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ANALYTICAL AND NUMERICAL TREATMENT OF FALKNER-SKAN EQUATION VIA A TRANSFORMATION AND ADOMIAN'S METHOD

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Abstract. The main feature of the boundary layer flow problems, such as Blasius and Falkner-Skan problems, is the inclusion of the boundary conditions at infinity. As a well known fact, such boundary conditions cause difficulties for any of the series methods. This is because the boundary conditions at infinity can not be imposed directly into the series solution, where Padé approximant should be first constructed before applying such conditions. To overcome this difficulty, an approach has been suggested recently by Ebaid and Al-Armani [Abstr. Appl. Anal., 753049 (2013)], which is based on changing the boundary conditions at infinity to classical ones by using a proper transformation. This approach is applied in the present paper to solve a class of Falkner-Skan equation analytically and numerically. Moreover, an exact solution is deduced at a certain value of the velocity ratio parameter. In addition, the current numerical results are compared with the other existing solutions, and good agreement has been achieved. Indeed, the main advantage of the present approach is the complete avoidance of Padé approximant to deal with the boundary condition at infinity.

Key words: Falkner-Skan equation, Adomian decomposition method, infinite domain.

1. INTRODUCTION

The boundary layer flow problems over a continuously stretching surface have significant applications in many industrial processes, such as cooling of a metallic plate in a cooling bath, an aerodynamic extrusion of plastic sheets, drawing of plastic films, metal spinning, metallic plates, insulating materials and in applications of glass and polymer studies. In these applications, the stretching sheet moves with a constant stretching speed and parallel to its plane. A few notable examples of the boundary layer flow problems are Blasius and Falkner-Skan equations. Blasius equation describes the velocity profile of the fluid in the boundary layer theory [1, 2] on a half-infinite interval and it is one of the basic equations in fluid dynamics. Also, the Falkner-Skan equation plays a considerable role in the development of boundary layer theory in fluid mechanics. Several analytical and numerical methods have been proposed in Refs. [1-11] to handle Blasius and Falkner-Skan equations. The main feature of such problems is the inclusion of the boundary conditions at infinity. As a well known fact, such boundary conditions cause difficulties for any of the series methods such as Adomian decomposition method [12-17] and the differential transformation method (or Taylor series method) [18-19]. This is because the boundary condition at infinity can not be implemented directly in the series, where Padé approximant should be established before applying the boundary condition at infinity. Besides, many authors [20-28] have been resorted to some analytical and numerical methods to solve different types of nonlinear differential equations and partial differential equations. Although the obtained results were accurate in many cases, a massive computational work was needed to obtain accurate approximate solutions. A possible way to avoid Padé technique is to change the boundary conditions at infinity into classical conditions. Therefore, a suggestion has been proposed very recently in [29] to transform the domain of the problem from unbounded domain into a bounded one with the help of a reliable transformation. Accordingly, the original differential equation is transformed into a new differential equation with classical boundary conditions. This new approach [29] will be applied in the current paper to solve the following class of Falkner-Skan problem [10]:

$$f'''(\eta)+f(\eta)f''(\eta)+\beta\left[\varepsilon^2-(f'(\eta))^2\right]=0, \qquad (1)$$

with the boundary conditions:

$$f(0)=0, \qquad f'(0)=1-\varepsilon, \qquad f'(\infty)=\varepsilon, \qquad (2)$$

where β refers to the pressure gradient parameter while ε refers to the velocity ratio parameter, $\varepsilon = U_\infty/(U_\infty + U_w)$. Equation (1) with the boundary conditions (2) is a new version of Falkner-Skan equation relating free stream velocity U_∞ to composite reference velocity, i.e., sum of the velocities of stretching boundary U_w and free stream U_∞ [10]. In order to apply the approach presented in [29] to solve the equations (1) and (2) we first transform the governing equation (1) into the following system of differential equations:

$$f'(\eta)=u(\eta), \qquad (3)$$

$$u''(\eta)+f(\eta)u'(\eta)+\beta[\varepsilon^2-(u(\eta))^2]. \qquad (4)$$

Here, we may indicate that in the theory of the boundary layer it is usually important to get information about three quantities: the skin-friction coefficient $f''(0)$, the fluid velocity $f'(\eta)$ and the stream function $f(\eta)$. Also, it is well known that at $\beta = 0,$ the problem reduces to a class of the Blasius problem, which has been studied very recently by Ebaid and Al-Armani [29].

2. A TRANSFORMATION AND A NEW SYSTEM

The unbounded domain of the independent variable $\eta \in [0,\infty)$ can be changed into a bounded one by using a new independent variable t (say) $\in [0;1)$ using the transformation $t = 1 - e^{-\eta}$. Accordingly, the governing system should be expressed in terms of the new variable t. In order to do that, we introduce the following relations between the derivatives with respect to η and the derivatives with respect to t [29]:

$$\frac{d}{d\eta}(W) = (1-t)\frac{d}{dt}(W), \tag{5}$$

$$\frac{d^2}{d\eta^2}(W) = (1-t)^2\frac{d^2}{dt^2}(W) - (1-t)\frac{d}{dt}(W). \tag{6}$$

The relations given by Eqs. (5-6) are obtained by using the chain rule in the differential calculus. Therefore, the system (3-4) becomes

$$f'(t) = \left(\frac{1}{1-t}\right)u(t), \tag{7}$$

$$u''(t) = \left(\frac{1}{1-t}\right)u'(t) - \left(\frac{1}{1-t}\right)f(t)u'(t) - \frac{\beta}{(1-t)^2}[\varepsilon^2 - (u(\eta))^2], \tag{8}$$

subject to the following set of boundary conditions

$$f(0) = 0 \qquad u(0) = 1-\varepsilon, \qquad u(1) = \varepsilon. \tag{9}$$

Equation (7) with the initial condition $f(0) = 0$ can be easily integrated as an initial value problem, while Eq. (8) with the boundary conditions given in (9) should be solved as a two-point boundary value problem. In this regard, the improved Adomian decomposition method [15, 29] is suggested to deal with such singular two-point boundary value problem.

