ROGUE WAVES OF THE $(3+1)$-DIMENSIONAL POTENTIAL YU-TODA-SASA-FUKUYAMA EQUATION

WEI LIU[1,*]

[1]College of Mathematic and Information Science, Shandong Technology and Business University, Yantai, 264005, P. R. China
Email: *liuweicc@mail.ustc.edu.cn*

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General high-order rogue waves of the $(3+1)$-dimensional potential Yu-Toda-Sasa-Fukuyama equation are derived by employing the bilinear method. These rogue wave solutions are given in terms of determinants whose matrix elements have simple algebraic expressions. It is shown that fundamental rogue waves in the (x,z) plane are line rogue waves, which arise from the constant background with a line profile and then disappear into the constant background again. The typical dynamics in other planes have also been illustrated by three dimensional plots. It is also shown that high-order rogue waves in the (x,z) plane are parallel line rogue waves, which also arise from the constant background and then decay back to it. Besides, dynamical behaviours of these high-order rogue waves in the (x,y) and (x,t) planes are also illustrated.

Key words: $(3+1)$-dimensional Yu-Toda-Sasa-Fukuyama Equation, Rogue waves, Bilinear transformation method.

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1. INTRODUCTION

It is well known that nonlinear evolution equations (NLEEs) depict many physical scenarios occurring in diverse areas of physics, such as fluid mechanics, plasma physics, optical fibers, and solid state physics. Hence solving nonlinear problems plays an important and significance role in nonlinear science, since they can provide much physical information and more insight into the physical aspects and then lead to further applications. Indeed, various effective methods have been developed to derive exact solutions to NLEEs, such as the Darboux transformation method [1, 2], the inverse scattering method [3], the Hirota bilinear method [4], the homogeneous balance method [5, 6], the Lie group method [7, 8] and so on [9–11].

Rogue waves, a special type of solitary waves, also known as monster waves, killer waves, extreme waves or gaint waves, have attracted a lot of attention in several physical settings [12]. The term rogue wave was initially coined for vivid description of the mysterious and monstrous ocean waves. In addition to observing them in the

*On leave from Royal Belfast Mathematical Institute open ocean, these extreme wave events were also observed in a wide class of physical systems including oceanography [12], hydrodynamics [13, 14], plasma physics [15], and nonlinear optics [16–20]. Mathematically, rogue waves are modeled as transient wave packets localized in both space and time, to mimic the episodic giant waves that seemingly “appear from nowhere and disappear without a trace” [21]. The simplest (first-order) rogue wave solution was first reported in the study of the nonlinear Schrödinger (NLS) equation by Peregrine [22]. Higher-order rogue waves in the NLS equation were reported in many articles [23–29]. Up to now, a lot of nonlinear soliton equations have been verified possessing rogue wave solutions [30–54].

In this paper, we mainly study rogue wave solutions of the $(3+1)$-dimensional potential Yu-Toda-Sasa-Fukuyama (YTSF) equation:

$$\begin{aligned}(-4u_t+\Phi(u)u_z)_x+3u_{yy}=0,\\ \Phi(u)=\partial_x^2+4u+2u_x\partial_x^{-1},\end{aligned} \tag{1}$$

where $u: R_x\times R_y\times R_z\times R_t\longrightarrow R$, and the operator ∂_x^{-1} is the inverse operator of ∂_x and satisfies $\partial_x^{-1}\partial_x=1$ and $\partial_x^{-1}(\bullet)=\int_{-\infty}^{+\infty}(\bullet)dx$. This equation is an extension of the Bogoyavlenskii-Schif (BS) equation in higher dimension [55], and it is not an integrable system. Recently, lots of intensive studies, including N-soliton solutions, and nontravelling wave solutions have been done on this equation [56–62]. To the best of author’s knowledge, general high-order rogue waves of equation (1) have not been investigated before. Hence investigating new types of rogue waves and general high-order rouge waves for the $(3+1)$-dimensional potential YTSF equation (1) is an important motivation for the present study.

In the present work, we derive general high-order rogue wave solutions for the $(3+1)$-dimensional potential YTSF equation, which are expressed in term of determinants based on the Hirota’s bilinear method [4] and the KP hierarchy reduction method [63]. The basic idea is to treat the $(3+1)$-dimensional potential YTSF equation as a constrained KP hierarchy. Then, we derive rational solutions of the $(3+1)$-dimensional potential YTSF equation from rational solutions of the KP hierarchy. In particular, these rational solutions can be given in a simple representation. Furthermore, we investigate dynamical behaviors of high-dimensional rogue waves in the $(3+1)$-dimensional potential YTSF equation.

The outline of the paper is as follows. In Sec. 2, the bilinear form and rational solutions of the $(3+1)$-dimensional potential YTSF equation are given. In Sec. 3, typical dynamics of the obtained rogue wave solutions of the $(3+1)$-dimensional potential YTSF equation are analyzed and illustrated. Section 4 contains a summary and the discussion of the results.

2. RATIONAL SOLUTIONS OF THE $(3+1)$-DIMENSIONAL POTENTIAL YTSF EQUATION

In this Section, we proceed to derive rational solutions of $(3+1)$-dimensional potential YTSF equation (1).

