

DISSIPATIVE CYLINDRICAL MAGNETOSONIC SOLITARY WAVES IN A MAGNETIZED QUANTUM DUSTY PLASMA

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Abstract. The propagation of nonlinear magnetosonic wave in electron-ion-dust (complex) plasmas, considering the effects of Bohm potential in the presence of an external magnetic field is reported. By means of the quantum hydrodynamics model and applying the reductive perturbation method, a cylindrical Kadomtsev-Petviashvili-Burgers (CKPB) equation is derived. The CKPB equation can be solved analytically using a suitable coordinate transformation with the *tanh* technique. The numerical results reveal that the plasma density, the magnetic field strength, and the dust kinematic viscosity strongly affect on the profile of magnetoacoustic shocks. It is found that the shock structures decrease with the increase of electron number density. Moreover, increasing the value of magnetic field intensity, the strength of the cylindrical magnetoacoustic shock diminishes at a specific time and the strength of the shock increases with the enhancement of the dust kinematic viscosity.

Key words: dusty plasma; magnetosonic shock wave; the cylindrical Kadomtsev-Petviashvili-Burgers equation; external magnetic field; the *tanh* technique.

1. INTRODUCTION

Nonlinear structures (e.g., solitary waves, shock waves, cnoidal waves, rogue waves, etc.) have been noticed in numerous physical systems, optics, including ocean, plasma, etc. [1-13]. These waves play a pivotal role in the investigation of plasma physics especially in dusty plasma. Dusty plasma is a medium in which solid particles “dust” are immersed within a plasma environment [14]. Dust grains in plasma have negative charge of several thousand elementary charges, because of collection of electrons (and ions flowing onto their surfaces). This type of plasma is omnipresent in the different astrophysical bodies such as magnetosphere of the Earth [15], pulsar magnetospheres [16], cemetery tails [17], the radial structure of Saturn’s rings [18] and the polar regions of neutron stars [19]. Many researchers have proven that this new state of matter has unique

thermodynamic properties radically different from those of ordinary electron-ion plasmas [20].

It is well known that the plasma is treated as a degenerate fluid when the charge carriers in plasmas are extremely dense and quantum mechanical effects play an important role in the dynamics (i.e. when the de Broglie thermal wavelength of the charged particles is equal or larger than the average inter-particle distance or the Fermi temperature is greater than the temperature of the system) [21-24]. The linear and nonlinear electrostatic and electromagnetic waves are studying by using the quantum hydrodynamic (QHD) model, which is beneficial in studying quantum systems, which have dimension larger than the Fermi lengths of the system [25-28] due to its applications in a variety of physical systems, such as ordinary metals, semiconductors, super dense astrophysical environments (e.g. neutron stars, white dwarfs etc.), nano-devices, and in laser-plasma experiments [25, 26, 29-32]. The QHD model can be generalized by adding the quantum statistical pressure term (the Fermi-Dirac distribution) and the quantum diffraction term (the Bohm potential) to the fluid model. The results of studying the nonlinear wave phenomenon in these systems have shown that the quantum effects have important role in studying of the nonlinear wave phenomenon in quantum plasma [25, 26].

Magnetoacoustic wave is a coherent nonlinear wave with low frequency mode in magnetized plasmas, which propagate perpendicular to both electric field and the surrounding magnetic field. Recently, researchers in theoretical and laboratory plasma [33, 34] have been studied the magnetosonic waves in plasma due to their applications in space plasmas, Earth's magnetotail, and in fusion plasmas. The nonlinear magnetosonic waves in quantum plasmas were studied by many authors [35]. Masood *et al.* [36] have investigated the linear and nonlinear properties of an obliquely propagating magnetosonic wave in a three-component dusty plasma. They found that not only retaining the electron pressure term gives rise to novel features in the dust magnetosonic wave, but also the slow dust magnetosonic wave is found to be the damped mode and, therefore, the only propagating mode in a dusty plasma is the fast magnetosonic mode. Mushtaq and Vladimirov [37] studied the magnetosonic waves in magnetized degenerate plasmas and found that the wave amplitude increases with the decrease in the Zeeman energy values. Liu *et al.* [38] have investigated the nonlinear fast magnetoacoustic solitary waves in a cylindrical dusty plasma and derived cylindrical Kadomtsev-Petviashvili (CKP) equation. They have shown that the dust cylindrical fast magnetoacoustic solitary waves in warm plasmas may disappear slowly by the increase in dust mass. The head-on collisions of dust magnetoacoustic solitary waves (DMASWs) in a magnetized electron-ion-dust plasma have been studied by Ruan *et al.* [39]. They found that this system admits a