3. ANALYSIS AND APPLICATION

On integrating Eq. (7) with respect to t form 0 to t, it then follows

$$f(t) = f(0) + \int_0^t \left(\frac{1}{1-z} \right) u(z) dz. \qquad (10)$$

Based on Adomian's method, the solutions $u(t)$ and $f(t)$ of system (7-8) are assumed in the following form

$$u(t) = \sum_{n=0}^{\infty} u_n(t) \qquad f(t) = \sum_{n=0}^{\infty} f_n(t). \qquad (11)$$

Inserting these series into Eq. (10), we obtain the following recurrence scheme for $f(t)$:

$$f_0(t) = 0 \qquad f_{n+1}(t) = \int_0^t z^{n-i} u_i(z) dz, \qquad n \geq 0. \qquad (12)$$

In order to establish an algorithm for the two-point boundary value problem given by Eq. (8) with the boundary conditions (9), we follow Ebaid and Al-Armani [29] and hence we shall use the following inverse operator:

$$L^{-1}[.] = \int_0^t dt' \int_c^{t'} [.] dt'' - t \int_0^1 dt' \int_c^{t'} [.] dt'', \qquad c \neq 1. \ (c = 0 \text{ for simplicity}) \qquad (13)$$

Operating both sides of Eq. (8) with this inverse operator, we have

$$u(t) = 1 - \varepsilon + (2\varepsilon - 1)t + L^{-1}[(\frac{1}{1-t})u'(t) - (\frac{1}{1-t}) f(t)u'(t) - \frac{\beta}{(1-t)^2}[\varepsilon^2 - (u(t))^2]]. \qquad (14)$$

Or

$$u(t) = 1 - \varepsilon + (2\varepsilon - 1)t - L^{-1}[\frac{\beta\varepsilon^2}{(1-t)^2}] + L^{-1}[(\frac{1}{1-t})u'(t) - (\frac{1}{1-t}) f(t)u'(t) + \frac{\beta}{(1-t)^2}(u(t))^2]. \qquad (15)$$

In order to establish the required algorithm, we use the following identities:

$$\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n, \qquad 0 < t < 1,$$

$$\frac{1}{(1-t)^2} = \sum_{n=0}^{\infty} (n+1) t^n,$$

$$\left(\frac{1}{1-t} \right) u'(t) = \sum_{n=0}^{\infty} \left(\sum_{i=o}^{\infty} t^{n-i} u_i'(t) \right), \qquad (16)$$

$$\left(\frac{1}{1-t} \right) f(t)u'(t) = \sum_{n=0}^{\infty} \left(\sum_{j=0}^{n} \sum_{i=0}^{j} t^{n-j} f_i(t) u_{j-i}'(t) \right),$$

$$\frac{1}{(1-t)^2}(u(t))^2 = \sum_{n=0}^{\infty}\left(\sum_{j=0}^{n}\sum_{i=0}^{j}(n-j+1)t^{(n-j)}u_i u_{j-i}\right).$$

According to these identities, Eq. (15) becomes

$$\sum_{n=0}^{\infty}u_n(t) = 1-\varepsilon+(2\varepsilon-1)t-\sum_{n=0}^{\infty}L^{-1}[\beta\varepsilon^2(n+1)t^n]+$$

$$\sum_{n=0}^{\infty}L^{-1}[\sum_{i=0}^{n}t^{n-i}u_i^{'}+\sum_{j=0}^{n}\sum_{i=0}^{j}(\beta(n-j+1)u_i u_{j-1}-f_i u_{j-i}^{'})t^{n-j}]. \quad (17)$$

Therefore, we obtain the recurrence relation

$$u_0(t) = 1-\varepsilon,$$

$$u_1(t) = (2\varepsilon-1)t-L^{-1}[\beta\varepsilon^2(n+1)t^n]+$$

$$L^{-1}[\sum_{i=0}^{\infty}t^{n-i}u_i^{'}+\sum_{j=0}^{n}\sum_{i=0}^{j}(\beta(n-j+1)u_i u_{j-1}-f_i u_{j-i}^{'})t^{n-j}], \qquad n=0, \quad (18)$$

$$u_{n+1}(t) = -L^{-1}[\beta\varepsilon^2(n+1)t^n]+$$

$$L^{-1}[\sum_{i=0}^{\infty}t^{n-i}u_i^{'}+\sum_{j=0}^{n}\sum_{i=0}^{j}(\beta(n-j+1)u_i u_{j-1}-f_i u_{j-i}^{'})t^{n-j}], \qquad n\geq 1.$$

The desired m^{th} order approximate solution $\phi_m(\eta)$ obtained by Adomian's method is expressed as

$$\phi_m(\eta) = \sum_{n=0}^{m-1}f_n(\eta). \quad (19)$$

On using the algorithms (12) and (18), we obtain the five term approximate solution, i.e., $\phi_5(\eta)$ for the stream function $f(\eta)$ at any values for ε and β in terms of the original variable η as

$$\phi_5(\eta) = (1-\varepsilon)(1-e^{-\eta})+\frac{1}{60480}[-2520(4\varepsilon^2-4\varepsilon-5)+84\beta(2\varepsilon-1)(16\varepsilon^2-$$

$$168\varepsilon-181)+6\beta^4(1-\varepsilon)(1-2\varepsilon)(17\varepsilon^2-30\varepsilon+30)-21\beta^2(1-2\varepsilon)$$

$$(124\varepsilon^2-60\varepsilon-185)-\beta^3(1-2\varepsilon)(496\varepsilon^3-1092\varepsilon^2-870\varepsilon+102585)]\times$$

$$(1-e^{-\eta})^2+\frac{1}{1440}[60(1-8\varepsilon-4\varepsilon^2)+120\beta(4\varepsilon^2-1)+3\beta^3(2\varepsilon-1)\times (4\varepsilon^2-10\varepsilon+5)+2\beta^2(2\varepsilon-1)(28\varepsilon^2-5\varepsilon-35)](1-e^{-\eta})^3+\ldots+ \frac{\beta^4}{45360}[(1-\varepsilon)(1-2\varepsilon)(\varepsilon^2-20\varepsilon+10)](1-e^{-\eta})^9. \quad (20)$$

