NEW HEAT AND MAXWELL'S EQUATIONS ON CANTOR CUBES

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Abstract. The fractal physics is an important research domain due to its scaling properties that can be seen everywhere in the nature. In this work, the generalized Maxwell's equations are given using fractal differential equations on the Cantor cubes and the electric field for the fractal charge distribution is derived. Moreover, the fractal heat equation is defined, which can be an adequate mathematical model for describing the flowing of the heat energy in fractal media. The suggested models are solved and the plots of the corresponding solutions are presented. A few illustrative examples are given to demonstrate the application of the obtained results in solving diverse physical problems.

Key words: Fractal heat equation, fractal wave equation, fractal calculus, fractal Cantor cubes, staircase function.

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1. INTRODUCTION

Fractional calculus is an old topic but now it has found many applications in science and engineering [1–6]. Fractional differential equations have been solved using numerical methods [1]. The classical mechanics involves fractional derivatives used to model non-conservative systems [2, 4, 5]. The fractional calculus is used to model a series of physical processes on the fractal spaces and curves. The fractional derivatives on real-line are non-local and for the case of small orders near the zero violate the causality principle in physics [6]. The fractional derivatives are suitable to model the processes with memory effects [7]. The anomalous diffusion is modeled by fractional derivatives in Refs. [8, 9]. Recently, new fractional derivatives without singular kernel are defined and applied in thermodynamics [10, 11]. The fractal geometry is a new subject and has many applications in the real world [12]- [20]. The fractal analysis using different methods was applied in many branches of science and engineering [13–15]. In many relevant papers, the fractal calculus was built and applied in solving a lot of physics problems [16–26]. The Maxwell's equations are generalized involving fractal local derivatives as a new framework for the electromagnetic theory [27]. The fractal Fourier transformation is used to model Fraunhofer diffraction from Cantor set grating [28]. New non-local derivatives are defined on fractal sets that can be used in constructing a mathematical model for the physical processes with memory effects [29, 30].

The plan of the paper is as follows. In Sec. 2 we study and define the mathematical tools we need in the paper. The generalized fractal calculus on the Cantor cubes is handled to obtain Maxwell's equations on the Cantor cubes in Sec. 3. The wave and heat equations on the fractal Cantor cubes time-space are given in Sec. 4 and Sec. 5, respectively. In Sec. 6 we have solved illustrative examples as applications in physics settings. Finally, we give our conclusions in Sec. 7.

2. BASIC TOOLS IN THE FRACTAL CALCULUS

In this Section, we define some of the basic definition in $\mathfrak{F}^{\alpha}$-calculus on the Cantor cubes .

2.1. THE INTEGRAL STAIRCASE ON CANTOR CUBES

Let F be the triadic Cantor set [16, 28]. We define $\mathfrak{F} = F \times F \times F \subset \Re^3$ as a fractal Cantor cubes set that is the subset of $I = [a,b] \times [c,d] \times [e,f]$, $a,\ b,\ c,\ e,\ f\ \in \Re$ (Real-line). We plot the Cantor cubes with fractal dimension $\frac{\log 8}{\log 3}$ in Fig. 1. The

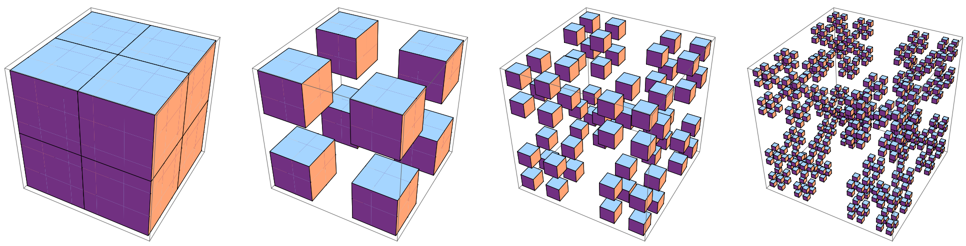

Fig. 1 – The finite iterations that create the fractal Cantor cubes $\mathfrak{F}$.