The $(3+1)$-dimensional potential YTSF equation defined in (1) can be transformed into the following bilinear form

$$(-4D_\zeta D_t - D_\zeta^4 + 3D_y^2) f \cdot f = 0, \tag{2}$$

through the dependent variable transformation

$$u = (2\log f)_{\zeta\zeta}. \tag{3}$$

Here $\zeta = x - z$, f is a real function with respect to variables x, y, z, and t, and the operator D is the Hirota's bilinear differential operator [4] defined by

$$\begin{aligned} &P(D_x, D_y, D_t,)F(x,y,t\cdots)\cdot G(x,y,t,\cdots)\\ =&P(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_t - \partial_{t'}, \cdots)F(x,y,t,\cdots)G(x^{'}, y^{'}, t^{'}, \cdots)|_{x'=x, y'=y, t'=t}, \end{aligned}$$

where P is a polynomial of D_x, D_y, $D_t, \cdots$.

Theorem 1. *The $(3+1)$-dimensional potential YTSF equation (1) has rational solutions*

$$u = 2(\log f)_{\zeta\zeta}, \tag{4}$$

where

$$f = \det_{1\le i,j\le N}(m_{2i-1,2j-1}), \tag{5}$$

and the matrix elements in f are defined by

$$m_{ij} = \sum_{k=0}^{i} \frac{a_k}{(i-k)!}(p\partial_p + \xi^{'})^{i-k} \times \sum_{l=0}^{j} \frac{a_l^*}{(j-l)!}(p^*\partial_{p^*} + \xi^{'*})^{j-l} \frac{1}{p+p^*} \tag{6}$$

with

$$\xi^{'} = p\zeta + 2ip^2 y - 3p^3 t, \zeta = x - z. \tag{7}$$

Here asterisk denotes complex conjugation, i in subscript denotes an integer, otherwise $i^2 = -1$, N, j are arbitrary positive integers, and p is an arbitrary complex constants.

By a scaling of m_{ij}, we can normalize $a_0 = 1$ without loss of generality, hereafter we set $a_0 = 1$ in this paper. Note that these rational solutions can also be expressed in terms of Schur polynomials as discussed in [29, 47]. What is more, these rational solutions are nonsingular if the real parts of wave numbers p_i $(1 \le i \le N)$ are all positive or negative. Next, we will provide a short proof for this theorem and the non-singularity of these rational solutions.

Lemma 1. *The bilinear equation in the KP hierarchy*

$$((D_{x_1}^4 - 4D_{x_1}D_{x_3} + 3D_{x_2}^2)\tau_n \cdot \tau_n = 0 \tag{8}$$

has the Gram determinant solutions

$$\tau_n = \det_{1\le i,j\le N}(m_{ij}^{(n)}), \tag{9}$$

with the matrix element $m_{ij}^{(n)}$ satisfying the following differential and difference relations,

$$\begin{aligned}
\partial_{x_1} m_{ij}^{(n)} &= \psi_i^{(n)}\phi_j^{(n)},\\
\partial_{x_2} m_{ij}^{(n)} &= \psi_i^{(n+1)}\phi_j^{(n)} + \psi_i^{(n)}\phi_j^{(n-1)},\\
\partial_{x_3} m_{ij}^{(n)} &= \psi_i^{(n+2)}\phi_j^{(n)} + \psi_i^{(n+1)}\phi_j^{(n-1)} + \psi_i^{(n)}\phi_j^{(n-2)},\\
\partial_{x_4} m_{ij}^{(n)} &= \psi_i^{(n+3)}\phi_j^{(n)} + \psi_i^{(n+2)}\phi_j^{(n-1)} + \psi_i^{(n+1)}\phi_j^{(n-2)} + \psi_i^{(n)}\phi_j^{(n-3)},\\
m_{ij}^{(n+1)} &= m_{ij}^{(n+1)} + \psi_i^{(n)}\phi_j^{(n+1)},\\
\partial_{x_v}\psi_i &= \psi_i^{(n+v)},\\
\partial_{x_v}\phi_j &= -\phi_j^{(n-v)} \quad (v = 1,2,3).
\end{aligned} \tag{10}$$

This Lemma can been proved by a similar way as the proof of the Lemma 1 in Ref. [29, 63], thus we omit the proof of this Lemma in this paper. Next we use this Lemma to prove Theorem 1.

Proof of Theorem 1. *In order to prove Theorem 1, we choose the following selections of functions $m_{ij}^{(n)}$, $\psi_i^{(n)}$ and $\phi_j^{(n)}$:*

$$\begin{aligned}
\psi_i^{(n)} &= A_i p^n e^{\xi},\\
\phi_j^{(n)} &= B_j(-q)^{-n}e^{\eta},\\
m_{ij}^{(n)} &= A_i B_j \frac{1}{p+q}\left(-\frac{p}{q}\right)^n e^{\xi+\eta},
\end{aligned} \tag{11}$$

where

$$A_i = \sum_{k=0}^{i} \frac{a_k}{(i-k)!}(p\partial_p)^{i-k}, \qquad B_j = \sum_{l=0}^{j} \frac{b_l}{(j-l)!}(q\partial_q)^{j-l},$$
$$\xi = px_1 + p^2x_2 + p^3x_3, \qquad \eta = qx_1 - q^2x_2 + q^3x_3.$$

For simplicity, functions $m_{ij}^{(n)}$ can be rewritten as

$$m_{i,j}^{(n)} = e^{\xi+\eta}\left(-\frac{p}{q}\right)^n \sum_{k=0}^{n_i} \frac{a_k}{(i-k)!}(p\partial_p + \xi' + n)^{n_i-k} \sum_{l=0}^{n_j} \frac{b_l}{(j-l)!}(q\partial_q + \eta' - n)^{n_j-l} \frac{1}{p+q}, \tag{12}$$

where

$$\xi_i^{'} = px_1 + 2p^2x_2 + 3px_3\,, \quad \eta_j^{'} = qx_1 - 2q^2x_2 + 3q^3x_3.$$