It can be easily observed from Eq. (20) that each term includes β^r, $(r=1,2,3,...)$ multiplied by the factor $\pm(1-2\varepsilon)$ that vanishes when $\varepsilon=\frac{1}{2}$. Here, we want to say that the approximate series solution, obtained above, leads to an exact solution at a special case , i.e., at $\varepsilon=\frac{1}{2}$. Accordingly, the approximate solution $\phi_5(\eta)$ becomes

$$\phi_5(\eta)=\frac{1}{2}(1-e^{-\eta})+\frac{1}{4}(1-e^{-\eta})^2+\frac{1}{6}(1-e^{-\eta})^3+\frac{1}{8}(1-e^{-\eta})^4+\frac{1}{10}(1-e^{-\eta})^5 = \frac{1}{2}\sum_{r=1}^{5}\frac{1}{r}(1-e^{-\eta})^r. \quad (21)$$

Therefore, the $m-$term series solution is given by

$$\phi_m(\eta)=\frac{1}{2}\sum_{r=1}^{m}\frac{1}{r}(1-e^{-\eta})^r, \quad (22)$$

and thus the following exact solution is obtained as $m\to\infty$:

$$\begin{aligned} f(\eta) &= \lim_{n\to\infty}\phi_m(\eta) \\ &= \frac{1}{2}\sum_{r=1}^{\infty}\frac{1}{r}(1-e^{-\eta})^r \\ &= -\frac{1}{2}\ln[1-(1-e^{-\eta})]=\frac{\eta}{2}. \end{aligned} \quad (23)$$

This exact solution satisfies the boundary conditions and can be easily verified by direct substitution. For more validation, the numerical results obtained by the present technique at different values for ε and β are checked in a subsequent section via a comparison with those published in the literature. In order to do that we evaluated below the five term approximate series solution at $\varepsilon = \frac{1}{10}$, which is given as

$$\phi_5(\eta) = \frac{9}{10}(1-e^{-\eta}) + (\frac{67}{300} - \frac{549\beta}{2500} + \frac{593\beta^2}{11250} - \frac{12883\beta^3}{1050000})(1-e^{-\eta})^2 +$$
$$(\frac{11}{150} - \frac{2\beta}{25} + \frac{587\beta^2}{15000} - \frac{101\beta^3}{15000})(1-e^{-\eta})^3 +$$
$$(\frac{17}{300} + \frac{29\beta}{2000} - \frac{27\beta^2}{4000} + \frac{587\beta^3}{100000} - \frac{101\beta^4}{100000})(1-e^{-\eta})^4 +$$
$$(-\frac{37}{375} + \frac{29\beta}{1500} - \frac{393\beta^2}{7500} + \frac{143\beta^3}{15000} - \frac{\beta^4}{1250})(1-e^{-\eta})^5 +$$
$$(\frac{1967\beta}{22500} - \frac{617\beta^2}{22500} + \frac{397\beta^3}{150000} + \frac{67\beta^4}{100000})(1-e^{-\eta})^6 +$$
$$(\frac{3691\beta^2}{157500} - \frac{1033\beta^3}{105000} + \frac{\beta^4}{1750})(1-e^{-\eta})^7 +$$
$$(\frac{6977\beta^3}{2100000} - \frac{801\beta^4}{1400000})(1-e^{-\eta})^8 + (\frac{89\beta^4}{700000})(1-e^{-\eta})^9. \quad (24)$$

At $\varepsilon = \frac{4}{10}$, the series solution given by Eq. (20) becomes

$$\phi_5(\eta) = \frac{3}{5}(1-e^{-\eta}) + (\frac{149}{600} - \frac{2047\beta}{30000} + \frac{4729\beta^2}{360000} - \frac{7417\beta^3}{4200000} + \frac{109\beta^3}{1050000})(1-e^{-\eta})^2 +$$
$$(\frac{89}{600} - \frac{3\beta}{100} + \frac{271\beta^2}{30000} - \frac{41\beta^3}{60000})(1-e^{-\eta})^3 +$$
$$(\frac{8}{75} - \frac{17\beta}{6000} - \frac{\beta^2}{500} + \frac{271\beta^3}{300000} - \frac{41\beta^4}{600000})(1-e^{-\eta})^4 +$$
$$(-\frac{139}{3000} + \frac{7\beta}{750} - \frac{421\beta^2}{60000} + \frac{17\beta^3}{15000} - \frac{\beta^4}{30000})(1-e^{-\eta})^5 +$$
$$(\frac{2867\beta}{90000} - \frac{331\beta^2}{45000} + \frac{\beta^3}{75000} + \frac{11\beta^4}{300000})(1-e^{-\eta})^6 +$$
$$(\frac{949\beta^2}{157500} - \frac{9\beta^3}{10000} + \frac{\beta^4}{42000})(1-e^{-\eta})^7 +$$
$$(\frac{331\beta^3}{1050000} - \frac{9\beta^4}{350000})(1-e^{-\eta})^8 + (\frac{\beta^4}{175000})(1-e^{-\eta})^9. \quad (25)$$

