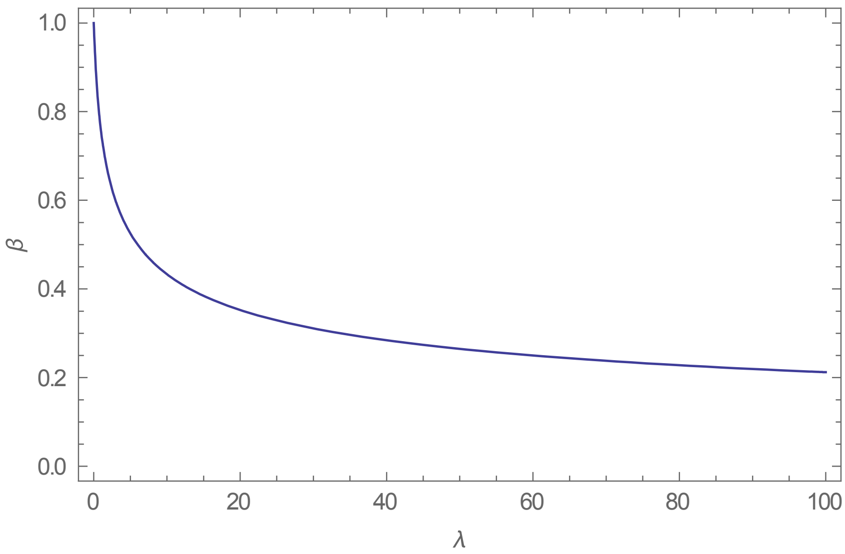

Fig. 1 – Variation of β against λ.

In Figs. 2–5, the 2-term approximate solutions of the dependent similarity variables $\theta(\eta)$ and $\phi(\eta)$ are depicted at the same selected values of the physical parameters chosen in [24]. However, without repeating the physical interpretation of the effects of these parameters on the variation of $\theta(\eta)$ and $\phi(\eta)$, it is observed that the results depicted in these figures are very close or may be identical to those obtained by [24] by using the 40-term approximate solution of the homotopy analysis method. In view of these comparisons, it may be concluded that Adomian's method requires less computational work when compared with the homotopy analysis method. Finally, the present analysis may be extended to cover many scientific models in computational physics [28–35].

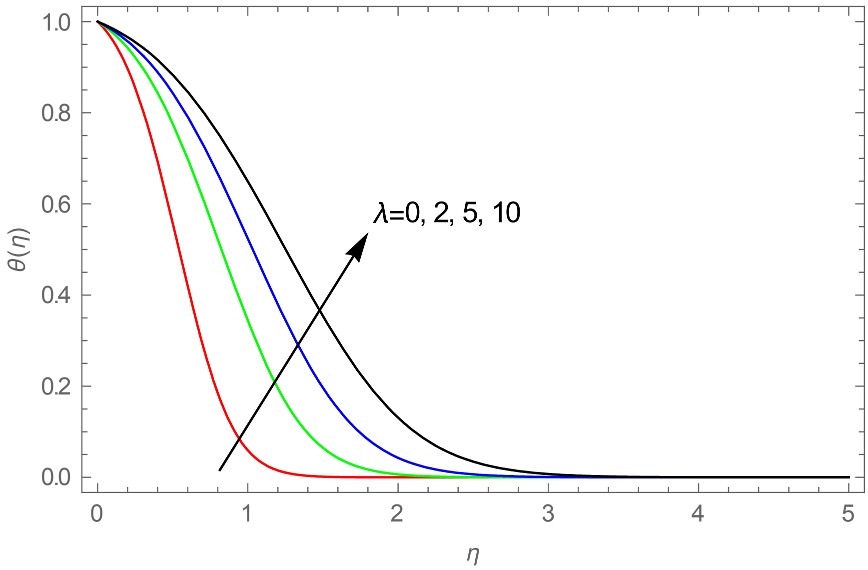

Fig. 2 – Effect of slip factor on dimensionless temperature at $\Pr = Le = 10$ and $Nb = Nt = 0.2$.