Here p, q, $a_k\,,b_l$ *are arbitrary complex constants, and* i, j, n_i, N *are arbitrary positive integers.*

Further, taking the parameter constraints

$$q = p^*, \qquad b_k = a_k^* \tag{13}$$

and assuming $x_1\,,x_3$ *are real,* x_2 *is pure imaginary, we have*

$$\eta^{'} = \xi^{'*}, \quad m_{ij}^*(n) = m_{ji}(-n), \quad \tau_n^* = \tau_{-n}. \tag{14}$$

Applying the change of independent variables $x_1 = \zeta\,, x_2 = i\,y\,, x_3 = t$*, and setting* $m_{ij} = m_{ij}(0), \tau_0 = f$*, the bilinear equation* (8) *can be transformed into the bilinear equation* (2)*. Under the gauge transformation* (3)*, rational solutions of* $(3+1)$*-dimensional potential YTSF equation* (1) *given in Theorem 1 can be obtained from the rational solutions of equation* (8)*. Thus the Theorem 1 has been proved.*

Next, we concentrate on commenting that the obtained solutions are nonsingular by using equation (10) ,(11), and (13). Note that $f = \tau_0$ is given by the determinant

$$f = \det_{1\le i,j\le N}(m_{2i-1,2j-1}(0))\,. \tag{15}$$

Indeed, for any non-zero column vector $\mu = (\mu_1\,,\mu_2\,...\mu_N)^T$ and $\overline{\mu}$ being its complex transpose, we have

$$\begin{aligned}
\overline{\mu} f \mu &= \sum_{i,j=1}^{N} \overline{\mu_i} m_{2i-1,2j-1}^{'}(0)\mu_j = \sum_{i,j=1}^{N} \overline{\mu_i}\mu_j A_{2i-1}B_{2j-1}\frac{1}{p+q}e^{\xi+\eta}\,|_{q=p^*} \\
&= \sum_{i,j=1}^{N} \overline{\mu_i}\mu_j A_{2i-1}B_{2j-1}\int_{-\infty}^{x} e^{\xi+\eta}dx\,|_{q=p^*} \\
&= \int_{-\infty}^{x} (\sum_{i,j=1}^{N} \overline{\mu_i}\mu_j A_{2i-1}B_{2j-1}e^{\xi+\eta}\,|_{q=p^*})dx \\
&= \int_{-\infty}^{x} |\sum_{i=1}^{N} \overline{\mu_i} A_{2i-1}e^{\xi}|^2 dx > 0\,,
\end{aligned} \tag{16}$$

thus we have proved that f is positive definitive. Therefore, rational solutions u given in Theorem 1 are non-singular.

3. DYNAMICS OF ROGUE WAVES IN THE $(3+1)$-DIMENSIONAL POTENTIAL YTSF EQUATION

In this Section, we focus on the typical features and asymptotic behaviors of rogue waves of the $(3+1)$-dimensional potential YTSF equation (1). First, we derive the fundamental rogue waves (i.e., first-order rogue waves) to the $(3+1)$-dimension potential YTSF equation (1) from the Theorem 1.

3.1. FUNDAMENTAL ROGUE WAVES OF THE $(3+1)$-DIMENSIONAL POTENTIAL YTSF EQUATION

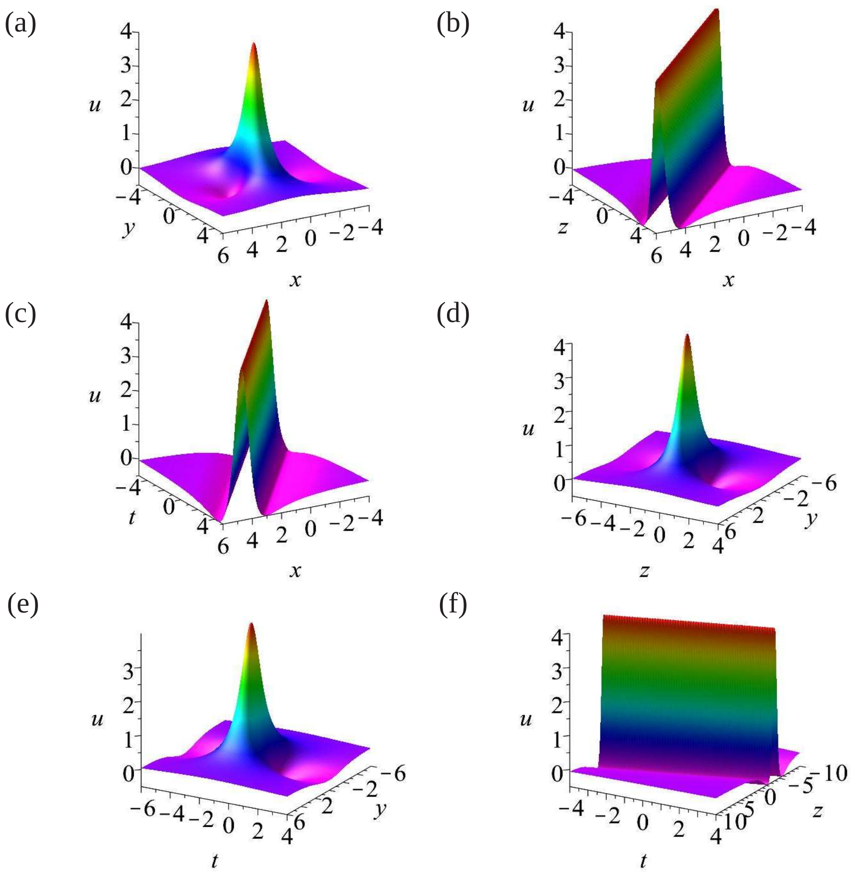

Fig. 1 – (Color online) A fundamental rogue wave u, defined by (21), of the $(3+1)$-dimensional potential YTSF equation with parameters $a_0 = 1, a_1 = 0, p_1 = 1$: $(a) z = 0, t = 0$; $(b) y = 0, t = 0$; $(c) y = 0, z = 0$; $(d) x = 0, t = 0$; $(e) x = 0, z = 0$; $(f) x = 0, y = 0$.