At $\beta = \dfrac{1}{2}$, the series solution given by Eq. (20) becomes

$$\phi_5(\eta) = (1-\varepsilon)(1-e^{-\eta}) + \frac{1}{4838400}(46819 + 135247\varepsilon + 10856\varepsilon^2 + 4883\varepsilon^3 + 1036\varepsilon^4)\times(1-e^{-\eta})^2 + \frac{1}{11520}(125 + 3640\varepsilon - 568\varepsilon^2 + 232\varepsilon^3)\times \varepsilon^2(1-e^{-\eta})^3 + \frac{1}{138240}(7565 + 9395\varepsilon + 21832\varepsilon^2 + 2720\varepsilon^3 - 232\varepsilon^4)\times (1-e^{-\eta})^4 + \frac{1}{11520}(1659 - 5265\varepsilon - 1254\varepsilon^2 + 160\varepsilon^3)(1-e^{-\eta})^5 + \ldots + \ldots + \frac{1}{725760}(10 - 50\varepsilon + 30\varepsilon^2 + 59\varepsilon^3 - 4\varepsilon^4)(1-e^{-\eta})^9. \quad (26)$$

4. RESULTS AND DISCUSSION

In this section we present some numerical results derived from the analytical series solutions introduced in the previous section for the purpose of comparisons with those reported in literature. As mentioned in the introduction section, in the field of boundary layer flow, it is usually important to get information about the skin-friction coefficient $f''(0)$, the fluid velocity $f'(\eta)$, and the stream function $f(\eta)$. In this regard, we begin with calculating the skin-friction coefficient by using a sequence of the approximate solutions $\phi''_{17}(0)$, $\phi''_{22}(0)$, and $\phi''_{27}(0)$ at different values for the pressure gradient parameter β for $\varepsilon = 0.1, 0.2, 0.3, 0.4,$ and 0.5 as shown in Table 1 in which the exact values obtained by Kudenatti [30] are also listed at the same values for β and ε. It can be easily seen from Table 1 that the approximate values of the skin-friction coefficient converge and are very close to the exact values in most cases. Moreover, the accuracy of the value of the skin-friction coefficient increases with increasing the number of the terms used in the Adomian's series. The results presented in Table 1 may refer to the effectiveness of the current approach in obtaining good numerical results for the Falkner-Skan equation without any need to Padé approximations.

Figures 1, 2, 3, 4, 5, and 6 present the velocity profiles at several values for the pressure gradient parameter β and at different values for ε using few terms of the current approach. In view of Figures 1, 2, 4, and 6 of the current paper and those obtained by Kudenatti [30] in Figures $1a$, $1b$, $1c$, and $1d$ at the same values of β it can be concluded that good agreement has been achieved. Figures 3 and 5, show more additional numerical results for the velocity profiles at $\beta = 0.5$ and $\beta = 1.5$ respectively. In addition, at $\beta = 0$, the current problem reduces to Blasius problem and the numerical results obtained in Figure 2 coincide also with those reported in [29].

Table 1

Comparison of skin friction coefficient *f''(0)* obtained by using different number of terms of Adomian's series with the exact values [30].

β	ε	Exact [30]	$\phi_{17}''(0)$	$\phi_{22}''(0)$	$\phi_{27}''(0)$
0.0	0.1	-0.492625	-0.486535	-0.488719	-0.489882
	0.2	-0.363901	-0.355043	-0.357600	-0.359056
	0.3	-0.237219	-0.229665	-0.232004	-0.233421
	0.4	-0.115811	-0.11107	-0.112551	-0.113507
	0.5	0.0	0.0	0.0	0.0
0.5	0.1	-0.675918	-0.672955	-0.673768	-0.674239
	0.2	-0.513980	-0.514422	-0.515376	-0.515232
	0.3	-0.346194	-0.353201	-0.354042	-0.354462
	0.4	-0.175403	-0.189345	-0.189900	-0.190149
	0.5	0.0	0.0	0.0	0.0
1.0	0.1	-0.823981	-0.822577	-0.822906	-0.823096
	0.2	-0.633671	-0.633632	-0.633866	-0.633966
	0.3	-0.432951	-0.433242	-0.433234	-0.433174
	0.4	-0.221982	-0.221819	-0.221666	-0.221542
	0.5	0.0	0.0	0.0	0.0
1.5	0.1	-0.951872	-0.951696	-0.951795	-0.951848
	0.2	-0.736812	-0.737287	-0.737189	-0.737099
	0.3	-0.505918	-0.506387	-0.506074	-0.505868
	0.4	-0.259021	-0.260145	-0.25986	-0.259703
	0.5	0.0	0.0	0.0	0.0
2.5	0.1	-1.169812	-1.17005	-1.16995	-1.16988
	0.2	-0.909281	-0.910587	-0.910247	-0.91004
	0.3	-0.626233	-0.627477	-0.627117	-0.626937
	0.4	-0.323034	-0.323241	-0.323111	-0.323064
	0.5	0.0	0.0	0.0	0.0

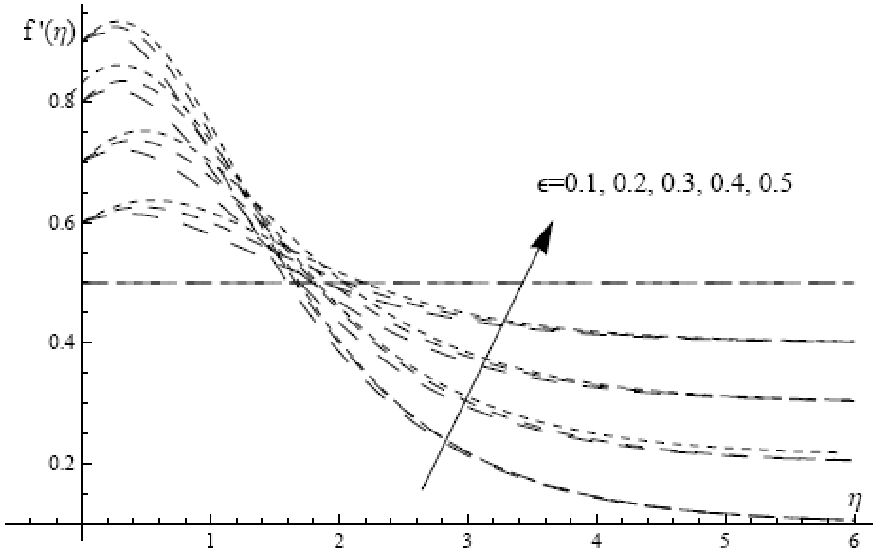

Fig. 1. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = -1$ and different values for velocity ratio parameter ε .

