

# STUDY ON PERIODIC INTERACTIONS OF OPTICAL SOLITONS IN DISPERSIVE OPTICAL FIBERS

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The quality of optical communication systems can be effectively improved by controlling soliton interactions. In this paper, periodic interactions of optical in optical fibers are studied. Analytic two-soliton solutions of a variable-coefficient higher-order nonlinear Schrödinger equation are presented, and the influences of the corresponding model parameters are discussed. Both the in-phase and out-of-phase interactions of optical solitons are analyzed. The numerical results indicate that the periodic interactions of optical solitons can be controlled in order to decrease the bit error rate of fiber communication systems.

*Key words:* Solitons, optical solitons, dispersive soliton interaction, soliton propagation, variable-coefficient nonlinear Schrödinger equation.

## 1. INTRODUCTION

Optical solitons in dispersive dielectric fibers have been first put forward by Hasegawa and Tappert in a pioneering paper published in 1973 [1]; see also a comprehensive historical review on applications of optical fiber solitons for high speed communications systems [2]. During the past two decades, a plethora of localized optical structures have been widely investigated in a series of relevant physical settings, including multidimensional ones [3–26].

Temporal optical solitons can maintain their shape, amplitude, and speed when they are transmitted over long distances using fiber communication systems [27–29]. The solitons have the particle-like feature when they collide against each other. Because of the unavoidable dispersion effect in optical fibers, the propagation speeds

of optical pulses with different wavelengths are not equal, which can make optical pulses expand in the time domain [30, 31]. The nonlinear effect in optical fibers can generate self-phase modulation, which can cause phase changes along the front and back edges of optical pulses; see the recent experimental works on erbium-doped fiber laser systems [32–38]. Because the frequency and propagation speed at the back edge of optical pulses are higher than at the front edge, the pulse width is compressed. When the pulse broadening and the pulse compression effects are exactly balanced, optical solitons are formed and are stable during propagation [39–42].

The properties of optical solitons that keep their shape constant enable them to realize the propagation over ultra-long distances and with ultra-large capacity [43]. The bit rate of an optical soliton communication system is determined by the time interval between optical solitons. To increase the bit rate, the time interval between them must be reduced [44]. But when the interval is reduced to a certain extent, optical solitons will strongly interact with each other. The interaction between adjacent optical solitons in single-mode fibers limits the potential bit rate of the optical soliton communication systems, and leads to an increase of the bit error rate [27]. Therefore, it is necessary to study the interaction of optical solitons in nonlinear optical fibers, especially when the dispersion and nonlinear effects depends on the normalized propagation distance  $x$ .

In optical fibers, the propagation and interaction of solitons can be described by the variable-coefficient higher-order nonlinear Schrödinger (vcHNLS) equation as follows [27–29, 39],

$$i \frac{\partial u}{\partial x} + \frac{1}{2} \beta_2(x) \frac{\partial^2 u}{\partial t^2} + \sigma(x) |u|^2 u + i s(x) \frac{\partial(|u|^2 u)}{\partial t} - i \tau_R(x) - i \beta_3(x) \frac{\partial^3 u}{\partial t^3} - i \Gamma u = 0. \quad (1)$$

Here  $u$  is a complex function of  $x$  and  $t$  that denotes the amplitude of optical solitons, and  $x$  and  $t$ , respectively, represent the propagation distance and the retarded time.  $\beta_2(x)$  is the group velocity dispersion coefficient,  $\sigma(x)$  is the nonlinear coefficient,  $s(x)$  is the stimulated Raman scattering coefficient,  $\tau_R(x)$  is the self-steepening coefficient,  $\beta_3(x)$  is the third-order dispersion coefficient, and  $\Gamma$  is the attenuation or gain constant. For Eq. (1), the effects of soliton amplification, reshaping, fission, and annihilation have been studied in Ref. [39], where the influences of related model parameters on optical solitons have been also discussed.

However, the periodic interactions of optical solitons in dispersive nonlinear optical fibers modeled by the vcHNLS equation (1) have not been investigated before, to the best of our knowledge. In this work, the periodic interactions of optical solitons will be studied analytically and numerically. Both in-phase and out-of-phase interactions of solitons will be presented by selecting the numerical values of the relevant free parameters of the governing model.