flag function for $\mathfrak{F}$ is defined as

$$\Theta(\mathfrak{F}, I) = \begin{cases} 1 & \text{if} \quad \mathfrak{F} \cap I \neq \emptyset \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Let us consider a subdivision of $I = [a,b] \times [c,d] \times [e,f]$ as follows

$$P_{[a,b]\times[c,d]\times[e,f]} = \{x_0 = a, x_1, x_2, ..., x_n = b\} \times \{y_0 = c, y_1, y_2, ..., y_n = d\}$$
$$\times \{z_0 = e, z_1, z_2, ..., z_n = f\} \tag{2}$$

We define $\gamma^{\xi}(\mathfrak{F},a,b,c,d,e,f)$ the mass function as

$$\gamma^{\xi}(\mathfrak{F},a,b,c,d,e,f)=\lim_{\delta\to 0}\inf_{P_{[a,b]\times[c,d]\times[e,f]}:|P|\leq\delta}\sum_{i=1}^{n}\frac{(x_i-x_{i-1})^{\alpha}}{\Gamma(\alpha+1)}\frac{(y_i-y_{i-1})^{\beta}}{\Gamma(\beta+1)}\frac{(z_i-z_{i-1})^{\mu}}{\Gamma(\mu+1)}$$
$$\times\Theta(F,[x_{i-1},x_i])\Theta(F,[y_{i-1},y_i])\Theta(F,[z_{i-1},z_i]), \quad (3)$$

where $\xi=\alpha+\beta+\mu$ and $0<\alpha\leq 1,\ 0<\beta\leq 1,\ 0<\mu\leq 1$. So that for the case of fractal Cantor cubes we have $\xi=0.6+0.6+0.6=1.8$.
The integral staircase function for the fractal Cantor cubes $S_{\mathfrak{F}}^{\xi}(x,y,z)$ of order ξ for a fractal set $\mathfrak{F}$ is defined

$$S_{\mathfrak{F}}^{\xi}(x,y,z)=\begin{cases}\gamma^{\xi}(\mathfrak{F},a_0,c_0,e_0,x,y,z) & \text{if} \quad x\geq a_0,\ y\geq c_0,\ z\geq e_0\\ -\gamma^{\xi}(\mathfrak{F},a_0,c_0,e_0,x,y,z) & \text{otherwise,}\end{cases} \quad (4)$$

where $a_0,\ c_0,\ e_0$ are arbitrary real numbers. The integral staircase shape is drawn for the fractal Cantor cubes in Fig. 2. The ξ-dimension of $\mathfrak{F}\cap[a,b]\times[c,d]\times[e,f]$ is

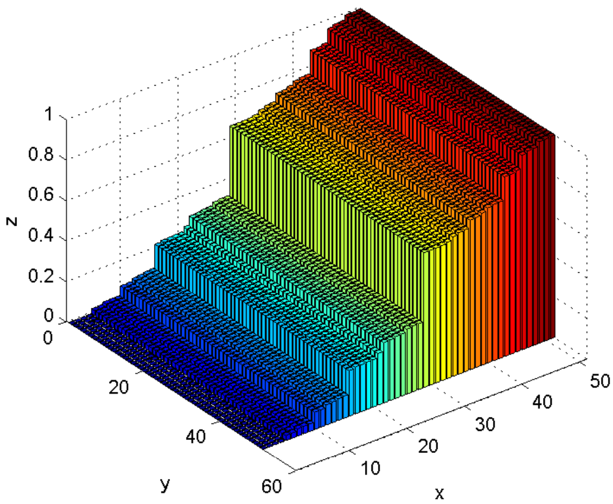

Fig. 2 – The integral staircase $S_{\mathfrak{F}}^{1.8}(x,y,z)$ is presented for the fractal Cantor cubes $\mathfrak{F}$ with dimension $\frac{\log 8}{\log 3}$.

defined as

$$\dim_{\xi}(\mathfrak{F}\cap[a,b]\times[c,d]\times[e,f]) = \inf\{\xi:\gamma^{\xi}(\mathfrak{F},a,b,c,d,e,f)=0\} \quad (5)$$
$$= \sup\{\xi:\gamma^{\xi}(\mathfrak{F},a,b,c,d,e,f)=\infty\} \quad (6)$$