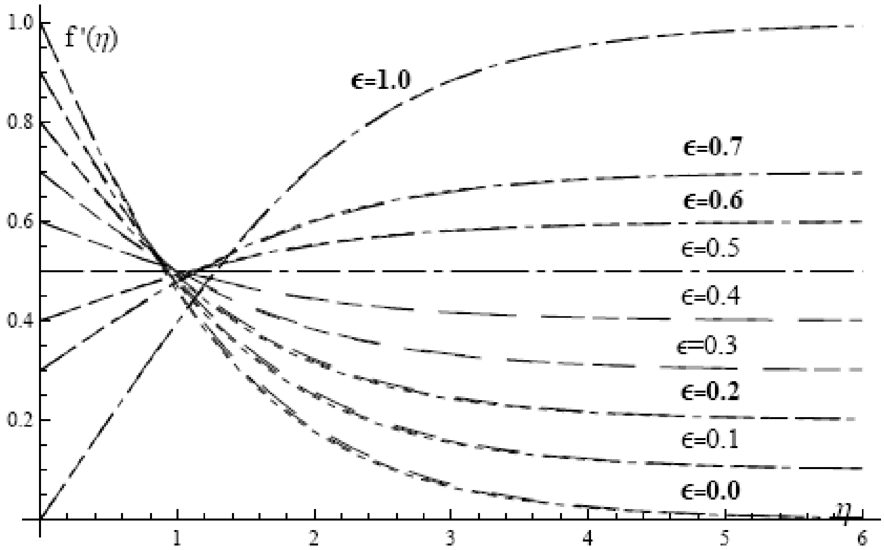

Fig. 2. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = 0$ and different values for velocity ratio parameter ε .

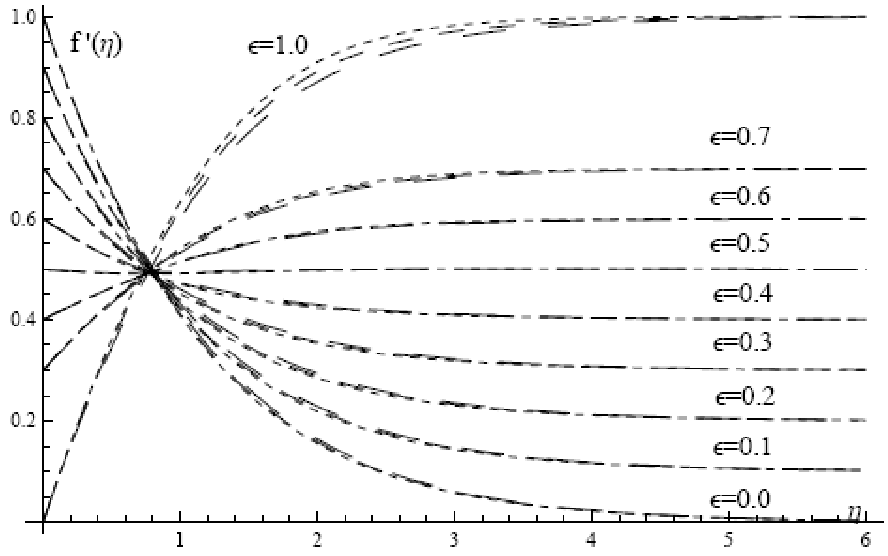

Fig. 3. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = 0.5$ and different values for velocity ratio parameter ε .

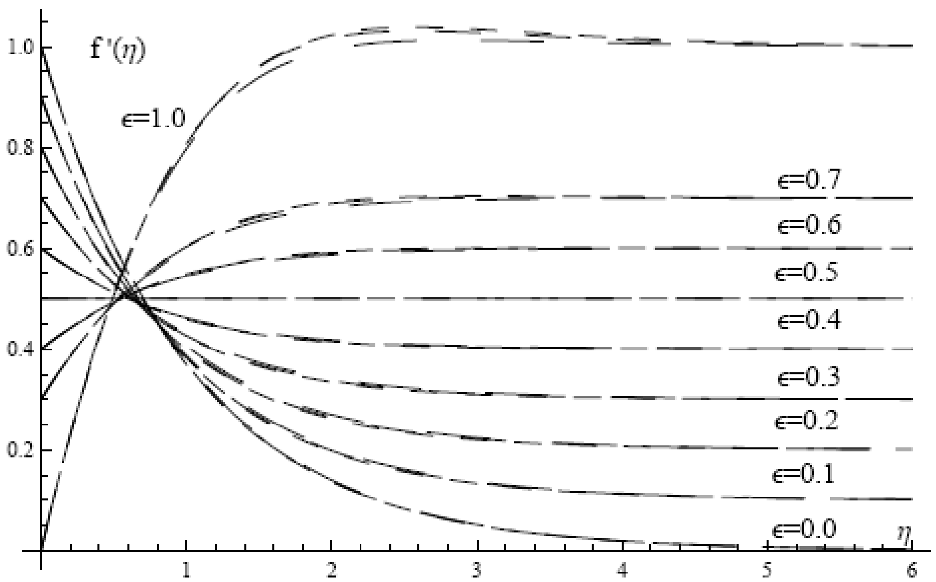

Fig. 4. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = 1$ and different values for velocity ratio parameter ε .