solution with two solitary waves, they travel toward each other. The head-on collisions of two magnetosonic solitary waves in quantum plasma was investigated by Li and Han [40]. They showed that both the compressive and rarefactive magnetosonic solitary waves exist in the system and only negative phase shifts for both compressive and rarefactive wave collisions is obtained. Hussain and Mahmood [41] studied low frequency nonlinear magnetosonic wave propagation in electron-ion quantum plasmas and they found that the hump magnetosonic solitons move with supermagnetosonic wave speed, but the dip solitons move with submagnetosonic wave speed.

The linear and nonlinear low-frequency waves in quantum plasma such as ion acoustic waves drift waves etc. have been studied by several authors [42-46]. Many researchers have studied propagation of the nonlinear waves in planar [47-52] and nonplanar [53-55] geometries. There are many practical importance cases such as capsule implosion (spherical geometry), shock tube (cylindrical geometry), star formation, and supernova explosions where planar geometry is not suitable and should be considered a nonplanar geometry.

The paper is organized as follows. In Sec. 2, the mathematical model and the basic equations for a degenerate quantum dusty plasma are described and using the reductive perturbation method, the KP equation is derived. Section 3 is devoted to the equilibrium points that are obtained by bifurcation theory. The possibility of the existence of solitary wave structures and periodic travelling wave solutions is discussed in Sec. 4. Section 5 reports the numerical analysis and the discussion of results. Finally, the conclusion is presented in Sec. 6.

2. MATHEMATICAL MODEL AND DERIVATION OF THE EVOILUTION EQUATION

We assume three-fluid magnetized quantum plasma in cylindrical geometry, in the presence of external magnetic field directed along the z -axis, i.e. $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. Further, we assume that all variables lie in the $r - \theta$ plane. The dynamic equations for studying low frequency magnetosonic waves in magnetized dusty plasmas, are given by

$$\frac{\partial n_l}{\partial t} + \frac{\partial (rn_l u_l)}{\partial r} + \frac{1}{r} \frac{\partial (n_l v_l)}{\partial \theta} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial r} + \frac{v_d}{r} \frac{\partial u_d}{\partial \theta} - \frac{v_d^2}{r} = -(E_r + \Omega_d v_d B) + \\ \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_d}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_d}{\partial \theta^2} \right), \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v_d}{\partial t} + u_d \frac{\partial v_d}{\partial r} + \frac{v_d}{r} \frac{\partial v_d}{\partial \theta} + \frac{u_d v_d}{r} = -(E_\theta - \Omega_d u_d B) + \\ \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_d}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_d}{\partial \theta^2} \right), \end{aligned} \quad (3)$$

$$\begin{aligned} [E_r + \Omega_d v_i B] - \Gamma_i \frac{1}{n_i} \frac{\partial n_i^{5/3}}{\partial r} + \\ \frac{1}{9} H_i \frac{\partial}{\partial r} \frac{1}{\sqrt{n_i}} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \sqrt{n_i} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \sqrt{n_i} \right) = 0, \end{aligned} \quad (4)$$

$$\begin{aligned} [E_r - \Omega_d u_i B] - \Gamma_i \frac{1}{n_i} \frac{1}{r} \frac{\partial n_i^{5/3}}{\partial \theta} + \\ \frac{1}{9} \frac{H_i}{r} \frac{\partial}{\partial \theta} \frac{1}{\sqrt{n_i}} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \sqrt{n_i} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \sqrt{n_i} \right) = 0, \end{aligned} \quad (5)$$

$$\begin{aligned} E_r + \Omega_d v_e B + \Gamma_e \frac{1}{n_e} \frac{\partial n_e^{5/3}}{\partial r} - \\ \frac{1}{9} H_e \frac{\partial}{\partial r} \frac{1}{\sqrt{n_e}} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \sqrt{n_e} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \sqrt{n_e} \right) = 0, \end{aligned} \quad (6)$$

$$E_r - \Omega_d u_e B + \Gamma_e \frac{1}{n_e} \frac{1}{r} \frac{\partial n_e^{5/3}}{\partial \theta} - \frac{1}{9} \frac{H_e}{r} \frac{\partial}{\partial \theta} \frac{1}{\sqrt{n_e}} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \sqrt{n_e} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \sqrt{n_e} \right) = 0, \quad (7)$$