A point (x,y,z) is a point of change of a $f(x,y,z)$ if it is not constant over any open set $(a,b)\times(c,d)\times(e,f)$ involving (x,y,z). The set of all points of change of a function is denoted by $Schf$. The $Sch(S_{\mathfrak{F}}^{\xi}(x,y,z))$ is called ξ-perfect if $S_{\mathfrak{F}}^{\xi}(x,y,z)$ is finite for all $(x,y,z)\in\Re$.

2.2. F^{ξ}-INTEGRATION

Consider $f(x,y,z)$ as a bounded function on $\mathfrak{F}$ so we define

$$M[f,\mathfrak{F},I]=\begin{cases}\sup_{x\in\mathfrak{F}\cap I}f(x,y,z), & \text{if } \mathfrak{F}\cap I\neq 0;\\ 0, & \text{otherwise.}\end{cases} \tag{7}$$

and similarly

$$m[f,\mathfrak{F},I]=\begin{cases}\inf_{x\in\mathfrak{F}\cap I}f(x,y,z), & \text{if } \mathfrak{F}\cap I\neq 0;\\ 0, & \text{otherwise.}\end{cases} \tag{8}$$

Now, we define the upper U^{ξ}-sum and the lower L^{ξ}-sum for function $f(x,y,z)$ on the subdivision P as [16,28].

$$\begin{aligned}U^{\xi}[f,\mathfrak{F},P]= & \sum_{i=1}^{n}M[f,F,[(x_{i-1},y_{i-1},z_{i-1}),(x_i,y_i,z_i)]]\\ & \times(S_{\mathfrak{F}}^{\xi}(x_i,y_i,z_i)-S_{\mathfrak{F}}^{\xi}(x_{i-1},y_{i-1},z_{i-1})),\end{aligned} \tag{9}$$

and

$$\begin{aligned}L^{\xi}[f,\mathfrak{F},P]= & \sum_{i=1}^{n}m[f,\mathfrak{F},[(x_{i-1},y_{i-1},z_{i-1}),(x_i,y_i,z_i)]]\\ & \times(S_{\mathfrak{F}}^{\xi}(x_i,y_i,z_i)-S_{\mathfrak{F}}^{\xi}(x_{i-1},y_{i-1},z_{i-1})).\end{aligned} \tag{10}$$

The $f(x,y,z)$ is $\mathfrak{F}^{\xi}$-integrable on $\mathfrak{F}$ if we have

$$\begin{aligned}&\underline{\int_{(a,c,e)}^{(b,d,f)}}f(x,y,z)d_F^{\alpha}xd_F^{\beta}yd_F^{\mu}z=\sup_{P_{[a,b]\times[c,d]\times[e,f]}}L^{\xi}[f,\mathfrak{F},P]\\ &=\overline{\int_{(a,c,e)}^{(b,d,f)}}f(x,y,z)d_F^{\alpha}xd_F^{\beta}yd_F^{\mu}z=\inf_{P_{[a,b]\times[c,d]\times[e,f]}}U^{\xi}[f,\mathfrak{F},P].\end{aligned} \tag{11}$$

The $\mathfrak{F}^{\xi}$-integral is denoted by $\int_{(a,c,e)}^{(b,d,f)}f(x,y,z)d_F^{\alpha}xd_F^{\beta}yd_F^{\mu}z$.

2.3. F^{α}-DIFFERENTIATION

Let us consider $\mathfrak{F}$ as a ξ-perfect set. Then the $\mathfrak{F}^{\xi}$-partial derivative of $f(x,y,z)$ with respect to x is defined as

$${}^{x}D_{\mathfrak{F}}^{\alpha}f(x,y,z)=\begin{cases}\mathfrak{F}-\lim_{(x',y,z)\to(x,y,z)}\frac{f(x',y,z)-f(x,y,z)}{S_{\mathfrak{F}}^{\xi}(x',y,z)-S_{\mathfrak{F}}^{\xi}(x,y,z)} & \text{if} \quad (x,y,z)\in\mathfrak{F}\\ 0, & \text{otherwise,}\end{cases} \tag{12}$$

if the limit exists. In addition, ${}^{y}D^{\alpha}_{\mathfrak{F}}f(x,y,z),{}^{z}D^{\alpha}_{\mathfrak{F}}f(x,y,z)$ can be defined as Eq. (12).