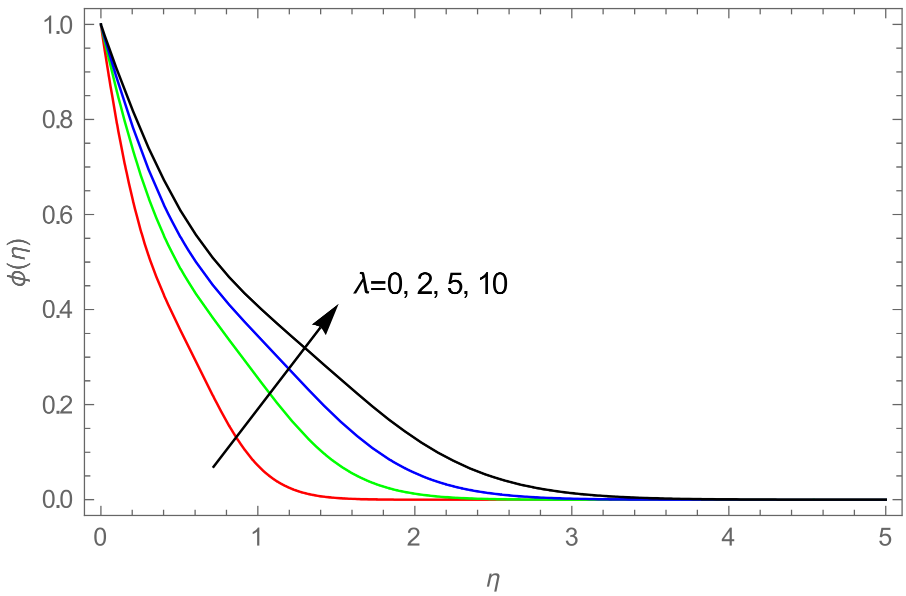

Fig. 3 – Effect of slip factor on dimensionless concentration at $\Pr = Le = 10$ and $Nb = Nt = 0.2$.

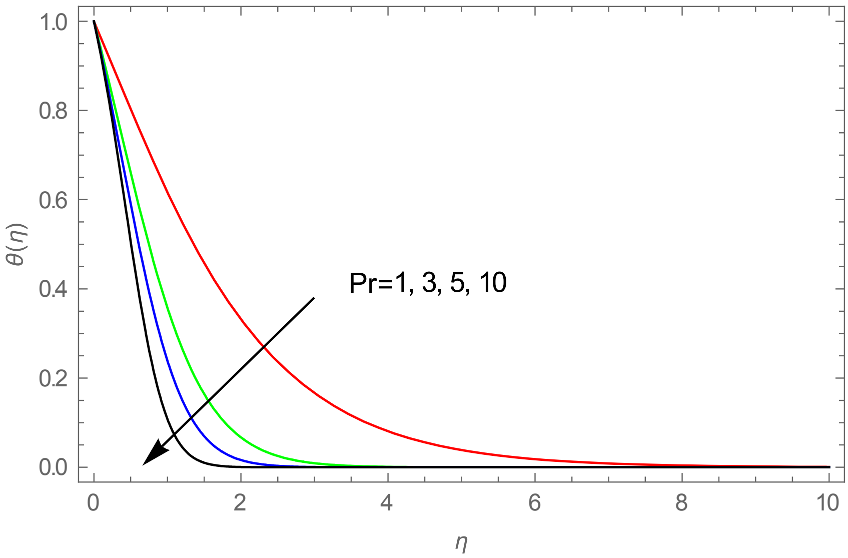

Fig. 4 – Effect of Prandtl number on dimensionless temperature at $\lambda = 1, Le = 5$ and $Nb = Nt = 0.1$.

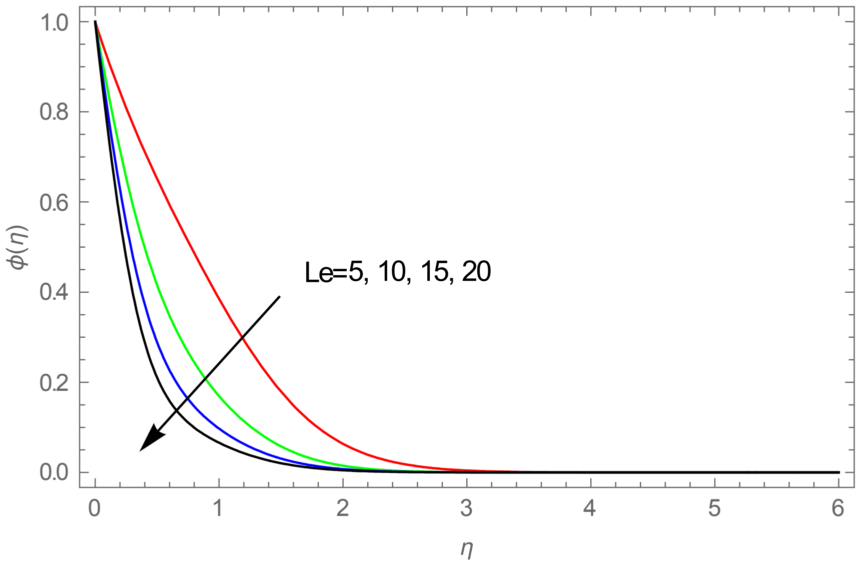

Fig. 5 – Effect of Lewis number on dimensionless concentration at $\lambda = 1, Le = 5$ and $Nb = Nt = 0.1$.

6. CONCLUSIONS

In this paper, an approach is proposed by using the ADM to solve a system of coupled nonlinear ordinary differential equations describing boundary layer flow of a nanofluid with partial slip over a stretching sheet. The obtained approximate analytical solutions were expressed in terms of the generalized incomplete Gamma function. The present analysis directly used the boundary conditions at infinity. In addition, the current numerical results coincide with those obtained by applying the homotopy analysis method in Ref. [24] by using forty iterations. Therefore, the ADM requires less computational work, which is the main advantage of this method.

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