DETERMINATION HALF-LIVE OF HEAVY NUCLEI USING FERMI GAS MODEL

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Abstract. Calculations on the α-decay half-lives of heavy nuclei are performed through Fermi gas model. The Wood Saxon potential is employed to calculate half–lives through a coulomb barrier. The present study is initially restricted to even-even nuclei in the heavy mass region with N>126. Then the study is extended to the heaviest nuclei including super heavy elements. The main consideration in this model is V_0b^2=100 MeV, where b is the force range between two nucleons which represents the attractive well, and $-V_0$ is depth. The calculated α-decay half lives are found to be in good agreement with the experimental data for heavy nuclei with approximately spherical shape. The results are independent from deformation parameters for super heavy nuclei.

Key words: Fermi gas model, heavy and Super-heavy nuclei, half lives.

1. INTRODUCTION

The study of alpha decay dates back to the early days of nuclear physics, even to the first observation of unknown radiation by Becquerel in 1896. With the foundation and development of quantum mechanics, Gamow [1], Condon and Gurney [2] independently described α-decay as a quantum tunneling problem for the first time in 1928. These pioneering works were the first successful applications of quantum mechanics to nuclear physics. Different from the cluster model based on the Gamow picture, some other theoretical models, such as the shell model and fission like model, have also been proposed in the pursuit of a microscopic description of α-decay. Consequently, the absolute α-decay width has been estimated by many theoretical calculations [3-17], which employ various approaches such as WKB method [4-7], coupled channel approach [14, 15] and phenomenological methods [16, 17]. On the experimental side, as an engaging topic in contemporary nuclear physics, the observation of α-decay chains from unknown parent nuclei to known nuclei has been a reliable method used to identify different super heavy elements (SHEs) and isometric states [18-21]. Moreover, α-decay, as one of the most important decay modes for unstable nuclei, has been a useful and precise tool in the investigation of nuclear structure for a long time. In the present research, the Fermi gas model is employed to calculate α-decay half–lives for heavy and super heavy nuclei. The features of this nuclear model has been used to solve the Schrodinger equation numerically to evaluate wave functions of alpha particles at attractive nuclear potential and the beyond region of the coulomb barrier between alpha particle and the rest of nuclei as daughter nuclei.

2. THEORETICAL FRAMEWORK

The Fermi Gas model has two approaches about the parts of potential, including nuclear attraction at r<R and Coulomb potential at $r \geq R$, for which the exclusion Pauli principle is considered [23, 27]. Following sections reveal the details of calculations in this article.

2.1 DEPTH AND WIDTH FOR NUCLEAR POTENTIAL WELL

In our attempt to account for some nuclear properties with Fermi gas model as simple and crude model, we should concentrate initially on the parameters empirically observed in the Weizsacker formula. Consider a system of a number of neutrons and protons put in a cubic box with linear dimension. The Schrodinger equation for a single particle in this box reduces to

$$-\frac{\hbar^2}{2M}\nabla^2\psi = E\psi \tag{1}$$

The solution of equation (1) with boundary conditions is given by

$$\psi(x,y,z) = A.sink_x.sink_y y.sink_z.z\ , \tag{2}$$

where A is a normalization factor and

$$k_x a = n_x\pi \quad , \quad k_y a = n_y\pi \qquad and \qquad k_z a = n_z\pi \quad , \tag{3}$$

where n_x, n_y, and n_z are all positive integers [23].
Each set of positive integers (n_x, n_y, n_z) defines a different solution corresponding to an energy

$$E(n_x, n_y, n_z) = \frac{\hbar^2}{2M}\left(k_x^2, k_y^2, k_z^2\right) = \frac{\hbar^2}{2M}\boldsymbol{k}^2 \tag{4}$$

It is obvious from relation (4) as well as equation (2) that **k**= $\left(k_x^2, k_y^2, k_z^2\right)$ is the momentum of the particle in the box. Because of the Pauli principle, a given momentum state can be occupied mostly by four nucleons. The number of permissible solutions n(k) with the magnitude of **k** between k and k+dk is given by

$$dn(k) = \frac{1}{8}4\pi k^2\, dk.\frac{1}{(\frac{\pi}{a})^3} \tag{5}$$

afterwards, the highest occupied momentum state, k_f, will be given according to relation (6)

$$\frac{A}{4} = \frac{4\pi}{3}\frac{k_f{}^3}{8(\pi/a)^3} \text{ or } A = \frac{2\Omega}{3\pi^2}k_f^3 \tag{6}$$

