

# SPATIAL AND TEMPORAL DYNAMICS OF FEW-CYCLE LASER BEAMS IN DISPERSIVE MEDIA

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**Abstract.** The spatio-temporal equivalence  $s=c \cdot t$ , where  $c$  is the speed of light and  $s$  is the spatial extent of ultra-short laser pulses of duration  $t$  is investigated after propagation through dispersive media using 2D modeling of the electromagnetic pulses. The spatial extension of the ultra-short pulses has been quantified after propagation through different media in the presence of pulse duration variation. The result is explained in correspondence with the extension of the Rayleigh range and it is relevant for a wide range of ultra-short pulse laser applications where tightly-focused few-cycle pulses (TFP) are required.

*Keywords:* Gaussian beam; Rayleigh range; Electromagnetic field; Ultra-short laser pulses.

## 1. INTRODUCTION

The spatial and temporal structure of the electromagnetic field in focus was extensively studied in the last years [1-4]. It is commonly assumed that the temporal properties of the pulse are the same for every spatial position. This hypothesis is often wrong; several recent studies concerning the spatio-temporal coupling have already proved the interdependence of temporal and spatial coordinates exhibited by the ultra-short laser pulses [5, 6]. Investigating the behavior of few-cycle laser pulses focused to diffraction limited spots represents a central point of interest for many research areas such as ultra-short pulse laser – matter interaction or experiments that combine ultra-short and ultra-intense laser pulses and relativistic particle beams or other types of radiation (e.g. vacuum birefringence studies, Compton scattering and radiation reaction of a single electron at high intensities, photon-photon interactions) [7-16].

The present work reports our recent numerical study of the electromagnetic field focusing in four media: air, quartz, ZnO, and TiO<sub>2</sub>, using the finite difference time domain (FDTD) method. A comparative analysis has been made between the numerically obtained beam waist values and the analytical evaluations calculated with the complex Gaussian formalism providing a particular description of spatial and temporal aspects of focused few-cycle laser beams in the four different dispersive media previously mentioned. The FDTD method contains numerical libraries for simulations and visualization of specific graphs offering the data necessary to define materials properties used in the numerical computations.

The approach described in this paper aims to contribute to ultra-short pulse laser experiments by offering necessary details concerning the overview on the dynamics of the electromagnetic field propagation in predefined conditions.

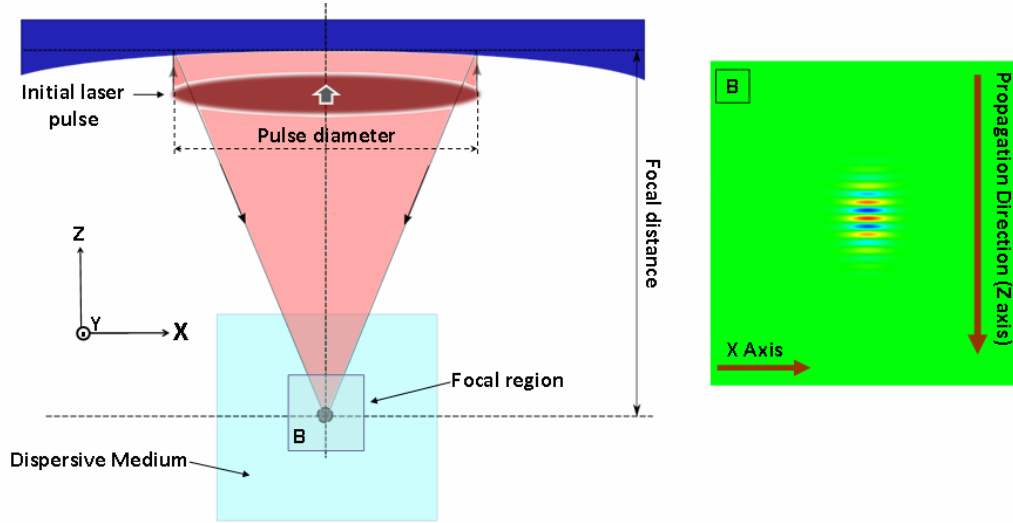
## 2. THEORETICAL APPROACH

Numerical simulations of few-cycle laser beams propagation were performed in order to investigate the electromagnetic field distribution in the focal region after propagation through dispersive media. The 2D numerical simulation had been developed using FullWAVE, a package of the commercial software RSoft [17], which solves the Maxwell equations using the FDTD method. In this way, we obtained the beam intensity distribution at different moments of time. The geometry of the problem is depicted in Fig. 1.