where $\ell = e(i, d)$ is for electron (ion, dust), $\Gamma_j = (m_d Z_{d0} / m_d)(V_{Fj}^2 / c_d^2)$ is the dimensionless coefficients of the j-th Fermi pressure terms, $c_d = \sqrt{2Z_{d0}k_B T_{Fi} / m_d}$ is the Fermi dust acoustic speed, $V_{Fj} = \sqrt{2\varepsilon_{Fj} / m_e}$ is the j-th Fermi speed and $\varepsilon_{Fj} = \hbar^2(3\pi^2 n_{j0})^{2/3} / 2m_e$ is the Fermi energy of the j-th species. $H_j = \hbar^2 Z_{d0} / 2m_j m_d \lambda_d^2 c_d^2$ is the j-th dimensionless of the quantum diffraction effects, μ is the normalized kinematic viscosity, $\Omega_d = q_d B_0 / m_d c$ is the dust gyro-frequency normalized by the dust plasma frequency $\omega_{pd} = \sqrt{4\pi n_{d0} Z_{d0}^2 e^2 / m_d}$, $q_d = -Z_{d0} e$ is the charge residing on the dust grains, and $\lambda_d = \sqrt{2k_B T_{Fi} / 4\pi n_{d0} Z_{d0} e^2}$ is the Fermi dust wave length. The densities are normalized by their respective equilibrium values, the velocities are normalized by the Alfvén speed V_A ($V_{AD} = B_0 / \sqrt{4\pi m_d n_{d0}}$), the magnetic field is normalized by the applied static field B_0 , and the electric field is normalized by $q_d / m_d \omega_{pd} c_d$. The time and space variables are normalized by dust plasma frequency ω_{pd}^{-1} and Fermi dust wavelength λ_d , respectively.

The Maxwell's equations for the electric and magnetic fields are given as

$$\Omega_d \frac{\partial B}{\partial t} = \frac{1}{r} \frac{\partial E_r}{\partial \theta} - \frac{E_\theta}{r} - \frac{\partial E_\theta}{\partial r}, \quad (8)$$

$$\frac{1}{r} \frac{\partial B}{\partial \theta} = \Omega_d \beta \left((1 + \alpha) n_i u_i - \alpha n_e u_e - n_d u_d + \frac{\partial E_r}{\partial t} \right), \quad (9)$$

and

$$-\frac{\partial B}{\partial r} = \Omega_d \beta \left((1 + \alpha) n_i v_i - \alpha n_e v_e - n_d v_d + \frac{\partial E_\theta}{\partial t} \right). \quad (10)$$

Here $\alpha = n_{e0} / Z_{d0} n_{d0}$ and $\beta = c_d^2 / V_{AD}^2$ is the quantum plasma beta value.

To investigate the solitary waves in such a plasma, we use the standard reductive perturbation method. The independent variables are stretched as: $\xi = \varepsilon^{1/2}(r - \lambda t)$, $\eta = \varepsilon^{-1/2}\theta$, and $\tau = \varepsilon^{3/2}t$, where ε is a real and small ($0 < \varepsilon \ll 1$) expansion parameter characterizing the nonlinearity strength and λ is the phase velocity of the propagating wave. Considering the kinematic viscosity to be small, we can assume $\mu = \varepsilon^{1/2}\kappa$ and the other perturbed quantities are expanded in terms of power series of ε about their equilibrium values as follows

$$n_l = 1 + \varepsilon n_l^{(1)} + \varepsilon^2 n_l^{(2)} + \varepsilon^3 n_l^{(3)} \dots, \quad (11)$$

$$u_l = \varepsilon u_l^{(1)} + \varepsilon^2 u_l^{(2)} + \varepsilon^3 u_l^{(3)} \dots, \quad (12)$$

$$v_l = \varepsilon^{3/2} v_l^{(1)} + \varepsilon^2 v_l^{(2)} + \varepsilon^{5/2} v_l^{(1)} \dots, \quad (13)$$

$$B = 1 + \varepsilon B_z^{(1)} + \varepsilon^2 B_z^{(2)} + \varepsilon^3 B_z^{(3)} \dots, \quad (14)$$

$$E_r = \varepsilon^{3/2} E_r^{(1)} + \varepsilon^{5/2} E_r^{(2)} \dots, \quad (15)$$

$$E_\theta = \varepsilon E_\theta^{(1)} + \varepsilon^2 E_\theta^{(2)} + \varepsilon^3 E_\theta^{(3)} \dots, \quad (16)$$