where $\Omega=a^3$ is the volume of the box. Under these conditions, the momentum of the highest occupied state depends only on the density $\rho = A/\Omega$ of nucleons in the box, and is given by $\rho = \frac{2}{3\pi^2}k_f^3$. Thus far the interaction among the nucleons is disregarded. Within this limit the momentum distribution per unit volume in momentum space is a step function with a constant value for $k<k_f$ and zero for $k>k_f$. This momentum distribution is referred to as the Fermi distribution. From the observed density of nuclei, $\rho = 1.72 \times 10^{38}\text{particle/cm}^3$, which, as known, is practically the same for all nuclei with $A \geq 12$, we obtain [23],

$$k_f = 1.36fm^{-1} \text{ with corresponding energy of } \epsilon_f = 38\ MeV \tag{8}$$

k_f and ϵ_f are called the Fermi momentum and Fermi energy of the degenerate gas model respectively [23]. To calculate the depth of nuclear well potential according to Fermi gas model and Bethe Weizsacker semi empirical mass formula, we can consider the binding energy per nucleon which is added to Fermi energy, as

$$V_0 = \epsilon_f + B/A \tag{9}$$

Since heavy and super-heavy nuclei have surplus of neutrons, the Fermi level of the protons and neutrons in a stable nucleus has to be equal. The increase in kinetic energy with density is easy to understand if we recall the *Pauli repulsion* among nucleons. Because of this repulsion, an increase in density is accompanied by an increase in the density of nodes in the wave function and therefore an increase in its curvature. Therefore the average curvature is of the order of magnitude of distance between the nucleons, that is, $\rho^{-1/3}$. The kinetic energy is quadratic in the inverse curvature of the wave function and, hence, the factor $\rho^{2/3}$.

To obtain an estimation of the equilibrium density of nuclei, we have to know also the density dependence of the potential energy. We can estimate this in the following way. Take the interaction between two nucleons to be represented by an attractive *square well of range b* and depth $-V_0$ ($V_0>0$). At a given density, there is a probability, p, for a nucleon to be the range of forces of other prescribed nucleons. The total contribution to the potential energy will, therefore, be

$$< V > = \frac{-A(A-1)}{2} pV_0 \tag{10}$$

The probability, p, can be estimated for large nuclei, to the extent that we can neglect surface effects. It is just the ratio of the interaction volume to the total volume, that is,

$$p = \frac{(\frac{4}{3})\pi b^3}{\Omega} = \frac{4}{3}\pi\frac{1}{A}b^3\rho \quad for \quad \Omega \gg b^3 \tag{11}$$

It seems that this conclusion is in disagreement with the experimental data. Nuclei are experimentally known to have more or less a constant density, but certainly not a constant radius. Additionally, their binding energy is roughly proportional to A, but certainly not proportional to A^2. We must take into account the repulsive parts of the nuclear interaction rather than the attractive square well adopted for the nucleon–

nucleon interaction. The equilibrium density observed in nuclei must be the combined the effects of the Pauli repulsion, the dominant nucleon-nucleon attraction, and the particular features of the nucleon-nucleon repulsion. As the kinetic energy is proportional to $\rho^{2/3}$ or inversely proportional to r_0^2 , $2r_0$ is the average spacing between nucleons. If r_c is the radius of the repulsive core and the space between the nucleons equals r_c, then the kinetic energy should go to infinity. For the average potential energy, we take the relation (11) with p again for finite nuclei. Noting that $R=r_0A^{1/3}$, we obtain the expression for the average potential energy

$$< V > = -V_0 \frac{A}{2}\left(\frac{b}{r_0}\right)^2\left[1 - \frac{9}{16}\frac{b}{R} + \frac{1}{32}\left(\frac{b}{R}\right)^3\right] \tag{12}$$

To obtain the equilibrium density, the $\langle T\rangle + \langle V\rangle$ has to minimize with respect to r_0.

$$V_0 b^2 = \frac{4\alpha}{3\left(\frac{b}{r_0}\right)\left[1-\frac{3}{4}\frac{b}{r_0A^{\frac{1}{3}}}+\frac{1}{16}\left(\frac{b}{r_0A^{\frac{1}{3}}}\right)^3\right]\left[1-\left(\frac{r_c}{r_0}\right)\right]^3} \tag{13}$$

The V_0b^2 is known from the low energy scattering data and a good approximation is $V_0b^2 = 100\ MeV\ (fm)^2$. On the basis of Fermi gas model, this relation is the main condition to determine the width of nuclear potential well [23].