The study implies a laser source (central wavelength  $\lambda$  of 800 nm, source diameter  $D$  of 20 and 90  $\mu\text{m}$ , respectively, vertical polarization and symmetrical position to the  $x$  axis) that propagates initially along the  $z$  axis. The numerical simulations have been done for pulse duration values of 5, 11, and 16 fs, which corresponds to 2, 4, and 6 temporal cycles, respectively. The laser pulse is focused by a spherical mirror that has the focal distance  $f$  of 75  $\mu\text{m}$  and the diameter of 100  $\mu\text{m}$ . In order to investigate the temporal parameters, units of  $c \cdot t$  are further used here, taking into consideration that 1  $\mu\text{m}$  corresponds to 3.33 fs. The laser source is Gaussian both in space and time described by the following expression:

$$\Phi(x, z, t) = A \exp[-x^2 / \delta^2] \exp\left[-\left(\frac{t}{\tau} - t_d\right)^2\right] \sin\left(\frac{2\pi}{\lambda} t + \phi_0\right), \quad (1)$$

where  $A$  is the electromagnetic field amplitude,  $\delta$  is the source size along the axis  $x$ ,  $\tau$  is the pulse duration in units of  $c \cdot t$ ,  $t_d$  is the delay time in units of  $\tau$ ,  $\lambda$  is the wavelength of the source, and  $\phi_0$  is the phase chosen to be 0 at the maximum of the Gaussian function.



**Fig. 1.** Schematic design of the pulse focusing and the electromagnetic field distribution in focus in the case of an extended source illustrating the Gaussian beam  $E_y$  distribution in the  $(xz)$  plane.

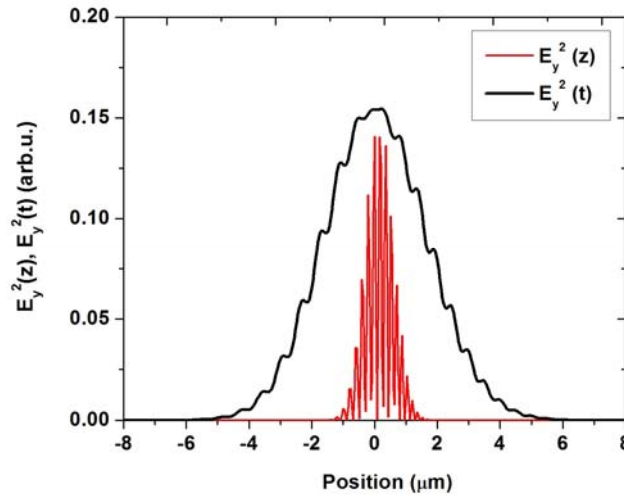
We investigated the electromagnetic field distribution in the focal region, both from spatial and temporal point of view after the propagation through dispersive media with different refractive indices. Previous studies [18, 19] indicate that the spatio-temporal aspects are not equivalent. In our simulations, the position of the focal point  $F$  where the electric field is the most intense for the simulated spatial range has been identified by involving successive sections along  $x$  axis, in the focal region, with an accuracy of  $\lambda/4$ . The monitoring of the electromagnetic field distribution in several points along the propagation axis  $z$  allowed the determination of the moment  $t_M$  when the maximum electric field is produced in the focal point  $F$ .

The detailed studies of the envelope full width at half maximum of the electric field indicate that the pulse duration is an essential parameter for the laser pulse description.

### 3. NUMERICAL RESULTS AND DISCUSSIONS

We analyzed the behavior of the resulted EM field distribution in the focal region in two cases:  $f$  number  $f\#=3.75$  and  $f\#=0.83$  in the presence of pulse duration variation. As shown in the inset image of Fig. 1, the electric field distribution has been illustrated using false color coded plots where the red color shows the positive part of the field, the green color represents the zero field while the blue color shows the negative values. The detailed field structure of the spatial extent of the electric field along the  $z$  axis is represented at the time when the pulse reaches the focus.