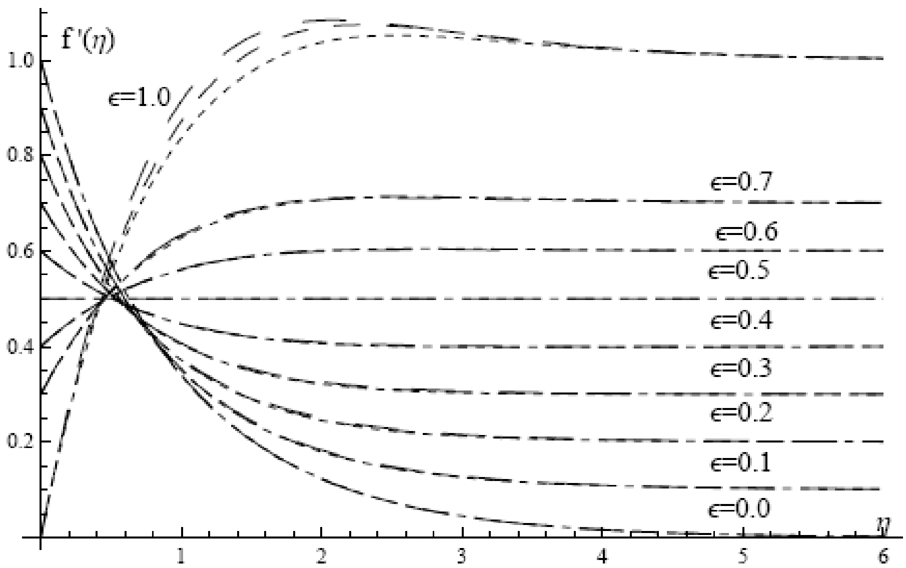

Fig. 5. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = 1.5$ and different values for velocity ratio parameter ε .

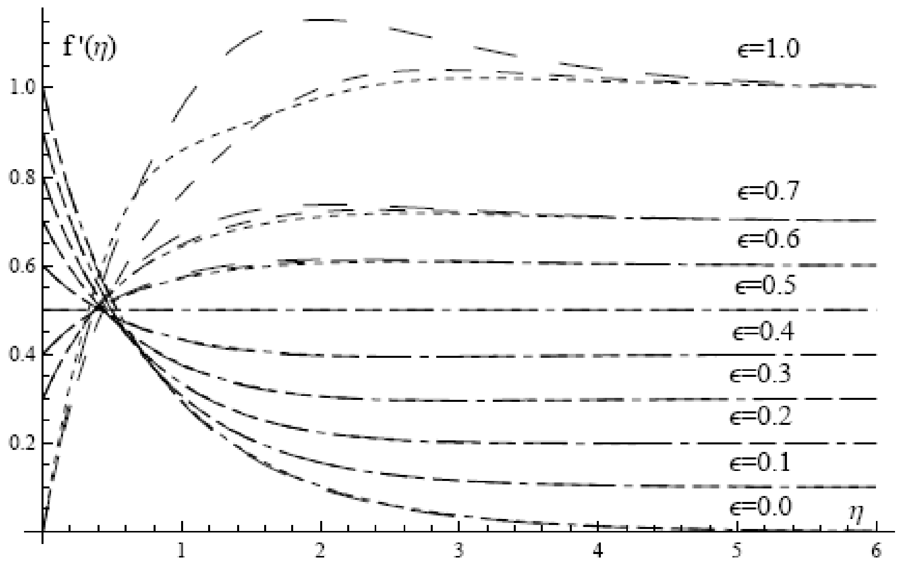

Fig. 6. Velocity profiles using 5, 7, and 9 terms of the current method at $\beta = 2.5$ and different values for velocity ratio parameter ε .

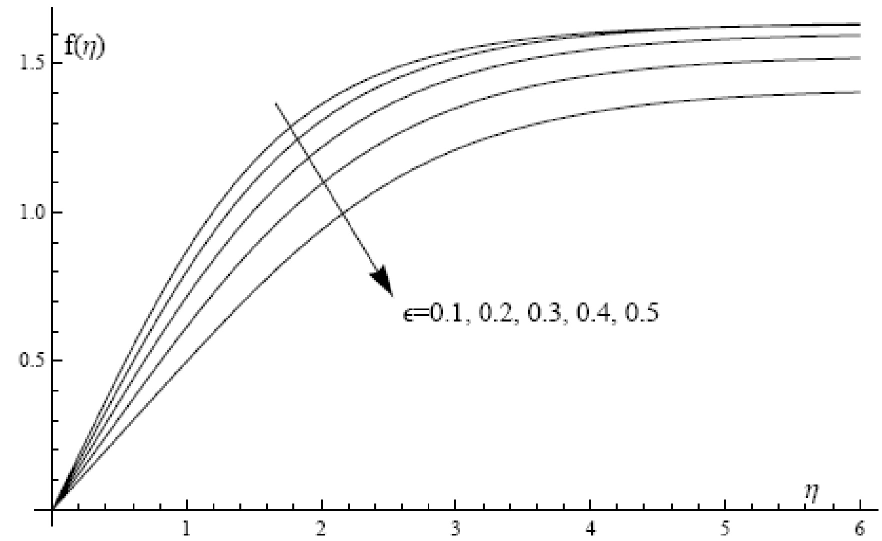

Fig. 7. The stream function using 9 terms of the current method at $\beta = -1$ and different values for velocity ratio parameter ε .