Using the stretched variables in Eqs. (1)-(10), we develop equations in different powers of ε . For the lowest order of ε , we obtain

$$n_e^{(1)} = n_i^{(1)} = n_d^{(1)} = B_z^{(1)}, \quad u_e^{(1)} = u_i^{(1)} = u_d^{(1)} = \lambda B_z^{(1)}, \quad E_\theta^{(1)} = \lambda \Omega_d B_z^{(1)},$$

$$\begin{aligned} v_i^{(1)} &= -\frac{1}{\Omega_d} \left\{ E_r^{(1)} - \frac{5}{3} \Gamma_i \frac{\partial B_z^{(1)}}{\partial \xi} \right\}, \\ v_e^{(1)} &= -\frac{1}{\Omega_d} \left\{ E_r^{(1)} + \frac{5}{3} \Gamma_e \frac{\partial B_z^{(1)}}{\partial \xi} \right\}, \\ v_d^{(1)} &= -\frac{1}{\Omega_d} \left(E_r^{(1)} - \lambda^2 \frac{\partial B_z^{(1)}}{\partial \xi} \right). \end{aligned} \quad (17)$$

Solving the system (17) yields the normalized phase velocity of the magnetosonic waves as follows

$$\lambda = \sqrt{\frac{3 + 5\beta\Gamma_i + 5\alpha\beta(\Gamma_e + \Gamma_i)}{3\beta(1 + \Omega_d^2)}} \quad (18)$$

From the next higher order in ε and exploiting relations (1)-(10), we obtain the relation between $E_r^{(1)}$ and $B_z^{(1)}$ as

$$\frac{\partial E_r^{(1)}}{\partial \xi} = -\frac{\Omega_d}{\tau} \frac{\partial B_z^{(1)}}{\partial \eta} + \frac{\lambda^2}{(1 + \Omega_d^2)} \frac{\partial^2 B_z^{(1)}}{\partial \xi^2}. \quad (19)$$

After some tedious and straightforward algebraic manipulations of the second order equations we get the following cylindrical Kadomstev-Petviashvili-Burgers (CKPB) equation:

$$\begin{aligned} & \frac{\partial}{\partial \xi} \left(\frac{\partial B_z^{(1)}}{\partial \tau} + P B_z^{(1)} \frac{\partial B_z^{(1)}}{\partial \xi} + Q \frac{\partial^3 B_z^{(1)}}{\partial \xi^3} - R \frac{\partial^2 B_z^{(1)}}{\partial \xi^2} + \frac{B_z^{(1)}}{2\tau} \right) + \\ & \frac{1}{2\lambda\tau^2} \frac{\partial^2 B_z^{(1)}}{\partial \eta^2} = 0, \end{aligned} \quad (20)$$

where

$$P = \frac{27\lambda^2 - 5[\alpha\Gamma_e + (1 + \alpha)\Gamma_i]}{18\lambda(1 + \Omega_d^2)}, \quad Q = \frac{18\lambda^4 - [\alpha H_e + (1 + \alpha)H_i](1 + \Omega_d^2)}{36\lambda(1 + \Omega_d^2)^2},$$

and $R = \frac{\kappa}{2(1 + \Omega_d^2)}$. Here P , Q , and R are the coefficients of the nonlinearity,

the dispersion, and the dissipation, respectively. The coefficient Q is modified due to the presence of the Bohm potential, whereas the coefficient R is proportional of the kinematic viscosity.

To study the nonlinear partial differential equation (20), some researchers [56-60] have proposed the solutions by different methods such as the inverse scattering method [61], the Hirota Bilinear formalism [62], the Backlund transformation [63], the *tanh* method [64], etc. In the following, we propose an analytical solution for Eq. (20) using *tanh* method [65-71]. To do this, the following transformations are introduced

$$X = \xi - \left(\frac{\lambda}{2} \eta^2 \tau + \eta \tau + \frac{1}{2\lambda^2} \tau \right), \quad \text{and} \quad B_z^{(1)} = B_z^{(1)}(X, \tau),$$

and substituting into Eq. (20), we obtain

$$\frac{\partial}{\partial X} \left(\frac{\partial B_z^{(1)}}{\partial \tau} + P B_z^{(1)} \frac{\partial B_z^{(1)}}{\partial X} - R \frac{\partial^2 B_z^{(1)}}{\partial X^2} + Q \frac{\partial^3 B_z^{(1)}}{\partial X^3} \right) = 0. \quad (21)$$