3. MAXWELL'S EQUATION ON THE FRACTAL TIME-SPACE

In this Section, we give the Maxwell's equation on the fractal Cantor cubes, which is a mathematical model for Maxwell's equation on fractal space-time.

3.1. GRADIENT, DIVERGENCE, AND CURL ON FRACTAL CANTOR CUBES

We define the gradient $\vec{\nabla}^{\xi}_{\mathfrak{F}}\varphi(x,y.z)$, divergence $\vec{\nabla}^{\xi}_{\mathfrak{F}}.\ \vec{A}(x,y,z)$, and curl $\vec{\nabla}^{\xi}_{\mathfrak{F}}\times \vec{A}(x,y,z)$ as follows

$$\begin{aligned}
&\vec{\nabla}^{1.8}_{\mathfrak{F}}\ \varphi = \hat{i}\ {}^{x}D^{0.6}_{F}\varphi + \hat{j}\ {}^{y}D^{0.6}_{F}\varphi + \hat{k}\ {}^{z}D^{0.6}_{F}\varphi \\
&\vec{\nabla}^{1.8}_{\mathfrak{F}}.\ \vec{A} = {}^{x}D^{0.6}_{F}\ A_x + {}^{y}D^{0.6}_{F}\ A_y + {}^{z}D^{0.6}_{F}\ A_z \\
&\vec{\nabla}^{1.8}_{\mathfrak{F}}\times\ \vec{A} = \hat{i}(\ {}^{y}D^{0.6}_{F}A_z - {}^{z}D^{0.6}_{F}A_y) + \hat{j}(\ {}^{z}D^{0.6}_{F}A_x - {}^{x}D^{0.6}_{F}A_z) \\
&+\hat{k}(\ {}^{x}D^{0.6}_{F}A_y - {}^{y}D^{0.6}_{F}A_x),
\end{aligned} \tag{13}$$

where $\vec{A}(x,y,z) = \hat{i}\ A_x + \hat{j}\ A_y + \hat{k}\ A_z$ and $\varphi(x,y,z)$ is a scalar function.

3.2. MAXWELL'S EQUATION ON THE FRACTAL CANTOR CUBES

We give the generalized Maxwell's equation on the fractal Cantor cubes on the vacuum as follows:

$$\vec{\nabla}^{1.8}_{\mathfrak{F}}.\ \vec{E}^{1.8}_{\mathfrak{F}} = \frac{\rho^{1.8}_{\mathfrak{F}}}{\epsilon^{\xi}_{F,0}}, \quad \text{Gauss's law} \tag{14}$$

$$\vec{\nabla}^{1.8}_{\mathfrak{F}}.\ \vec{B}^{1.8}_{\mathfrak{F}} = 0, \quad \text{Gauss's law for magnetism} \tag{15}$$

$$\vec{\nabla}^{1.8}_{\mathfrak{F}}\times\ \vec{E}^{1.8}_{\mathfrak{F}} = -{}^{t}D^{0.6}_{F}\ \vec{B}^{1.8}_{\mathfrak{F}}, \quad \text{Faraday's law} \tag{16}$$

$$\vec{\nabla}^{1.8}_{\mathfrak{F}}\times\ \vec{B}^{1.8}_{\mathfrak{F}} = \mu^{\xi}_{F,0}\ \vec{J}^{1.8}_{\mathfrak{F}} + \mu^{\xi}_{F,0}\ \epsilon^{\xi}_{F,0}\ {}^{t}D^{0.6}_{F}\vec{E}^{1.8}_{\mathfrak{F}}, \quad \text{Ampère-Maxwell's law} \tag{17}$$

where $\vec{E}^{1.8}_{\mathfrak{F}}$, $\vec{B}^{1.8}_{\mathfrak{F}}$, ϵ_0, μ_0, $\vec{J}^{1.8}_{\mathfrak{F}}$, and $\rho^{1.8}_{\mathfrak{F}}$ are fractal electric field, fractal magnetic field, permittivity of the fractal vacuum, permeability of the fractal vacuum, fractal electric current density, and fractal electric charge density, respectively.