This paper is organized as follows. In Sec. 2, the analytic two-soliton solutions

of Eq. (1) will be presented. The scenarios of periodic interactions of optical solitons will be obtained and analyzed in Sec. 3. Finally, in Sec. 4, the conclusions will be given.

## 2. ANALYTIC TWO-SOLITON SOLUTIONS OF EQ. (1)

The analytic two-soliton solutions of Eq. (1) can be expressed as [39]

$$u(x, t) = \frac{g_1(x, t) + g_3(x, t)}{1 + f_2(x, t) + f_4(x, t)}, \quad (2)$$

with

$$g_1(x, t) = e^{\theta_1(x, t)} + e^{\theta_2(x, t)}, \quad (3a)$$

$$g_3(x, t) = e^{2\Gamma x} \left[ \xi_1(x) e^{\theta_1(x, t) + \theta_2(x, t) + \theta_1^*(x, t)} + \xi_2(x) e^{\theta_1(x, t) + \theta_2(x, t) + \theta_2^*(x, t)} \right], \quad (3b)$$

$$f_2(x, t) = e^{2\Gamma x} \left[ \zeta_1(x) e^{\theta_1(x, t) + \theta_1^*(x, t)} + \zeta_2(x) e^{\theta_2(x, t) + \theta_1^*(x, t)} + \zeta_3(x) e^{\theta_1(x, t) + \theta_2^*(x, t)} + \zeta_4(x) e^{\theta_2(x, t) + \theta_2^*(x, t)} \right], \quad (3c)$$

$$f_4(x, t) = \eta(x) e^{4\Gamma x + \theta_1(x, t) + \theta_2(x, t) + \theta_1^*(x, t) + \theta_2^*(x, t)}. \quad (3d)$$

Here  $\theta_j(x, t) = a_j(x) + b_j t + k_j = [a_{j1}(x) + ia_{j2}(x)] + (b_{j1} + ib_{j2})t + (k_{j1} + ik_{j2})$  ( $j = 1, 2$ ),  $a_{j1}(x)$  and  $a_{j2}(x)$  are both real differentiable functions, and  $b_{j1}$ ,  $b_{j2}$ ,  $k_{j1}$ ,  $k_{j2}$  are real constants to be determined. The constraints on the model parameters are as follows

$$\begin{aligned} a_{j1}(x) &= \int b_{j1} \left[ (b_{j1}^2 - 3b_{j2}^2) \beta_3(x) - b_{j2} \beta_2(x) \right] dx, \\ a_{j2}(x) &= \int \left[ \frac{1}{2} (b_{j1}^2 - b_{j2}^2) \beta_2(x) + b_{j2} (3b_{j1}^2 - b_{j2}^3) \beta_3(x) \right] dx, \\ \xi_1(x) &= \frac{(b_1 - b_2)^2 \sigma(x)}{4(b_1^* + b_2)^2 b_{11}^2 \beta_2(x)}, \quad \xi_2(x) = \frac{(b_1 - b_2)^2 \sigma(x)}{4(b_1 + b_2^*)^2 b_{21}^2 \beta_2(x)}, \\ \zeta_1(x) &= \frac{\sigma(x)}{4b_{11}^2 \beta_2(x)}, \quad \zeta_2(x) = \frac{\sigma(x)}{(b_1^* + b_2)^2 \beta_2(x)}, \\ \zeta_3(x) &= \frac{\sigma(x)}{(b_1 + b_2^*)^2 \beta_2(x)}, \quad \zeta_4(x) = \frac{\sigma(x)}{4b_{21}^2 \beta_2(x)}, \\ \eta(x) &= \frac{[(b_{11} - b_{21})^2 + (b_{12} - b_{22})^2]^2 \sigma(x)^2}{16(b_1^* + b_2)^2 (b_1 + b_2^*)^2 b_{11}^2 b_{21}^2 \beta_2(x)^2}, \quad \beta_2(x) = e^{2\Gamma x} \sigma(x). \end{aligned}$$