The first-order rational solutions of the $(3+1)$-dimensional potential YTSF

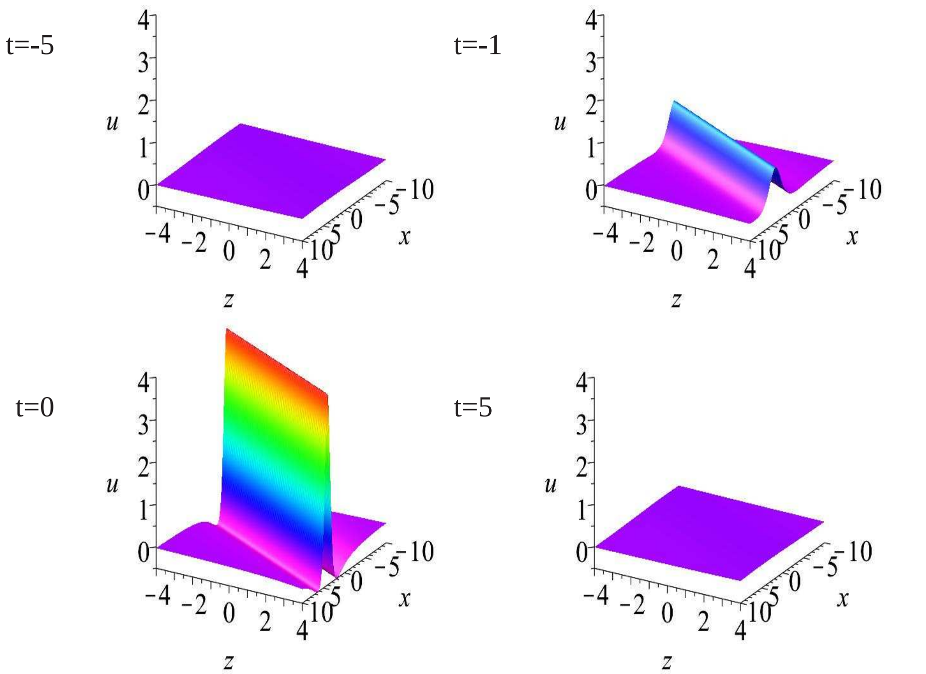

Fig. 2 – (Color online) Dynamics of fundamental rogue waves u, defined by (21), of the $(3+1)$-dimensional potential YTSF equation with parameters $a_0 = 1, a_1 = 0, p_1 = 1, y = 0$ in the (x,z) plane.

equation can be derived by taking $N = 1$ in Theorem 1, which can been given as the following form

$$u = (2\log f)_{\zeta\zeta}, \tag{17}$$

with

$$\begin{aligned} f &= \left(\sum_{k=0}^{1} \frac{a_k}{(1-k)!}(p\partial_p + \xi')^{1-k} \sum_{l=0}^{1} \frac{a_k^*}{(1-k)!}(p^*\partial_{p^*} + \xi'^*)^{1-l}\right)\frac{1}{p+p^*}, \\ &= (p\partial_p + \xi' + a_1)(p^*\partial_{p^*} + \xi'^* + a_1^*)\frac{1}{p+p^*}, \\ &= \frac{1}{p+p^*}[(\xi' - \frac{p}{p+p^*} + a_1)(\xi'^* - \frac{p^*}{p+p^*} + a_1^*) + \frac{pp^*}{(p+p^*)^2}], \end{aligned} \tag{18}$$

where

$$\xi' = p\zeta + 2ip^2y - 3p^3t, \zeta = x - z, \tag{19}$$

p and a_1 are arbitrary complex parameters. Besides, a_1 can be eliminated after a shift of space and time coordinates, thus we can set $a_1 = 0$ without loss of generality. We