These transformations enable us to search for analytic solutions in a simpler way. Combining the new above variables X and τ into the travelling variable $\zeta = kX - \omega\tau$ where k and ω are constants to be determined later, we would be able to find the possible solutions of the Eq. (21) by using symbolic software like *Mathematica* in the following solutions [68]

$$B_z^{(1)}(\zeta) = A \sec^2(\zeta) + B \tanh(\zeta) + C \quad (22)$$

where

$$A = \frac{3R^2}{25PQ}, \quad B = -\frac{6R^2}{25PQ}, \quad C = -\frac{6R^2}{25PQ}, \quad \omega = -\frac{3R^3}{125Q^2}, \quad \text{and } k = \frac{R}{10Q}.$$

3. RESULTS AND DISCUSSION

In this Section, we investigate numerically the effects of some physical parameters on the characteristics of dust magnetosonic shock wave such as the electron density n_{e0} , the amplitude of the magnetic field B_0 , and the kinematic viscosity μ . For this purpose, we choose some of the typical plasma parameters in the cgs system units [31, 69-72], $c = 3 \times 10^{10} \text{ cm/s}$, $e = 4.8 \times 10^{-10} \text{ esu}$, $k_B = 1.38 \times 10^{-16} \text{ erg/k}$, $\hbar = 1.05 \times 10^{-27} \text{ erg s}$, $n_{e0} \sim 9.9 \times 10^{29} \text{ cm}^{-3}$, $T_i = 10^8 \text{ k}$, $n_{i0} \sim 10^{30} \text{ cm}^{-3}$, $n_{d0} \sim 10^{24} - 10^{27} \text{ cm}^{-3}$, $Z_{d0} \sim 10^3$, $m_i = 1836 m_e$, $m_d = 10^{12} m_i$, and $B_0 = 10^{10} \sim 10^{12} \text{ G}$.

Figure 1 shows the time evolution of the dust magnetoacoustic shock wave profiles in cylindrical geometry. It is obvious that at starting time ($\tau = 0$) weaker magnetoacoustic shocks are observed whereas by increasing time, the strength of shocks are found to increase significantly.

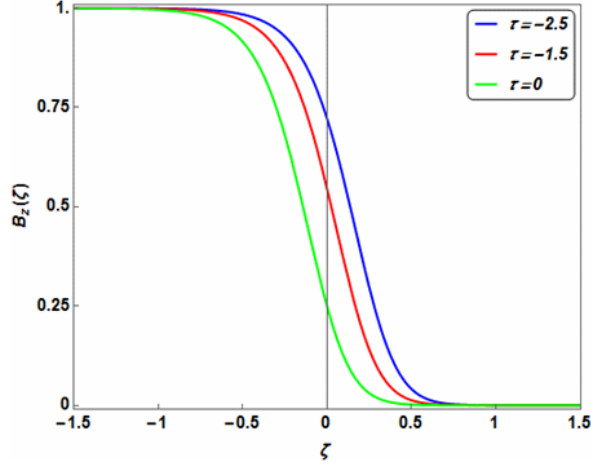


Fig. 1 – Shock strength $B_z^{(1)}$ versus ζ for different value of times. Here, $n_{e0} \sim 9.9 \times 10^{29} \text{ cm}^{-3}$, $n_{i0} \sim 10^{30} \text{ cm}^{-3}$, $B_0 = 10^{12} \text{ G}$, and $\mu = 0.2$.

The shock strength $B_z^{(1)}$ versus ζ is plotted in Fig. 2. It is clear that it decreases with the increase in electron number density. It is noticed that the propagation speed of the shock is enhanced remarkably by increasing the plasma density.

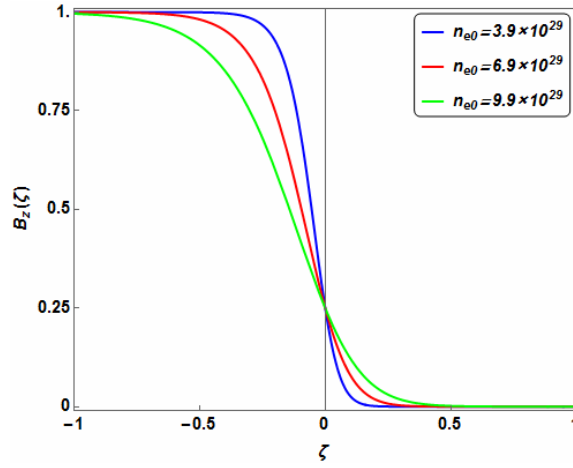


Fig. 2 – The behavior of the shock wave $B_z^{(1)}$ versus ζ for different values of n_{e0} . Here, $\tau = 0$, $n_{i0} \sim 10^{30} \text{ cm}^{-3}$, $B_0 = 10^{12} \text{ G}$, and $\mu = 0.2$.