2.2 COULOMB ENERGY (C.E) in FERMI GAS MODEL

At the liquid drop model, nucleus to be considered as a uniformly charged sphere of charge *Ze* and charge density $\rho = \frac{Ze}{\frac{4}{3}\pi R^3}$. The electrical energy of the nucleus is therefore [28],

$$V_{coulomb} = \int_0^R \frac{4}{3}\pi R^3 \rho . 4\pi r^2 dr \rho . \frac{1}{r} = \frac{3}{5}\frac{Z^2 e^2}{R} \tag{14}$$

In Fermi gas model approach the charge density is the function of charge radius and thickness of nucleus. Initially this function has approximately a trapezoidal shape from the function is drawn. In this approach the amount of thickness approximately equal to 3.5 fm. As the quoted empirical value of the surface thickness, the distance between the radii at which ρ = 0.9 and 0.1 of the central value, the thickness is predicted, according to trapezoidal distribution, 2.8 fm.

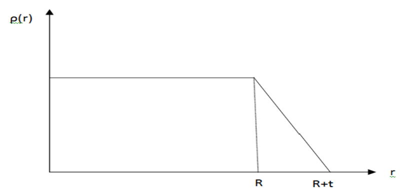

Fig. 2- The trapezoidal charge distribution assumed in deriving surface thickness

The effect of the Pauli repulsion and Pauli attraction is illustrated by the simple but important example of the coulomb energy for nuclear ground state in reference [23]. According to

$$C.E = \frac{A(A-1)}{2}\int \rho_{00}(r_1, r_2)\, v_{12}(r_1, r_2)\, dr_1\, dr_2 \qquad (15)$$

Where v_{12} is given by

$$v_{12} = \frac{e^2}{r_{12}}\left[\frac{1+\tau_3(1)}{2}\right]\left[\frac{1+\tau_3(2)}{2}\right]$$

$$r_{12} \equiv |r_1 - r_2|$$

the isospin factors guarantee that the interaction takes place only between protons.
By changing variables from r_1 and r_2 to $r_1$and **r** $(\equiv r_1 - r_2)$ and evaluating the integral, it becomes

$$(C.E)_{ex} = \frac{9\pi}{4}\frac{Z(Z-1)e^2}{\Omega k_f^2}\left[1 - j_0^2(k_f R) - j_1^2(k_f R)\right] \qquad (16)$$

Where j_0 and j_1 are the spherical Bessel function. These spherical Bessel function terms can be dropped as $k_f R$ becomes large. Hence a very good approximation is

$$(C.E)_{ex} = \frac{9\pi}{4}\frac{Z^2 e^2}{\Omega k_f^2} \qquad (17)$$

The total Coulomb energy is then

$$C.E. = \frac{3}{5}\frac{Z^2 e^2}{R} - (C.E)_{ex} \rightarrow C.E = \frac{0.77Z^2}{A^{\frac{1}{3}}}\left(1 - \frac{1.00}{A^{\frac{2}{3}}}\right)MeV \qquad (18)$$

For achievement to realistic charge distribution issue the Wood – Saxon function of charge distribution is utilized in calculations, substituted by what is considered in

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-\bar{R}}{a}\right)} \qquad (19)$$

Where, $\bar{R}$ is the average of radius and a is the diffuseness coefficient with constant amount equal to 0.54 fm [24]. Thus, the total radius of nucleus, R_{total} , can be calculated by accumulation of average radius, $\bar{R}$ and the thickness of nucleus with

$$R_{total} = \bar{R} + 2aLn(3) \qquad (20)$$

The thickness is considered as the change of nuclear charge distribution from 90% initial value to 10% of it [30]. The rms of radius is then conveniently written as,

$$\bar{R} \equiv \langle r^2 \rangle^{1/2} = \left[\frac{\int \rho(r) r^4 dr}{\int \rho(r) r^2 dr}\right]^{1/2} \qquad (21)$$