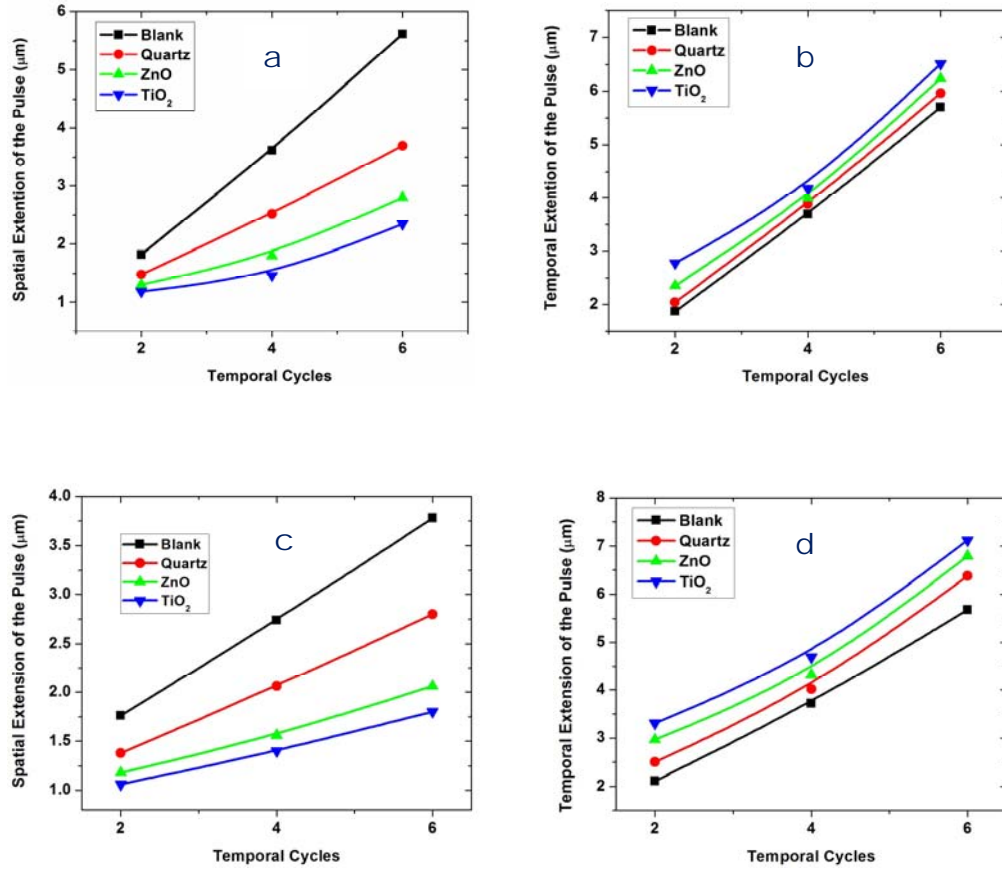
The temporal analysis of the focused laser beam is based on data provided by temporal monitors positioned along the  $z$  axis. In order to investigate the spatial-temporal analogy of the EM field distribution in focus, first we transformed the temporal axis into a spatial one using the space relation:  $s=c \cdot t$ . Then, we plotted the envelope of the temporal evolution of the EM field considering the spatial extension of the field along the propagation axis  $E(x=0; z)$  (Fig. 2). These calculations have been made for all four media considered in our study.



**Fig. 2.** Relation between spatial and temporal envelopes for the square of the electric field in the case of no dispersive medium at 2 cycles pulse duration and an  $f$ -number of 3.75. The full width at half maximum of the spatial envelope is  $1.85 \mu\text{m}$  while the temporal envelope has  $2.8 \mu\text{m}/c$  duration, where  $c$  is the speed of light.

In the case of no dispersive medium and pulse duration of 2 cycles, the temporal envelope is a factor of 1.5 larger than the spatial one at  $f\# = 3.75$ . Similar simulations

for  $f\# = 0.83$  lead to a factor of 1.56. This aspect was observed in the case of all four media considered and it is summarized in Fig. 3. As depicted in Figs. 3(a-d), the full width half maxima of the spatial and temporal envelopes of the EM pulse show a significant enlargement as function of the mentioned pulse duration values at  $f\# = 3.75$  and  $f\# = 0.83$ , respectively.