4. WAVE EQUATION ON THE FRACTAL TIME-SPACE

The generalized wave equation on the fractal time-space is given as

$$({}^{t}D^{\alpha}_{F})^2 y^{\xi}_{F}(x,y,z,t) = \frac{1}{(v^{\xi}_{\mathfrak{F}})^2}\vec{\nabla}^{\xi}_{\mathfrak{F}}.\vec{\nabla}^{\xi}_{\mathfrak{F}}\ y^{\xi}_{F}(x,y,z,t), \tag{18}$$

where time changes on the fractal set F with dimension $\alpha = 0.6$ and the fractal space $F \times F \times F = \mathfrak{F}$ has dimension $\xi = 1.8$. Here, we arrive at the standard result by choosing $\alpha = 1$.

5. HEAT EQUATION ON THE FRACTAL TIME-SPACE

The heat equation has an important role in the study of Brownian motion and Fokker-Planck equation. It shows the distribution of heat in a given region over the time changes. Now, we suggest the generalized heat equation on the fractal time-space as follows

$$({}^{t}D_F^{\alpha})u_F^{\xi}(x,y,z,t) = \mathfrak{D}_f^{\xi}\,(\vec{\nabla}_{\mathfrak{F}}^{\xi})^2\; u_F^{\xi}(x,y,z,t) \tag{19}$$

where $\mathfrak{D}_f^{\xi}$ is the fractal coefficient of conductivity. Here $(\vec{\nabla}_{\mathfrak{F}}^{\xi})^2$ is the generalized Laplacian on fractal Cantor cubes.

6. EXAMPLES

In this Section, we present some illustrative examples to clarify the introduced models for possible applications in physics.

Example 1. We consider the uniformly distributed charge Q_F^{α} on the fractal Cantor set. The electric field due to this distribution at point P is shown in Fig. 3.

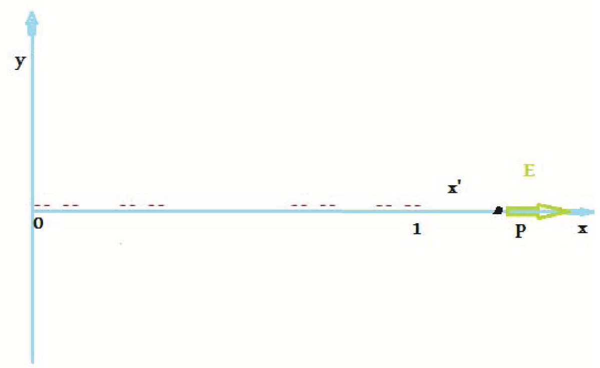

Fig. 3 – The fractal Cantor set and the point P where we want to calculate the electric field.

Then, in view of Fig. 3 and using the Coulomb's law on fractal sets we obtain theelectric field as

$$
\begin{aligned}
E_F^{\alpha}(x') &= \int_{S_F^{\alpha}(0)}^{S_F^{\alpha}(1)} \frac{k_F^{\alpha}\ \lambda_F^{\alpha}\ d_F^{\alpha}x}{(S_F^{\alpha}(1)+S_F^{\alpha}(x')-S_F^{\alpha}(x))^2},\\
E_F^{\alpha}(x') &= k_F^{\alpha}\lambda_F^{\alpha}\left[\frac{1}{S_F^{\alpha}(x')}-\frac{1}{S_F^{\alpha}(x')+S_F^{\alpha}(1)-S_F^{\alpha}(0)}\right],\\
&= \frac{k_F^{\alpha}\lambda_F^{\alpha}S_F^{\alpha}(1)}{S_F^{\alpha}(x')(S_F^{\alpha}(x')+1)} = \frac{k_F^{\alpha}Q_F^{\alpha}}{S_F^{\alpha}(x')(S_F^{\alpha}(x')+1)},
\end{aligned}
\tag{20}
$$

where $S_F^{\alpha}(1)=1,\ S_F^{\alpha}(0)=0,$ and $\alpha=0.6.$

We have plotted Eq. (20) as a function of x' in Fig. 4 in order to compare with the standard result.