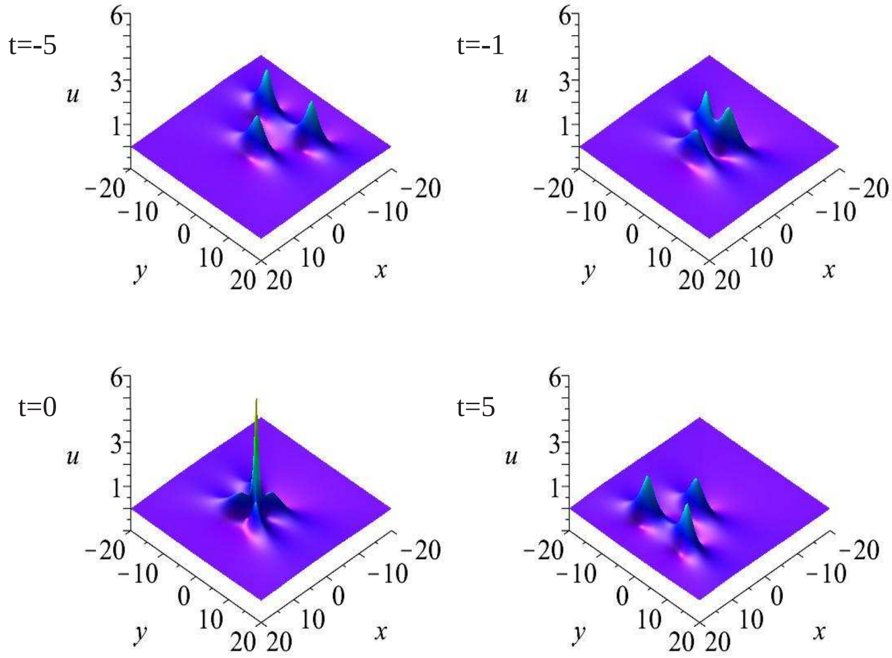

Fig. 3 – (Color online) Dynamics of second-order rogue waves u, defined by (22), of the $(3+1)$-dimensional potential YTSF equation with parameters $a_0 = 1, a_1 = 0, a_2 = 0, a_3 = -\frac{1}{12}, p_1 = 1, z = 0$ in the (x, y) plane.

assume $p = p_{1R} + i\,p_{1I}$ and then rewrite the above solutions as

$$f = \frac{1}{2p_{1R}}(\theta\,\theta^* + \theta_0), \tag{20}$$

where $\theta = l_1 + i\,l_2\,, l_1 = p_{1R}(x-z) - 4p_{1R}p_{1I}y - 3(p_{1R}^3 - 3p_{1R}p_{1I}^2)t, \theta_0 = \frac{p_{1R}^2+p_{1I}^2}{4p_{1R}^2}$ $l_2 = p_{1I}(x-z) + 2(p_{1R}^2 - p_{1I}^2)y - 3(3p_{1R}^2 p_{1I} - p_{1I}^3)t$. Then the final expression of the rational solutions is

$$u = 4\frac{(p_{1R}^2 + p_{1I}^2)(2l_1 l_2 - \theta_0) + (p_{1R}^2 - p_{1I}^2)(l_1^2 + l_2^2)}{(l_1^2 + l_2^2 + \theta_0)^2}. \tag{21}$$

As rogue waves of the generalized $(3+1)$-dimensional shallow water equation [64], rogue waves in the $(3+1)$-dimensional potential YTSF equation defined by (21) also possess two different dynamical behaviours:

(i) Lump solution. When $p_{1I} \neq 0$, it is not easy to find that rational solutions u given by (21) are constant in any planes along the trajectory
$p_{1R}(x-z) - 4p_{1R}p_{1I}y - 3(p_{1R}^3 - 3p_{1R}p_{1I}^2)t = 0,$

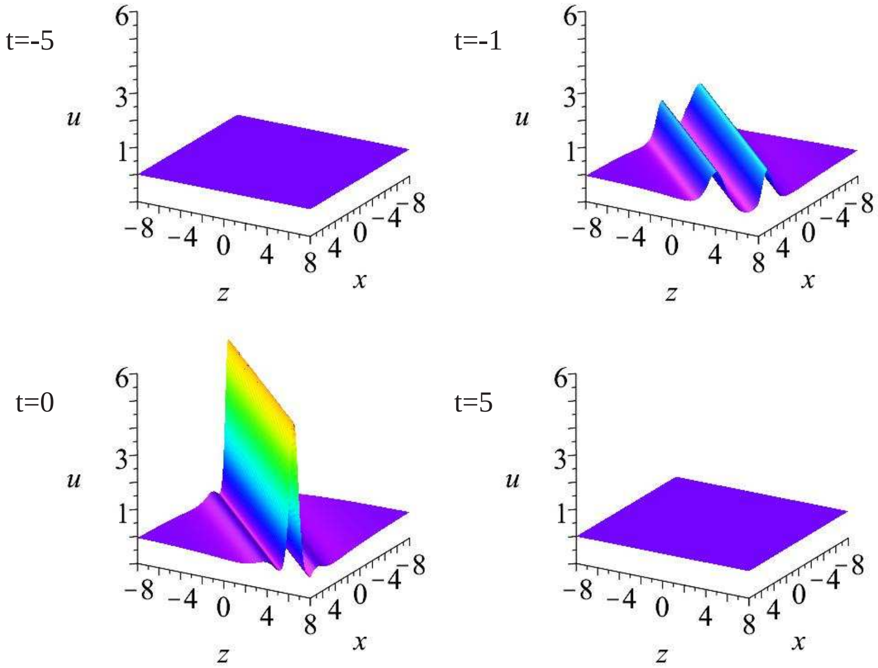

Fig. 4 – (Color online) Dynamics of second-order rogue waves u, defined by (22), of the $(3+1)$-dimensional potential YTSF equation with parameters $a_0 = 1, a_1 = 0, a_2 = 0, a_3 = -\frac{1}{12}, p_1 = 1, y = 0$ in the (x,z) plane.

$p_{1I}(x-z)+2(p_{1R}^2 - p_{1I}^2)y - 3(3p_{1R}^2 p_{1I} - p_{1I}^3) = 0.$
Besides, at any given time, $u \to 0$ when (x,y,z) turns to infinity. Thus rational solutions u defined in (21) are permanent lumps moving on the constant background when p_1 is complex.