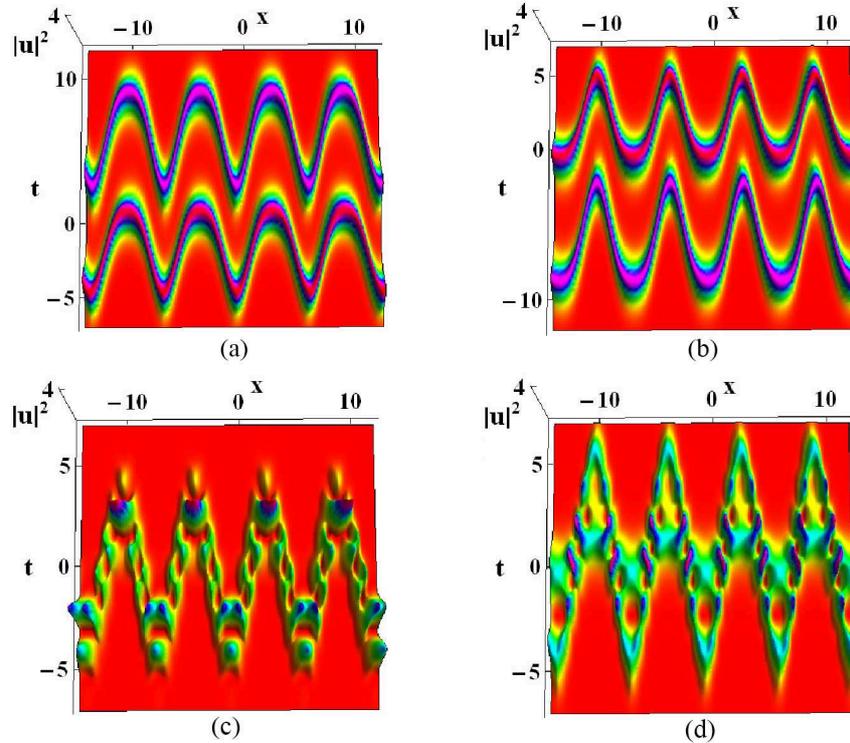


Fig. 1 – Periodic interactions of optical solitons in dispersive nonlinear fibers. The parameters are  $k_{11} = 1, k_{12} = 2, k_{21} = 2, k_{22} = 1, \Gamma = -0.001, \sigma(x) = \cos(2x)$  and  $\beta_3(x) = \cos(x) + \sin(x)$  with (a):  $b_{11} = 0.9, b_{12} = 1, b_{21} = 1, b_{22} = 1$ ; (b):  $b_{11} = -0.9, b_{12} = -1, b_{21} = -1, b_{22} = -1$ ; (c):  $b_{11} = 1, b_{12} = 1, b_{21} = 1, b_{22} = -1$ ; (d):  $b_{11} = 1, b_{12} = 1, b_{21} = -1, b_{22} = -1$ .

### 3. DISCUSSION OF NUMERICAL RESULTS

For Eq. (2), the parameters are chosen as  $k_{11} = 1, k_{12} = 2, k_{21} = 2, k_{22} = 1, \Gamma = -0.001, \sigma(x) = \cos(2x)$ , and  $\beta_3(x) = \cos(x) + \sin(x)$ . We mainly discuss the influences of  $b_{11}, b_{12}, b_{21}$ , and  $b_{22}$  on soliton propagation. In Figs. 1(a) and 1(b), the two solitons propagate periodically. They propagate with the same phase and separation interval. In the propagation process, they do not interact with each other, and keep their original characteristics moving forward periodically when the positive and negative signs in front of  $b_{11}, b_{12}, b_{21}$ , and  $b_{22}$  are all the same.

However, when the positive and negative signs in front of the above four parameters are different, the solitons strongly interact with each other. In Fig. 1(c),  $b_{22}$  decreases to  $-1$ , the interval between solitons becomes smaller, and this small interval makes them interact strongly. Moreover, changing the signs of  $b_{21}$  can change the phase of the soliton. As shown in Fig. 1(d), the two interacting solitons cause a