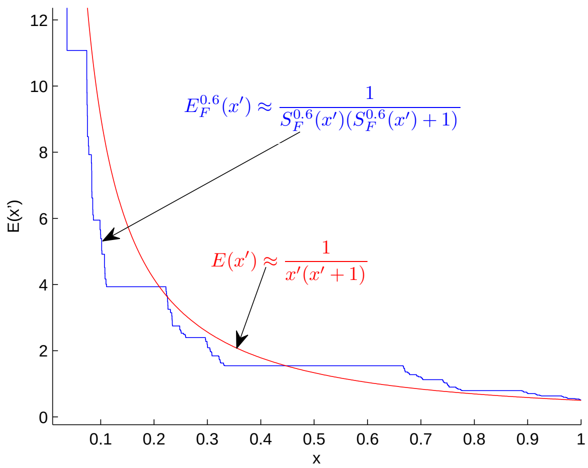

Fig. 4 – Plot of the electric field function due to Cantor set at point P.

Example 2. We consider the uniform charge distribution with density $\sigma_F^{1.2}$ on the Cantor dust; see Fig. 5 for the details.)

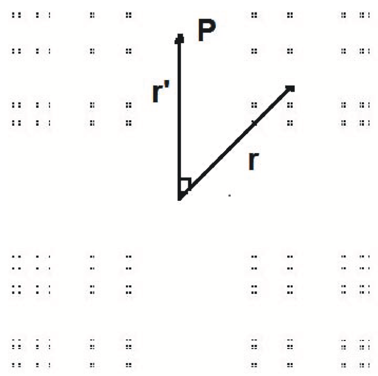

Fig. 5 – The charge distribution on the Cantor dust.

The electric field due to Cantor dust charge distribution at the point P using F^{α}-calculus is

$$d_{\mathfrak{F}}^{\xi}\vec{E}_{\mathfrak{F}}^{\xi}(\vec{r}_{\mathfrak{F}}^{\xi}) = \frac{k_{\mathfrak{F}}^{\xi}\,(\vec{r'}_{\mathfrak{F}}^{\xi} - \vec{r}_{\mathfrak{F}}^{\xi})}{|\vec{r'}_{\mathfrak{F}}^{\xi} - \vec{r}_{\mathfrak{F}}^{\xi}|^3} d_{\mathfrak{F}}^{\xi}x\; d_{\mathfrak{F}}^{\xi}y,$$

$$\vec{E}_{\mathfrak{F}}^{\xi}(\vec{r'}_{\mathfrak{F}}^{\xi}) = \int\int \frac{k_{\mathfrak{F}}^{\xi}\,(\vec{r'}_{\mathfrak{F}}^{\xi} - \vec{r}_{\mathfrak{F}}^{\xi})}{|\vec{r'}_{\mathfrak{F}}^{\xi} - \vec{r}_{\mathfrak{F}}^{\xi}|^3} d_{\mathfrak{F}}^{\xi}x\; d_{\mathfrak{F}}^{\xi}y, \tag{21}$$

where $\vec{r}_{\mathfrak{F}}^{\xi} = S_F^{\alpha}(x)\hat{i} + S_F^{\beta}(y)\hat{j},\;\; \vec{r'}_{\mathfrak{F}}^{\xi} = z\,\hat{k}$. Then, one can write

$$\vec{E}_{\mathfrak{F}}^{\xi}(\vec{r'}_{\mathfrak{F}}^{\xi}) = \int\int \frac{k_{\mathfrak{F}}^{\xi}\,(z\,\hat{k} - S_F^{\alpha}(x)\hat{i} + S_F^{\beta}(y)\hat{j})}{|z^2 + S_F^{\beta}(x)^2 + S_F^{\beta}(y)^2|^3} d_{\mathfrak{F}}^{\xi}x\; d_{\mathfrak{F}}^{\xi}. \tag{22}$$