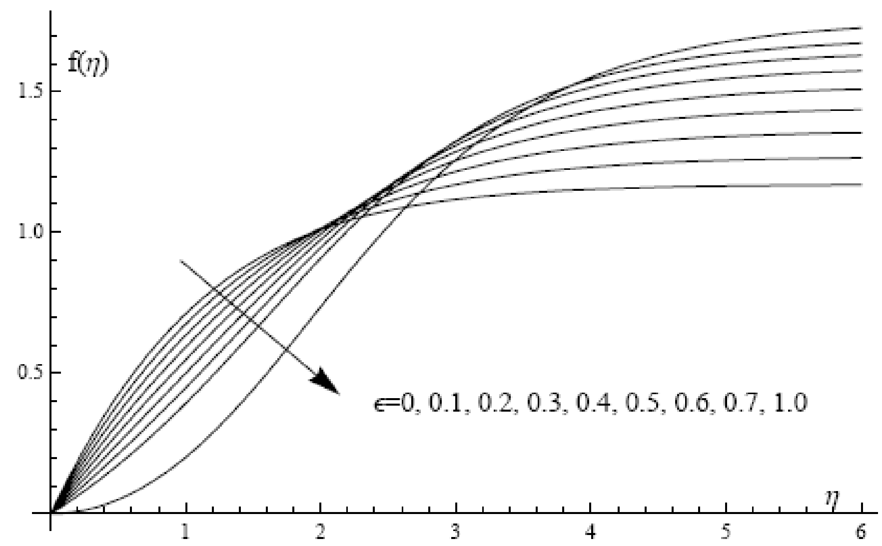

Fig. 8. The stream function using 9 terms of the current method at $\beta = 0$ and different values for velocity ratio parameter ε .

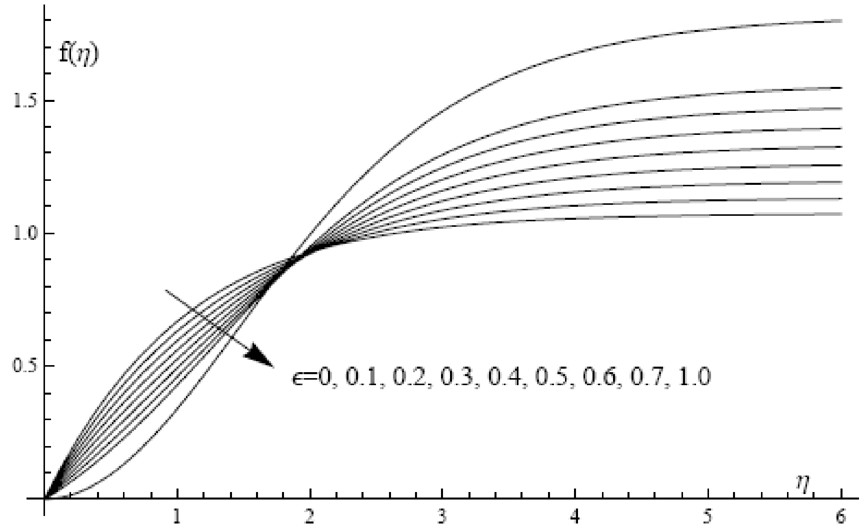

Fig. 9. The stream function using 9 terms of the current method at $\beta = 0.5$ and different values for velocity ratio parameter ε .

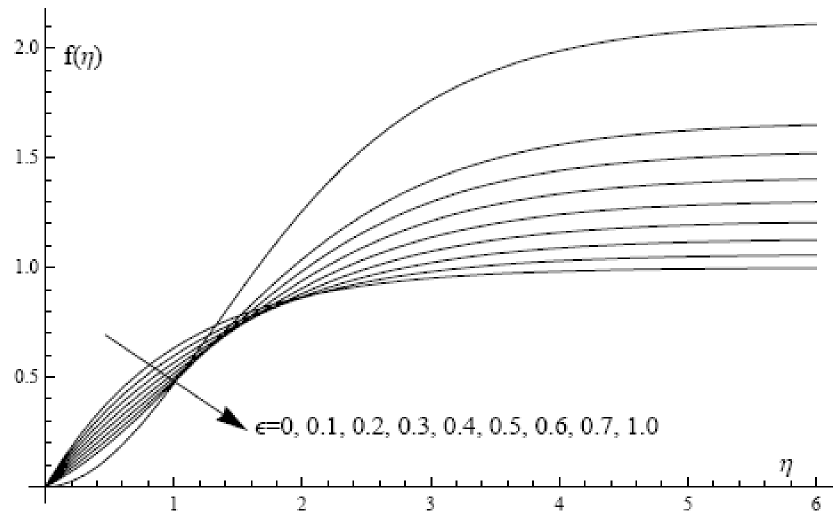

Fig. 10. The stream function using 9 terms of the current method at $\beta = 1$ and different values for velocity ratio parameter ε .

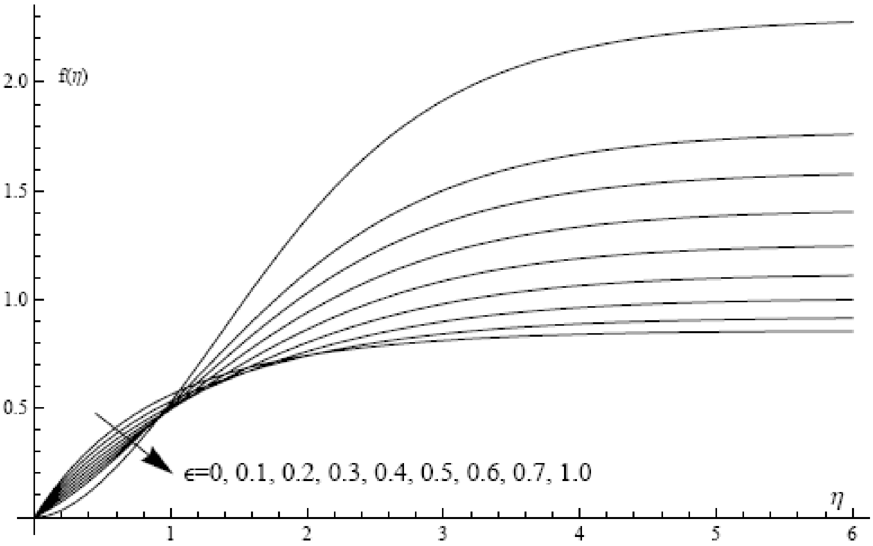

Fig. 11. The stream function using 9 terms of the current method at $\beta = 2.5$ and different values for velocity ratio parameter ε .