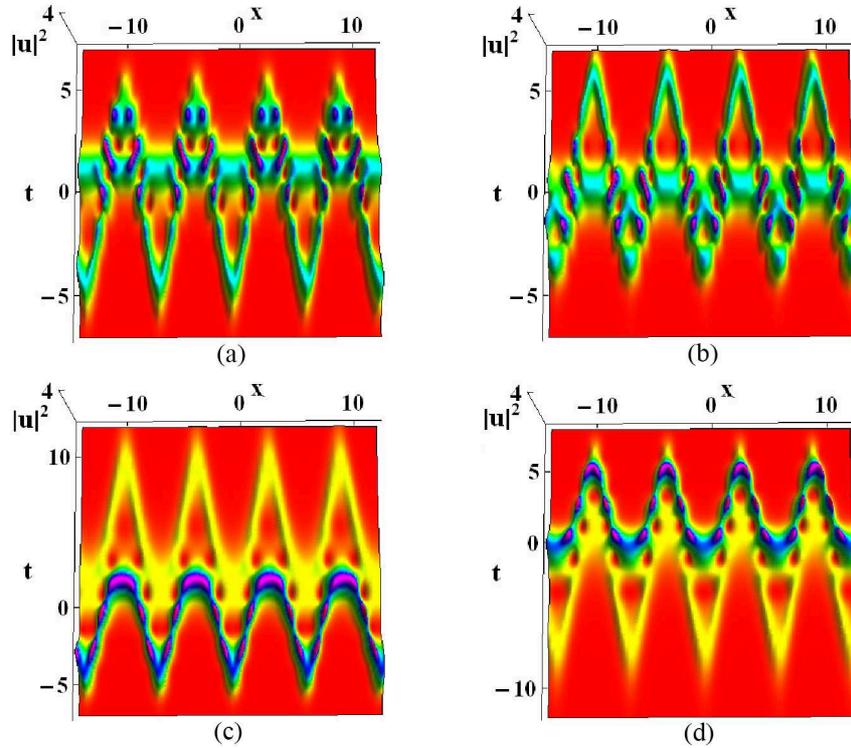


Fig. 2 – Periodic interactions of optical solitons in dispersive nonlinear fibers. The parameters are  $k_{11} = 1$ ,  $k_{12} = 2$ ,  $k_{21} = 2$ ,  $k_{22} = 1$ ,  $\Gamma = -0.001$ ,  $\sigma(x) = \cos(2x)$  and  $\beta_3(x) = \cos(x) + \sin(x)$  with (a):  $b_{11} = 1$ ,  $b_{12} = 1$ ,  $b_{21} = -1$ ,  $b_{22} = -0.85$ ; (b):  $b_{11} = 1$ ,  $b_{12} = 0.85$ ,  $b_{21} = -1$ ,  $b_{22} = -1$ ; (c):  $b_{11} = 1$ ,  $b_{12} = 1$ ,  $b_{21} = -0.5$ ,  $b_{22} = -1$ ; (d):  $b_{11} = 0.5$ ,  $b_{12} = 1$ ,  $b_{21} = -1$ ,  $b_{22} = -1$ .

backward propagation due to the opposite phases. The nearer parts of two solitons interact more strongly, whereas the far away parts of the two solitons are less affected by soliton interactions.

Compared to Fig. 1(d), when  $b_{22}$  increases from  $-1$  to  $-0.85$ , the interval between solitons becomes larger, and we obtain that the interaction between solitons is weakened, as shown in Fig. 2(a). To verify this result, we decrease  $b_{12}$  to  $0.85$ , and we get that the soliton interactions are also weakened, see Fig. 2(b).

By decreasing the phase difference between solitons, we can also weaken the soliton interactions. In Figs. 2(c) and 2(d), the values of  $b_{21}$  and  $b_{11}$  are changed in order to decrease the phase difference, and we see that one of the two interacting solitons has only a small impact on the other one. Thus, we can control the periodic interactions of solitons through the appropriate selection of the values of the model parameters  $b_{11}$ ,  $b_{12}$ ,  $b_{21}$ , and  $b_{22}$ .

#### 4. CONCLUSIONS

In conclusion, we have investigated, both analytically and numerically, the periodic interactions of optical solitons in dispersive nonlinear optical fibers described by the variable-coefficient higher-order nonlinear Schrödinger equation. When the group-velocity dispersion and third-order dispersion coefficients have been chosen to be either sine or cosine functions, the optical solitons propagate periodically. The two model parameters  $b_{11}$  and  $b_{21}$  have been used to control the soliton phase, whereas the other two model parameters  $b_{12}$  and  $b_{22}$  have been related to the soliton separation interval.

Both the in-phase and out-of-phase interactions of optical solitons have been studied. When the two interacting solitons have been launched in the same direction, they can propagate stably without significant interaction. Otherwise, significant soliton interactions occur in the case of reverse propagation of the two nonlinear pulses.

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