(ii) Rogue wave solution. When $p_{1I} = 0$, these rational solutions have different dynamics in different planes, which are shown in Fig. 1 with parameters $p_{1R} = 1, p_{1I} = 0$. As can be seen, the corresponding solution describes a line wave in $(x,z),(x,t)$, and (z,t) planes. Specially, this solution is different from the moving line solitons of the multi-dimensional soliton equations, since line solitons maintain a perfect profile without any decay during their propagation in the $(x,z),(x,t),(z,t)$ planes. In contrast to line solitons, note that u approaches the constant background as $|t| \gg 0$, whereas at intermediate t, it reaches a much higher amplitude, see Fig. 2. As this solution has similar behaviors in $(x,y),(y,z),(y,z)$ planes, or in $(x,t),(z,t)$ planes, hereafter we just focus on dynamics in (x,y) plane, (x,t) plane, and (x,z) plane in the next part of this paper.

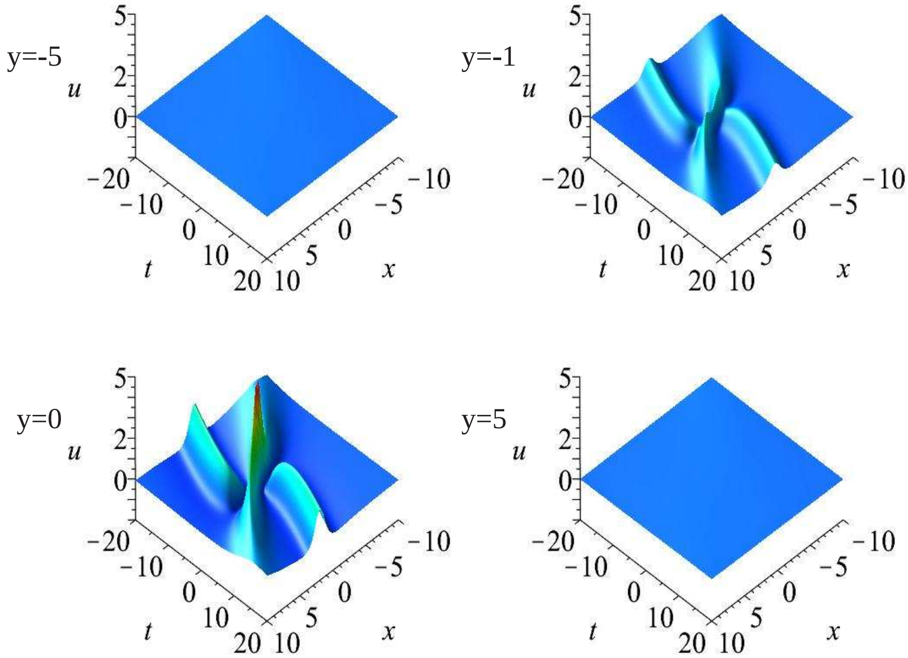

Fig. 5 – (Color online) Dynamics of second-order rogue waves u, defined by (22), of the $(3+1)$-dimensional potential YTSF equation with parameters $a_0=1, a_1=0, a_2=0, a_3=-\frac{1}{12}, p_1=1, z=0$ in the (x,t) plane.

The above discussion just focused on fundamental rogue waves of the $(3+1)$-dimensional potential YTSF equation. Next we derive nonfundamental rogue waves from Theorem 1 by taking $N>1$, and consider different patterns of nonfundamental rogue waves in the next part of this paper.

3.2. NONFUNDAMENTAL ROGUE WAVES OF THE $(3+1)$-DIMENSIONAL POTENTIAL YTSF EQUATION

Nonfundamental rogue waves of the $(3+1)$-dimensional potential YTSF equation can be derived by taking $N>1$ in the rational solutions given by (4), which describe the interaction of several fundamental rogue waves in different planes. In the (x,y) plane, the Nth-order rogue waves are composed of $\frac{N(N+1)}{2}$ fundamental rogue waves, which can generate some usual patterns of nonfundamental rogue waves as $(1+1)$-dimensional rogue waves [21, 65], such as fundamental patterns, triangular patterns, and circular patterns. In the (x,z) plane, at any given y, the nonfundamental rogue waves feature N parallel line rogue waves arising from the constant

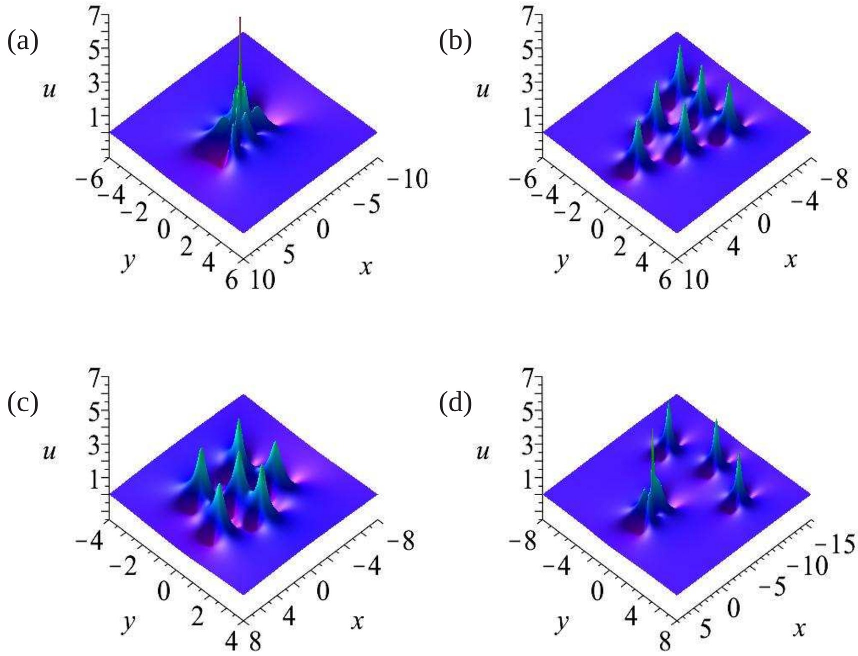

Fig. 6 – (Color online) Third-order rogue waves of the $(3+1)$-dimensional potential YTSF equation in the (x,y) plane with parameters given by (23) and $z=0, t=0$, and parameters (a_3,a_5) as: (a) $(\frac{1}{12},\frac{1}{240})$; (b) $(12,0)$; (c) $(0,12i)$; (d) $(2,6i)$.