The strength of magnetic field always plays a key role in the study of nonlinear magnetoacoustic waves. It is evident from Fig. 3 that increasing the

magnetic field intensity, the cylindrical magnetoacoustic shock diminishes for a specific time. The enhancement in the value of magnetic field intensity also results a decrement in the value of β .

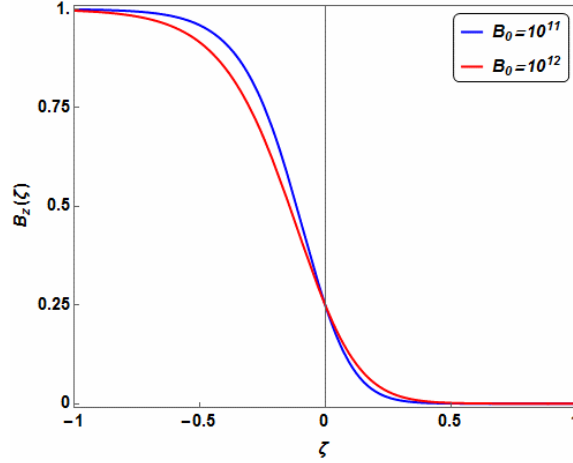


Fig. 3 – Variation of $B_z^{(1)}$ against ζ for different values of B_0 . Here, $\tau = 0$, $n_{e0} \sim 9.9 \times 10^{29} \text{ cm}^{-3}$, $n_{i0} \sim 10^{30} \text{ cm}^{-3}$, and $\mu = 0.2$.

In Fig. 4, it is observed that for small variations in the viscosity parameter, a significant change in the amplitude of shock structure takes place. The strength of the shock increases with the enhancement of the kinematic viscosity of the dust grains in the plasmas. The shock speed U_0 is found to increase with the enhancement in the value of kinematic viscosity due to ions.

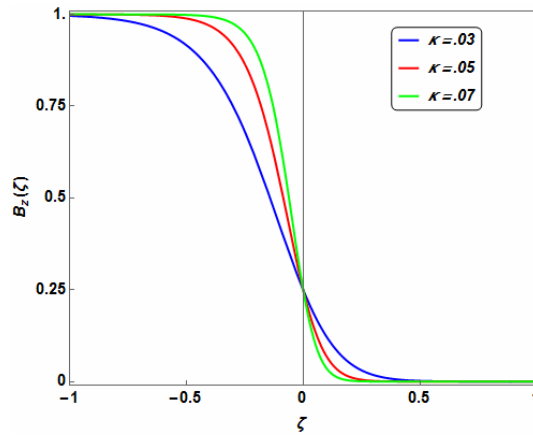


Fig. 4 – Variation of the shock wave $B_z^{(1)}$ versus ζ for the different values of viscosity parameter.

Here, $\tau = 0$, $n_{e0} \sim 9.9 \times 10^{29} \text{ cm}^{-3}$, $n_{i0} \sim 10^{30} \text{ cm}^{-3}$ and $B_0 = 10^{12} \text{ G}$.

4. CONCLUSION

We have investigated the nonlinear magnetosonic wave structures in a dense quantum dusty plasma in the presence of the external magnetic field (directed along the z -axis, i.e. $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$). Using the standard reductive perturbation technique and the QHD model for the dynamics of three-fluid magnetized quantum plasma, the cylindrical Kadomstev-Petviashvili-Burgers (CKPB) equation, which represents the dynamics of small as well as finite amplitudes of the quantum magnetosonic solitary wave is found. We have found the solution of the CKPB equation by the *tanh* technique. The effects of plasma density, magnetic field strength, and dust kinematic viscosity on the profile of magnetoacoustic shocks are plotted. It is shown that the shock structures decrease with the increase of electron number density. Also, it is exhibited that by increasing the value of magnetic field intensity the strength of the cylindrical magnetoacoustic shock diminishes for a specific time. The strength of the shock increases with the enhancement of the dust kinematic viscosity. Therefore, our present study of magnetosonic shock waves is useful in understanding the magnetosonic shock formation in many astrophysical plasma environments such as neutron stars, white dwarfs, and chromospheric and coronal heating phenomenon where the quantum effects are expected to play a vital role.

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