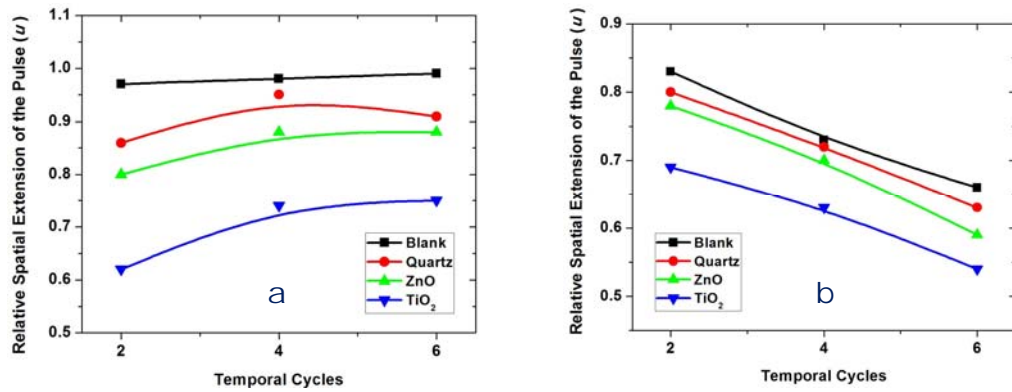


**Fig. 3.** Spatio-temporal analysis of the TFP extension for all four media investigated. a) Spatial extension of TFP at  $f\# = 3.75$ ; b) Temporal extension of TFP at  $f\# = 3.75$ ; c) Spatial extension of TFP at  $f\# = 0.83$ ; d) Temporal extension of TFP at  $f\# = 0.83$ .

Comparing the spatial case with the temporal one, we observe that the EM pulse FWHM value behaves in the opposite way by changing the dispersive media. Thus, the

ratio between the FWHM values of the temporal and spatial envelope of the pulse increases linearly with 1 unity from a dispersive medium to another, for both  $f$  number values. The highest ratio between the spatial and temporal FWHM values has been obtained in the case of  $\text{TiO}_2$  at pulse duration of 6 cycles corresponding to 16 fs. It can be observed that the spatial extension of the TFP slightly decreases for  $f\# = 0.83$  in comparison with the case of  $f\# = 3.75$ , while the temporal one presents a minor increase, extending the difference between the spatial and temporal envelope FWHM values.

The discrepancy between spatial and temporal extent of the tightly focused pulse has been characterized using the *relative spatial extension* (RSE) of the pulse described in [19]. The use of factor  $u = n \cdot L_{FWHM} / (c\tau_{FWHM})$ , where  $n$  is the refractive index of dispersive media,  $L_{FWHM}$  and  $\tau_{FWHM}$  represent the full width at half maximum of the spatial extension of the laser pulse and temporal duration measured in focus, made possible the quantification of the relative spatial extension of the TFP for all investigated cases. The results are represented in Fig. 4. For high  $f\#$ , RSE is slightly increasing with the pulse duration while for lower  $f\#$ , RSE decreases significantly for all cases, showing that shorter pulses may be obtained in certain conditions.



**Fig. 4.** Relative spatial extension of the pulse for all four media investigated in the case of pulse duration variation at a)  $f\# = 3.75$  and b)  $f\# = 0.83$ .

#### 4. CONCLUSIONS

The obtained results show that the spatial extent of the EM field in the focus of ultra-short pulses depends on Rayleigh range and it is shorter than the temporal duration of the pulse  $c \cdot t$  for all four media investigated. The spatio-temporal equivalence has been studied as function of the relative spatial extension  $u$  of the electromagnetic field of the laser pulse in the focal region and important details for extended lambda-cubed regime experiments have been related. Our study shows that shorter pulses can be obtained in certain conditions and, in order to provide a complete overview of the behavior of the few-cycle laser pulses focused to diffraction limited spots, essential parameters such as pulse duration and f-number should be carefully analyzed. This approach can be considered as a powerful and an efficient tool for further simulations of electromagnetic beam propagation in non-linear media.

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