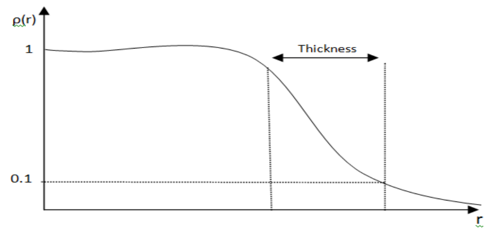

Fig. 3- Wood- Saxon function is supposed to charge distribution in nucleus

For large A, relation (18) should be compared with the semi empirical result of Myers and Swiatecki [24], whom considered Fermi, (Wood –Saxon), distribution as charge distribution in nucleus.

$$C.E = \frac{0.71Z^2}{A^{\frac{1}{3}}}\left(1 - \frac{1.69}{A^{\frac{2}{3}}}\right)MeV \tag{22}$$

In this article all Coulomb energies of alpha –decay nuclei are calculated by relation (22) which is modified by adding the effect of thickness and changing the charge distribution function from trapezoidal shape in relation (18) to Wood –Saxon.
There is no remarkable difference between charge distribution functions for calculation alpha decay half lives for heavy nuclei while the Wood - Saxon function could emerge results which are significantly closed to experimental data. Since all even – even nuclei and their isotopes are in 0^+ situations, the centrifugal term is eliminated from calculations.

2.3 NUMERICAL SOLUTION OF SCHRODINGER EQUATION

The half life for radioactive decay of the heavy and super-heavy nuclei can be computed by numerical solution of the Schrodinger equation, by "finite difference method" in MATLAB software [29]. The concept of this method is the approximation of the second derivative by the difference formula

$$\frac{d^2\psi(x)}{dx^2} = \frac{\psi(x+\Delta x) - 2\psi(x) + \psi(x-\Delta x)}{\Delta x^2} \tag{23}$$

The Schrodinger equation is

$$\frac{d^2\psi(x)}{dx^2} = -k(x)^2\psi(x), \text{ where } k(x) = \sqrt{\left(\frac{2m}{\hbar^2}\right)(E - U(x))} \tag{24}$$

In the finite difference method, it could be started with

$$x = [x(1), x(2), \dots, x(N)] \qquad k = [k(1), k(2), \dots, k(N)] \tag{25.a}$$

where N is the maximum number of x coordinates, $x(1)=x_{\min}$ and $x(N)=x_{\max}$, therefore

$$\psi(x_{min}) = \psi(x(1)) = 0 \text{ and } \psi(x(2)) = \psi(x(1) + \Delta x) = 1 \tag{25.b}$$

Then as x is incremented, the other values of $\psi(x)$ are calculated from the equation

$$\psi(x_{c+1}) = ((2 - (k_c\Delta x^2)^2)\psi(x_c) - \psi(x_{c-1}) \text{ for } c = 2 \text{ to } N-1 \tag{25.c}$$

When a physically accepted solution is found for n^{th} state, the wave function is normalized by numerically integrating the wave function using Simpson rule, so

$$\int_{x(1)}^{x(N)} |\psi(x)|^2 dx = A_n \;\rightarrow\; \psi_n(x) = \frac{\psi(x)}{\sqrt{A_n}} \tag{25.d}$$

In Schrodinger equation, energy E is given by Q_α, the energy of alpha particle, and potential energy U(x) is given by

$$U(x) = \begin{cases} -V_0 & r<b \\ \frac{ZZ'e^2}{r} & r > b \end{cases} \tag{26}$$

Moreover, for numerical solution of relation (24), it is necessary to determine dimensions of nuclear well potential and the Coulomb energy (C.E) of potential barrier which could be found in section 2.1 and 2.2. To determine the Q_α, it is necessary to notice tunneling process of alpha particle through the Coulomb barrier. In this case, two turning points are shown in figure 4. The first one, *b*, is on the edge of nuclear well potential where the particle proceeds to tunneling and the second turning point, b', is in the point of escaping particle from the Coulomb barrier. So the relation (27) determines the value of Q_α as;

$$C.E = \frac{ZZ'e^2}{b} \rightarrow ZZ'e^2 = b \times C.E \qquad (27.a)$$

$$V(b') = Q_\alpha = \frac{ZZ'e^2}{b'} \qquad (27.b)$$

$$Q_\alpha = \frac{b \times (C.E)}{b'} \qquad (27.c)$$

By using relations (27), (22) and (20) we can determine the first input parameter to solve the Schrodinger equation numerically. To calculate the half life for each nucleus, it is necessary to determine mass number, A, and atomic number, Z, for the parent nucleus in addition to the Q_α .