Therefore, the asymptotic solution is

$$\vec{E}_{\mathfrak{F}}^{\xi}(\vec{r'}_{\mathfrak{F}}^{\xi}) = \frac{\sigma_{\mathfrak{F}}^{1.2}}{2\epsilon_{\mathfrak{F}}^{1.2}}, \quad S_F^{\alpha}(x) \to \infty, \quad S_F^{\beta}(y) \to \infty. \tag{23}$$

Example 3. Let us consider a wave equation on the fractal time-space as follows

$$({}^{t}D_F^{0.6})^2 y_F^{\xi}(x,t) = \frac{1}{(v_F^{0.6})^2}({}^{x}D_F^{0.6})^2 y_F^{\xi}(x,t). \tag{24}$$

It is straightforward that the solution of Eq. (24) is

$$y_F^{1.2}(x,t) = A_F^{1.2}\cos(k_F^{0.6}S_F^{0.6}(x) - \omega_F^{0.6}S_F^{0.6}(t)), \quad v_F^{0.6} = \frac{\omega_F^{0.6}}{k_F^{0.6}}. \tag{25}$$

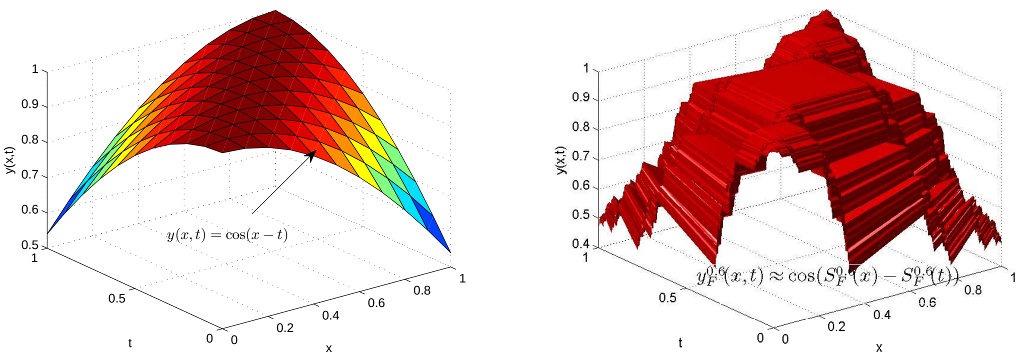

Fig. 6 – The plots of $y_F^{1.2}(x,t)$ and $y(x,t) \approx \cos(\omega - t)$.

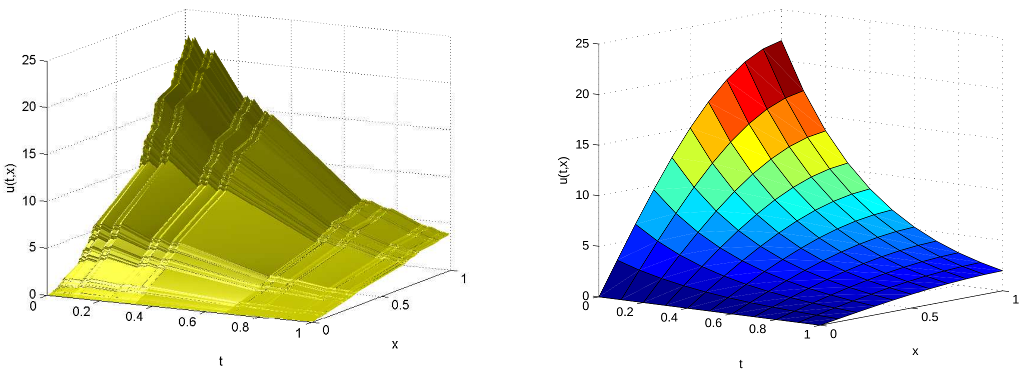

Fig. 7 – The plots of the solution of Eq. (26) and the standard solution in order to compare the results.