background and then decaying back to the constant background at larger t. Specifically, these waves still keep straight line waves in the whole process. In the (x,t) plane, the nonfundamental rogue waves describe the interaction of N individual line rogue waves. When $y\to\pm\infty$, the solution approaches to the constant background uniformly in the entire (x,t) plane. In the intermediate y, N line rogue waves arise from the constant background, interact with each other, and then disappear into the constant background again. In this process, the wavefronts of the solution are no longer lines, and interesting curve wave patterns would generate.

To illustrate the dynamics of these nonfundamental rogue waves in different planes, we first consider the case of $N=2$ (i.e., the second-order rogue waves). In this case, the solutions can be obtained from (4) as

$$u=(2\log(\begin{vmatrix} m_{11} & m_{13}\\ m_{31} & m_{33}\end{vmatrix}))_{\zeta\zeta}, \tag{22}$$

where m_{ij} is given by (6), $\zeta=x-z$, p is real, and $a_k\,(0\le k\le 3)$ are complex constants. Below we consider the dynamics of two-rogue waves given by (22) in

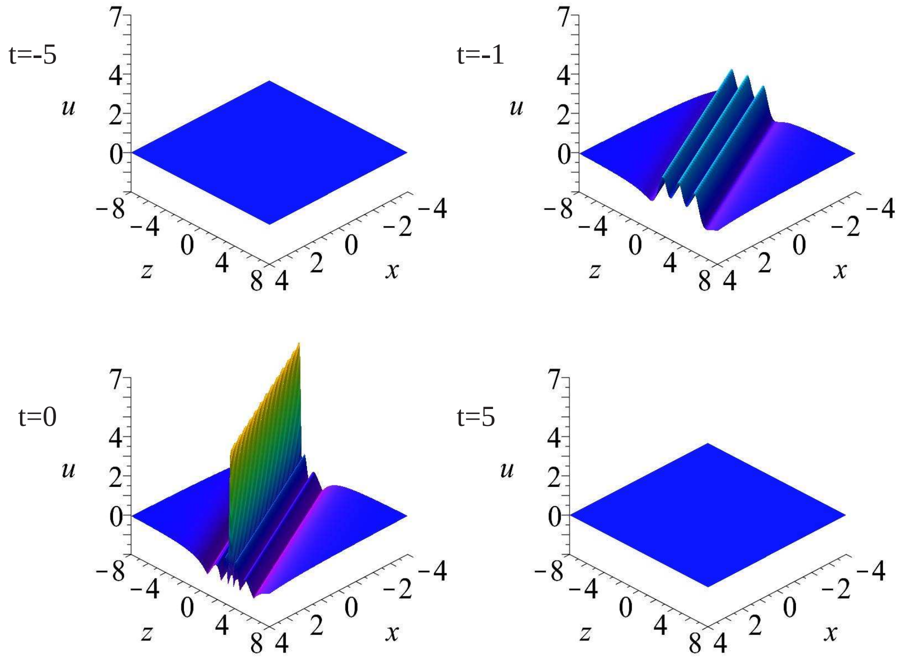

Fig. 7 – (Color online) Third-order rogue waves of the $(3+1)$-dimensional potential YTSF equation in the (x,z) plane with parameters given by (23) and parameters $y=0, a_3=-\frac{1}{24}, a_5=\frac{1}{1200}$.

different planes.

In the (x,y) plane, the two-rogue wave solution is made up of three fundamental rogue waves, see Fig. 3. The interaction of these three fundamental rogue waves can generate various types of wave patterns. When $|t| \gg 0$, these three fundamental rogue waves are far from each other and out of interaction. In this case, this two-rogue wave solution features triangular wave pattern (see the panel at $t=\pm5$). Besides, when $t \to 0$, these three fundamental rogue waves get closer, and begin to interact with each other, the fundamental pattern would appear (see the panel at $t=0$). It is noticed that the maximum value of the two-rogue wave solution defined in (22) can exceed 5 (see the panel at $t=0$), while does not exceed 2 under the triangular wave pattern. Thus the interaction between tree fundamental rogue waves in the (x,y) plane can generate very high peaks.