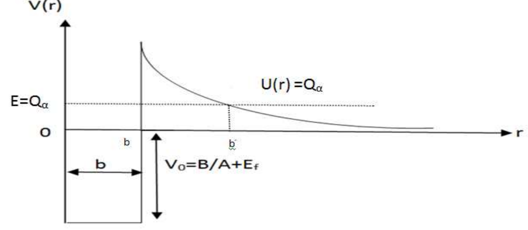

Fig. 4- Alpha particle tunneling through the Coulomb barrier.

The relation to calculate alpha decay half life of nuclei is

$$t_{1/2} = \frac{Ln(2)}{fP} \qquad (28)$$

In relation (28), *f* is frequency and is given by

$$f = \frac{v_\alpha}{2b} \qquad (29)$$

V_α is velocity of alpha particle between turning points and given by

$$V_\alpha = \sqrt{\frac{2Q_\alpha}{m_\alpha}} \qquad (30)$$

And the probability, *P*, is given by,

$$P = \left(\frac{A_{out}}{A_{in}}\right)^2 \qquad (31)$$

where, A_{in} and A_{out} are amplitudes of wave functions inside the potential well and outside the well respectively[31]. The Schrodinger equation must be solved for a potential energy function which represents the system of the parent nucleus and the alpha particle to find amplitudes.

3. RESULT AND DISCUSSION

After calculating alpha decay half life in even–even states for heavy nuclei, it is realized that there is a good agreement between theoretical assumptions on the basis of Fermi gas model and experimental data [34,35], just for heavy nuclei, having an approximately spherical shapes and a small amount of deformation β parameter, including Quadrupole deformation, with respect to heavier isotopes in each group of heavy nuclei. The higher deviation between theoretical and experimental data, the higher β deformation parameter in isotopes. For example one can see good agreement between theoretical and experimental data for alpha decay half life for $^{222}_{92}U$ with $\beta_2 = 0.008$, whereas by turning to heavier isotopes, the deformation parameter gets rise to $\beta_2 = 0.241$ for $^{238}_{92}U$, and the discrepancy between Fermi gas model calculation results and empirical data is obvious. In Po isotopes, the nuclear deformation is changing from oblate shape for the lightest isotopes into a rather spherical shape for the heaviest isotopes, so the discrepancy between theoretical calculations and experimental data is extremely likely due to the effect of shape coexistence [33]. Furthermore, due to similarity treatment between the isotopes at the end of table 1, for example No and Rf isotopes, and the super heavy nuclei, one can see the gradual independency of alpha decay half life to the deformation parameters.

Table 1.
Experimental and calculated α-decay half lives for even-even nuclei with neutron number N>126.