In Fig. 6 we have plotted $y_F^{1.2}(x,t)$ for the case of

$$A_F^{1.2} = k_F^{0.6} = \omega_F^{0.6} = \chi_F,$$

where χ_F is the characteristic function of the Cantor set. Here $y(x,t) \approx \cos(\omega - t)$ is the standard solution.

Example 4. We consider a heated fractal bar with the length $0 \leq x \leq 1$. The corresponding fractal heat time-space equation is

$$({}^{t}D_F^{0.6})u_{\mathfrak{F}}^{1.2}(x,t) = \mathfrak{D}_{\mathfrak{F}}^{1.2}\ ({}^{x}D_F^{0.6})^2\ u_{\mathfrak{F}}^{1.2}(x,t), \tag{26}$$

where the heat distribution $u_{\mathfrak{F}}^{1.2}(x,t)$ in the fractal bar has the boundary conditions

$$u_{\mathfrak{F}}^{1.2}(0,t) = 0, \qquad {}^{x}D_F^{0.6}u_{\mathfrak{F}}^{1.2}(1,t) = 0, \tag{27}$$

and initial condition as

$$u_{\mathfrak{F}}^{1.2}(x,0) = 2\sin(\frac{\pi}{2}\ x). \tag{28}$$

For solving Eq. (26) we suppose the solution is as

$$u_{\mathfrak{F}}^{1.2}(x,t) = f\left(S_F^{0.6}(t)\right)\sin\left(\frac{\pi\ S_F^{0.6}(x)}{2}\right). \tag{29}$$

Then by substituting Eq. (29) into Eq. (26) we have

$$({}^tD_F^{0.6})u_{\mathfrak{F}}^{1.2}(x,t) - \mathfrak{D}_{\mathfrak{F}}^{1.2}\ ({}^xD_F^{0.6})^2\ u_{\mathfrak{F}}^{1.2}(x,t) = \ {}^tD_F^{0.6} f\left(S_F^{0.6}(t)\right)\sin\left(\frac{\pi\ S_F^{0.6}(x)}{2}\right) \tag{30}$$

$$+\mathfrak{D}_{\mathfrak{F}}^{1.2}\frac{\pi^2}{4} f\left(S_F^{0.6}(t)\right)\sin\left(\frac{\pi\ S_F^{0.6}(x)}{2}\right) = 0. \tag{31}$$

Then we have

$${}^tD_F^{0.6} f\left(S_F^{0.6}(t)\right) + \mathfrak{D}_{\mathfrak{F}}^{1.2}\frac{\pi^2}{4} f\left(S_F^{0.6}(t)\right) = 0. \tag{32}$$

The solution for Eq. (32) is

$$f\left(S_F^{0.6}(t)\right) = f\left(S_F^{0.6}(0)\right)\exp\left(\frac{-\mathfrak{D}_{\mathfrak{F}}^{1.2}\ S_F^{0.6}(t)\pi^2}{4}\right) = 2\exp\left(\frac{-\mathfrak{D}_{\mathfrak{F}}^{1.2}\ S_F^{0.6}(t)\pi^2}{4}\right). \tag{33}$$

Finally, we arrive at

$$u_{\mathfrak{F}}^{1.2}(x,t) = 2\exp\left(\frac{-\mathfrak{D}_{\mathfrak{F}}^{1.2}\ S_F^{0.6}(t)\pi^2}{4}\right)\sin\left(\frac{\pi\ S_F^{0.6}(x)}{2}\right). \tag{34}$$

We have presented the graph of Eq. (34) and the standard solution in Fig. 7.

7. CONCLUSION

In this paper, we have generalized the F^α-calculus on the fractal Cantor cubes. The Maxwell's equation on the fractal Cantor cubes is given and the electric field due to Cantor dust is derived. We have also obtained the wave equation on the fractal time-space. Moreover, the heat equation on the fractal time-space is written as an application of the fractal calculus. Finally, using some illustrative examples, we have compared the obtained results with those given by the standard calculus.

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