In the (x,z) plane, the two-rogue wave consists of two parallel line rogue waves, see Fig. 4. When $t << 0$, this two-rogue wave solution approaches to the constant background (see $t=\pm5$ panel). In the intermediate time, two parallel line rogue waves arise from the constant background, and the region where the two rogue

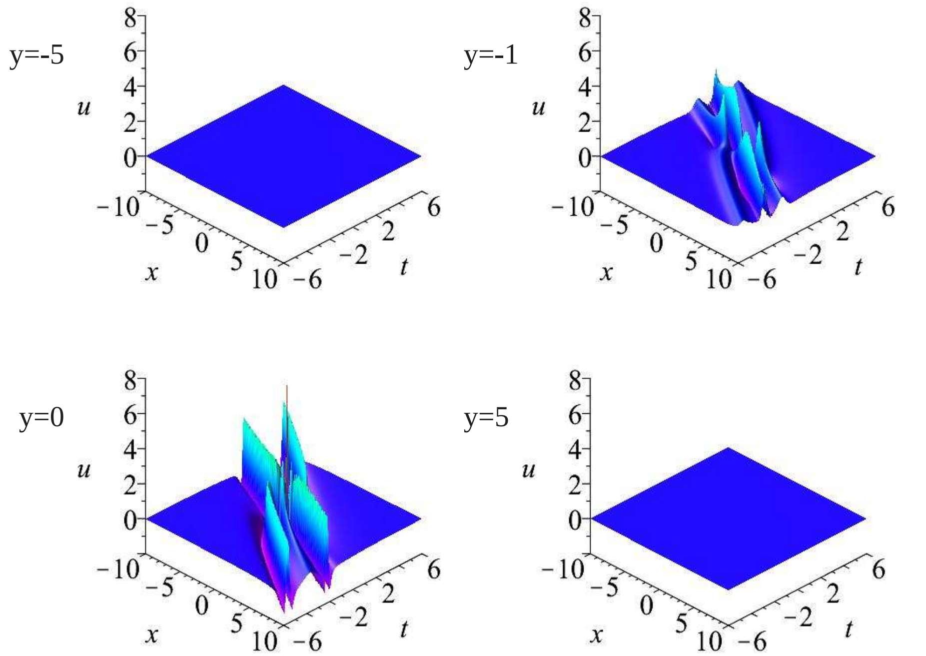

Fig. 8 – (Color online) Third-order rogue waves of the $(3+1)$-dimensional potential YTSF equation in the (x,t) plane with parameters given by (23) and parameters $z=0, a_3=-\frac{1}{24}, a_5=\frac{1}{1200}$.

wave appear rise to higher amplitudes. Then these two parallel line rogue waves combine into one possessing a much higher peak and two lower peaks (see the panel at $t=0$). Note that for all the time, the two individual line rogue wave still keep parallel lines. This type of nonfundamental rogue waves is completely different from the rogue waves in the DS systems [47, 48], which are no longer lines at the panel $t=0$. To the best of our knowledge, this type of rogue waves has not been reported in the $(3+1)$-dimensional potential YTSF equation.

In the (x,t) plane, the two-rogue wave is also composed of two line waves. However, it possesses different behaviors comparing to the corresponding one in the (x,z) plane; see Fig. 5. When $|y| >> 0$, the two-rogue wave solution approaches to the constant background. When $y \to 0$, two line waves arise from the constant background, and then interact with each other, see the panel at $y=\pm 5$. Due to the interactions, the two line waves are no longer lines, which is different from the parallel line rogue waves in the (x,z) plane. Besides, these two line waves are not completely separated for all times, see the panel at $y=0$.

For larger N, these nonfundamental waves behaves similarly, but more fundamental rogue waves arise from the constant background and interact with each other. Because of the interactions, more complicated waveforms would form in the interaction region. For instance, with parameter choices

$$N = 3, p = \frac{1}{2}, a_0 = 1, a_1 = 0, a_2 = 0, a_4 = 0, \tag{23}$$

the corresponding solution in (x, y), (x, z), and (x, t) planes are shown in Figs. 6, 7, and 8. As can be seen in Fig. 6, the third-order rogue wave is made up of six fundamental rogue waves in the (x, y) plane. The interaction of the six fundamental rogue waves could also generate various types of wave patterns, including fundamental patterns, triangular patterns, and circular patterns. The dynamical profiles in the (x, z) plane are shown in Fig. 7. It is seen that the third-order rogue wave is composed of three parallel line rogue waves in the (x, z) plane, which arise from the constant background and disappear into the constant background again. These line rogue waves just exist on the constant background for a short period. The typical behaviors in the (x, t) plane are shown in Fig. 8. As $y >> 0$, the three-rogue wave solution approaches to the constant background. In the intermediate y region, several line waves arise from the constant background and generate interesting wave patterns in the (x, t) plane.

4. SUMMARY AND DISCUSSION

In summary, general high-order rogue waves in the $(3 + 1)$-dimensional potential YTSF equation have been derived by employing the bilinear method and the KP reduction method, which are given in terms of determinants. The fundamental rogue waves (i.e., first-order rogue waves) possess different dynamics in different planes, see Fig. 1. In particular, these rogue waves are localized line waves in the (x, z) plane, which arise from the constant background with a line profile and then disappear into the constant background again, see Fig. 2. The dynamical behaviours of nonfundamental rogue waves in the $(x, y), (x, z), (x, t)$ planes are also discussed. In the (x, y) plane, the Nth-order rogue waves consist of $\frac{N(N+1)}{2}$ individual fundamental rogue waves. The interaction of these fundamental rogue waves generates several types of wave patterns, such as fundamental patterns, triangular patterns, and circular patterns, see Figs. 3 and 6. In the (x, z) plane, the Nth-order rogue waves are parallel line rogue waves, which consist of N parallel line rogue waves, see Figs. 4 and 7. Note that for all the time, these line waves keep parallel line waves. That is different from the line rogue waves in the DS systems [47, 48], as the wavefronts of the latter one are no longer lines. In the (x, t) plane, the rogue waves also behave as line waves, but they possess different dynamics comparing to that in the (x, z) plane, as the wavefront of the solutions are no longer line waves, see Figs. 5 and 8. As demonstrated, the interaction of these line waves can also generate interesting wave patterns.

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