Element	A	Z	B.E(MeV)	U_0(MeV)	b(fm)	E_c (MeV)	R_{total}(fm)	E_α(MeV)	β_2	$(T_{1/2})_{exp}$(S)	$(T_{1/2})_{cal}$ (S)
Po	212	84	1655.70	45.80	1.477	790.98	7.08	164.98	0.055	2.99×10^{-7}	3.08×10^{-7}
Po	214	84	1666.02	45.78	1.477	788.61	7.09	164.28	0.022	1.64×10^{-4}	1.80×10^{-6}
Po	216	84	1675.91	45.75	1.478	786.41	7.11	163.13	0.000	1.45×10^{-1}	3.88×10^{-7}
Po	218	84	1685.40	45.73	1.478	784.24	7.13	162.24	0.009	$1.86\times10^{+2}$	9.17×10^{-7}
Rn	214	86	1664.00	45.77	1.478	826.84	7.09	171.92	0.014	2.70×10^{-7}	4.40×10^{-6}
Rn	216	86	1675.88	45.75	1.478	824.24	7.11	170.99	0.000	4.50×10^{-5}	2.89×10^{-6}
Rn	218	86	1687.06	45.69	1.479	822.26	7.12	170.37	0.009	3.50×10^{-2}	4.71×10^{-6}
Rn	220	86	1697.80	45.71	1.479	820.00	7.14	169.38	0.020	$5.56\times10^{+1}$	1.69×10^{-7}
Rn	222	86	1708.10	45.69	1.479	817.77	7.16	168.46	0.039	$3.31\times10^{+5}$	4.41×10^{-7}
Ra	216	88	1671.00	45.73	1.478	863.56	7.11	179.51	0.000	1.82×10^{-7}	9.00×10^{-6}
Ra	218	88	1684.06	45.72	1.478	861.18	7.13	178.51	0.008	2.56×10^{-5}	6.65×10^{-6}
Ra	220	88	1696.58	45.71	1.479	858.81	7.14	177.34	0.008	1.81×10^{-2}	1.79×10^{-6}
Ra	222	88	1708.6	45.69	1.479	856.48	7.16	176.39	0.040	$3.92\times10^{+1}$	1.04×10^{-6}
Ra	224	88	1720.30	45.67	1.479	854.17	7.18	175.45	0.111	$3.33\times10^{+5}$	2.22×10^{-7}
Ra	226	88	1731.60	45.66	1.479	851.88	7.19	174.74	0.142	5.35×10^{10}	2.44×10^{-6}
Th	218	90	1676.70	45.69	1.479	901.00	7.13	186.27	0.008	1.17×10^{-7}	3.03×10^{-6}
Th	220	90	1690.60	45.68	1.479	898.52	7.14	185.56	0.008	9.70×10^{-6}	1.78×10^{-6}
Th	222	90	1704.20	45.67	1.479	896.08	7.16	184.54	0.095	2.29×10^{-3}	2.77×10^{-6}
Th	224	90	1717.57	45.66	1.479	893.66	7.17	183.79	0.103	1.33×10^{0}	4.09×10^{-5}
Th	226	90	1730.50	45.65	1.480	891.27	7.19	182.79	0.129	$2.43\times10^{+3}$	2.31×10^{-6}
Th	228	90	1743.08	45.64	1.480	888.91	7.20	182.06	0.179	$8.35\times10^{+7}$	8.30×10^{-6}
Th	230	90	1755.10	45.63	1.480	886.57	7.22	181.08	0.202	3.12×10^{12}	1.59×10^{-5}
Th	232	90	1766.99	45.61	1.480	884.25	7.24	180.14	0.217	5.67×10^{17}	2.70×10^{-5}
U	222	92	1695.50	45.63	1.480	936.58	7.16	192.92	0.008	1.25×10^{-6}	7.40×10^{-5}
U	224	92	1710.29	45.63	1.480	934.05	7.17	192.06	0.030	7.29×10^{-4}	8.38×10^{-5}
U	226	92	1724.81	45.63	1.480	931.55	7.19	191.00	0.153	4.12×10^{-1}	9.22×10^{-5}
U	228	92	1739.06	45.62	1.480	929.08	7.20	190.20	0.164	$8.45\times10^{+2}$	1.00×10^{-4}
U	230	92	1752.82	45.62	1.480	926.63	7.22	189.16	0.228	$2.67\times10^{+6}$	1.06×10^{-4}
U	232	92	1765.90	45.61	1.480	924.21	7.24	188.00	0.230	$3.19\times10^{+9}$	3.15×10^{-6}
U	234	92	1778.50	45.60	1.480	921.82	7.25	187.38	0.224	1.08×10^{13}	1.14×10^{-4}

U	236	92	1790.40	45.58	1.481	919.45	7.27	186.50	0.261	1.00×10^{15}	1.19×10^{-3}
U	238	92	1801.60	45.56	1.481	917.11	7.28	185.00	0.241	5.67×10^{17}	1.08×10^{-3}
Fm	246	100	1837.10	45.46	1.483	1073.70	7.34	215.58	0.224	1.49×10^{3}	0.007×10^{3}
Fm	248	100	1851.50	45.46	1.483	1071.10	7.34	215.00	0.234	4.81×10^{5}	0.001×10^{5}
Fm	250	100	1865.50	45.46	1.483	1068.40	7.37	213.70	0.234	2.65×10^{-2}	1.50×10^{-2}
Fm	252	100	1879.00	45.45	1.483	1065.90	7.39	212.00	0.235	1.09×10^{-3}	1.30×10^{-2}
Fm	254	100	1891.00	45.44	1.483	1063.30	7.40	211.00	0.299	1.34×10^{-4}	7.70×10^{-2}
Fm	256	100	1902.54	45.43	1.483	1060.80	7.42	210.84	0.304	1.35×10^{-5}	4.90×10^{-2}
No	252	102	1871.30	45.42	1.483	1109.10	7.39	221.36	0.235	4.71×10^{3}	0.065×10^{3}
No	254	102	1885.60	45.42	1.483	1106.50	7.40	220.40	0.235	7.08×10^{5}	0.80×10^{0}
No	256	102	1871.30	45.30	1.485	1103.80	7.42	219.68	0.245	3.36×10^{0}	0.02×10^{0}
Rf	256	106	1877.65	45.33	1.485	1192.60	7.42	237.13	0.252	2.38×10^{0}	0.05×10^{0}
Rf	258	106	1893.53	45.34	1.485	1189.70	7.43	236.00	0.237	9.23×10^{-2}	2.06×10^{-2}