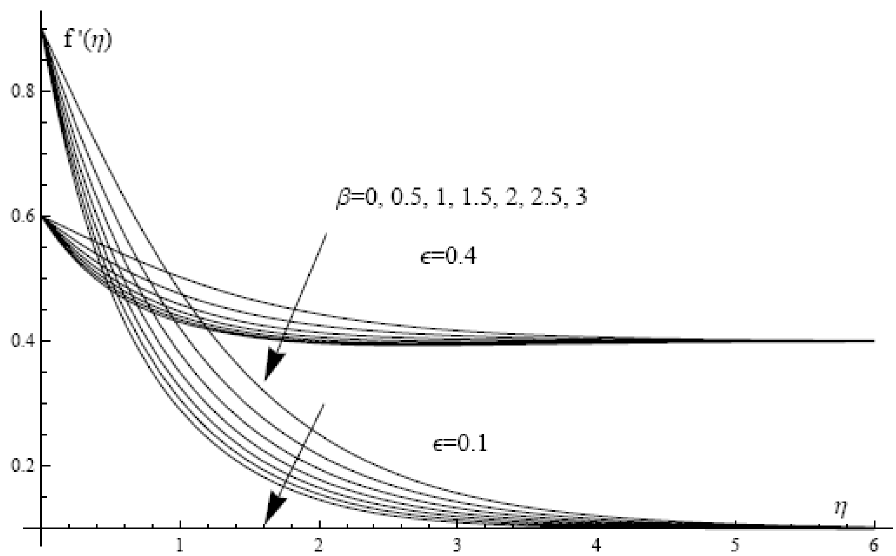

Fig .12. Velocity profiles for different sets of pressure gradient parameter β and velocity ratio parameter ε .

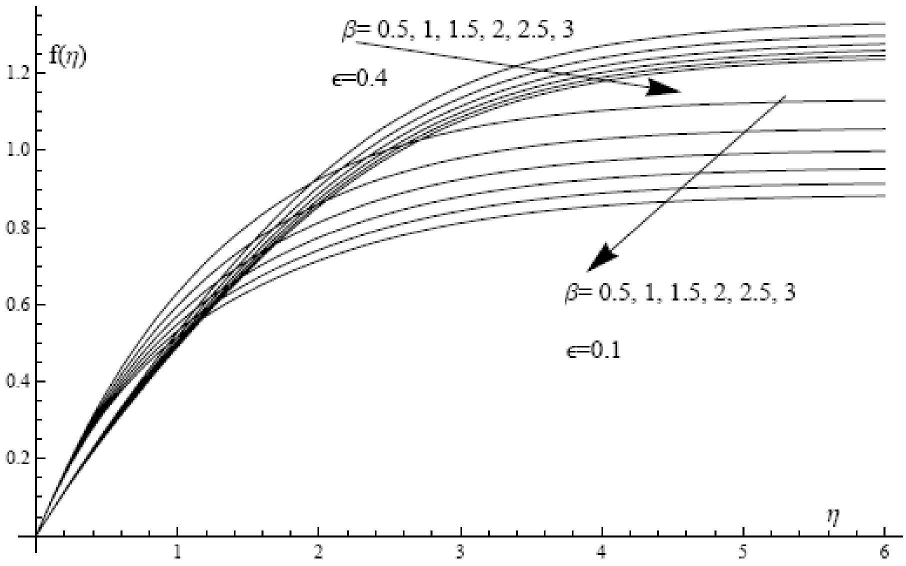

Fig. 13. The stream function using 9 terms of the current method at different sets of pressure gradient parameter β and velocity ratio parameter ε.

Regarding the stream function $f(\eta)$, it is plotted against the similarity variable η in Figures 7, 8, 9, 10, and 11 using 9-term approximate solutions at different values of the parameter β. Furthermore, the approximate series solution $\phi_9'(\eta)$ for the velocity profiles at $\varepsilon = 0.1$ and $\varepsilon = 0.4$ for different values of the pressure parameter $\beta = 0, 0.5, 1.5, 2, 2.5, 3$ are depicted in Figure 12. It is observed from Figure 12 that the obtained approximate solution for the velocity using a few terms of Adomian's series agree with the numerical results reported recently by Kudenatti (see Figure 2 in [27]). Moreover, the stream function is also illustrated in Figure 13 for the same data as in Figure 12.

5. CONCLUSION

A class of Falkner-Skan equations has been analytically and numerically treated in this paper using Adomian's method together with a reliable transformation. This transformation has been successfully used to change the boundary condition at infinity to a classical one. Accordingly, the transformed equations have been solved via an improved version of Adomain's method. Moreover, exact solutions are deduced at a certain value of the velocity ratio parameter ε. In addition, the current numerical results are compared with the other published solutions, where good agreement has been achieved. Indeed, the main advantage of the present approach is the complete avoidance of Padé approximation to deal with the boundary condition at infinity. The current approach may be extended in the near future to investigate similar problems in order to explore its power over some of the existing analytical and numerical approaches.

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