As the extension of calculations for super-heavy nuclei, the results show that there is a good agreement between the data concerning to theoretical calculation on the basis of Fermi gas model and experimental data [36-39]. The analysis of super-heavy nuclei data correlation, shown in table 2, illustrates the independency of theoretical calculations from deformation β parameters in various SHNs. therefore, it can produce the reliability of Fermi gas model application for super heavy nuclei structure to study as a crude and simple nuclear model in comparison with the others like Shell model and Nilsson model.

Table 2.

Comparison of the experimental and calculated α-decay half lives for super-heavy nuclei. At the rest two columns the agreement between half lives from theoretical and experimental data are shown.

Element	A	B.E(MeV)	U_0(MeV)	b(fm)	E_c (MeV)	R_{total} (fm)	E_α (MeV)	β_2	$(T_{1/2})_{exp}$ (S)	$(T_{1/2})_{cal}$ (S)
108	264	1926.57	45.29	1.485	1226.70	7.48	242.00	0.239	0.85×10^{-3}	16×10^{-3}
106	266	1951.96	45.33	1.485	1178.70	7.49	232.15	0.229	25.7000	0.084
108	270	1970.53	45.29	1.485	1218.30	7.52	238.88	0.230	3.6000	0.4503
108	266	1941.71	45.30	1.485	1223.80	7.49	241.07	0.229	0.0023	0.3800
110	270	1959.08	45.26	1.486	1264.00	7.52	248.00	0.230	1×10^{-4}	54×10^{-4}
111	278	2010.02	45.23	1.486	1275.80	7.57	248.69	0.222	0.0042	0.0466
113	282	2027.37	45.19	1.487	1316.60	7.60	255.70	0.064	0.073	5.200
111	280	223.70	45.22	1.487	1273.00	7.58	248.00	0.202	3.600	0.849
113	284	2041.34	45.18	1.487	1313.80	7.61	255.00	0.117	0.48	3.08
115	288	2058.37	45.14	1.488	1355.10	7.65	261.00	0.080	0.087	1.800
112	284	2047.30	45.2	1.487	1290.50	7.61	248.00	0.108	9.80	3.70
114	288	2064.90	45.17	1.487	1331.50	7.64	258.00	0.089	0.80	3.20
116	292	2081.93	45.13	1.488	1373.10	7.67	264.20	0.053	0.018	2.400
107	274	1999.20	45.29	1.485	1190.40	7.54	233.00	0.212	78.00	0.22
109	278	2018.40	45.26	1.486	1230.00	7.57	239.70	0.192	11.00	0.48
111	282	2036.95	45.22	1.487	1270.20	7.60	246.64	0.136	0.74	1.48

113	286	2054.87	45.18	1.487	1311.00	7.63	253.50	0.099	28.30	2.00
115	290	2072.19	45.14	1.488	1352.20	7.65	261.00	0.072	0.023	0.029
117	294	2088.90	45.10	1.489	1394.00	7.68	270.00	- 0.073	0.078	0.30
114	286	2051.25	45.17	1.488	1334.40	7.63	258.00	0.089	0.26	0.20
114	[1]289	2070.94	45.24	1.486	1330.10	7.65	258.00	0.089	2.10	4.10
113	[1]285	2048.80	45.18	1.487	1312.40	7.62	254.00	0.099	5.50	7.30
112	[1]285	2053.18	45.20	1.487	1289.20	7.62	249.50	0.108	29.00	4.40

4. CONCLUSION

α–decay half lives for even-even heavy nuclei are calculated on the basis of Fermi gas model approaches, and the results are in a good agreement with experimental data for spherical shape of heavy nuclei. Extending the calculation for SHNs, the results are in accordance with empirical data without any considerations for deformation parameters. This result shows that the Fermi gas model could be applied to study the structure of nuclei in SHNs and as a simple nuclear model among other nuclear structure models, it has the potential to be extended to further investigation of α-decay properties.

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