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MULTIDIMENSIONAL LOCALIZED STRUCTURES IN OPTICAL AND MATTER-WAVE MEDIA: A TOPICAL SURVEY OF RECENT LITERATURE

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Abstract. We perform a survey of some recent theoretical and experimental studies on multidimensional (two- and three-dimensional) localized structures in optical and matter-wave media in a broad set of physical settings. This article is structured as a resource letter that briefly outlines a large series of results in the areas of multidimensional solitons in optical media, nonlinear structures in parity-time-symmetric systems, rogue waves in multidimensional physical settings, and matter-wave localized structures.

Key words: localized optical structures, spatiotemporal optical solitons, optical vortices, parity-time-symmetric nonlinear waves, rogue waves, matter-wave localized structures.

1. INTRODUCTION

The propagation and self-trapping of multidimensional localized structures in optical and matter-wave media is a research field of broad interest in fundamental and applied sciences (see Refs. [1–38] for a series of relevant works). Common to both optical and matter-wave media are *i*) a theoretical framework that is amenable to both analytic and numerical treatments and *ii*) an almost unprecedented experimental robustness that turned these two generic media into a genuine test bed for nonlinear phenomena (see Ref. [1] on the side of optical media and Ref. [39] on the side of matter-wave media). The evolution equations that describe the dynamics of localized structures in both media are mathematically kindred to the celebrated nonlinear Schrödinger, Gross-Pitaevskii, and Ginzburg-Landau equations and therefore benefited from numerous existing results (see, for example, the comprehensive review [27], as well as Ref. [40]). Moreover, the advent of advanced scientific computing methods allowed for very efficient numerical treatments of these generic nonlinear partial differential equations across a wide range of computing infrastructures [41–48]. In particular, different variants of Crank-Nicolson finite-difference and semi-implicit algorithms were developed and carefully compared in the literature [41, 42]. Intensive effort was also devoted to parallelization of these algorithms, closely following significant development of available computing resources, including multi- and many-core CPUs, general-puropose graphical processing units (GPGPUs), and parallel clusters, comprising of vast numbers of compute nodes, each with one or more multi-core CPUs and, possibly, one or more GPGPUs. To make use of multi-core shared-memory environments, OpenMP C versions of programs were developed [43], while MPI C variant [44] is intended for use on parallel clusters. They employ hybrid architecture, where OpenMP is used on individual cluster compute nodes, while MPI is used to distribute the load between cluster nodes. Fortran version of OpenMP programs is also available [45]. Crank-Nicolson algorithms were further developed for nonlinear partial differential equations with convolution-type integral kernels, especially useful for treatment of dipole-dipole interaction in various physical systems [46–48]. CUDA version of programs [47] makes possible efficient use of a single GPGPU, while the software package described in Ref. [48] combines OpenMP, MPI, and CUDA versions of programs into a hybrid algorithm that is capable of using all available computing resources of a large-scale parallel cluster.

In this topical review we focus on some recent theoretical and experimental studies dedicated to localized structures in optical and matter-wave media, which expand our understanding of these systems and have the potential to catalyze new research directions. This paper is intended for scientists with a good exposure on these topics, but junior researchers can also benefit from it. The article is structured in six sections, the first one being of introductory nature, while the last one gathers the concluding remarks. The remaining sections are dedicated to multidimensional solitons in optical media (Sec. 2), nonlinear localized structures in parity-time-symmetric systems (Sec. 3), rogue waves in multidimensional physical settings (Sec. 4), and matter-wave localized structures (Sec. 5). The chapters are largely independent of one-another and can be used as resource letters on the specific subject of the chapter.

Among the plethora of localized structures investigated over the past few decades, two- and three-dimensional (2D and 3D) stand out due to their important role in diverse applications; see a recent viewpoint by Malomed *et al.* [36] on multidimensional solitons and their legacy in contemporary atomic, molecular, and optical physics. Also, a comprehensive review on well-established results and novel findings in the area of multidimensional solitons in nonlinear optics and atomic Bose-Einstein condensates (BECs) has been recently published by Malomed [38].

The unique phenomena of multidimensional soliton formation and robust propagation over long distances in optical media, i.e., over several diffraction/dispersion lengths, were investigated in detail in connection to the fascinating possibility of using spatiotemporal optical solitons (alias light bullets) as elementary bits of information in all-optical processing devices [4, 5]. Other practical applications of temporal, spatial, and spatiotemporal solitons consist of the use of dissipative solitons in mode-locked lasers [49], the supercontinuum generation in photonic crystal fibers [50, 51] and the use of ultrashort (few-cycle) solitons in supercontinuum generation processes [52, 53], the possibility of using temporal cavity solitons as bits in all-optical buffers [54] etc. Diverse applications of 3D matter-wave solitons are also envisaged, e. g. in highly precise interferometry [55–57].

The rest of the paper is structured as follows. In Sec. 2 we overview the recent research activity in the area of multidimensional solitons in optical media, namely: i) theoretical and experimental studies of linear and nonlinear light bullets, ii) theoretical studies of 2D and 3D solitons that form in carbon nanotubes, and iii) recent theoretical and experimental activity in the area of ultrashort (few-cycle) localized optical structures. Section 3 overviews recent results on nonlinear localized structures in parity-time-symmetric optical systems. In Sec. 4 we briefly review the recent works on rogue waves in multidimensional physical settings. Matter-wave multidimensional localized structures are discussed in Sec. 5. In Sec. 6 we present our conclusions.

2. MULTIDIMENSIONAL SOLITONS IN OPTICAL MEDIA

In nonlinear optical media the 2D and 3D solitons are self-trapped as a result of the competition between linear and nonlinear effects. The linear effects of diffraction (in one transverse direction for 2D solitons and in both transverse directions for 3D solitons) and group-velocity dispersion tend to broaden the optical wave packet in both spatial and temporal dimensions, whereas the nonlinear effects tend to induce the self-compression of the optical field. The one-dimensional (1D) solitons, e.g. the temporal solitons that form in monomode optical fibers, are usually stable physical objects [1]. However, the 2D and 3D optical solitons are stable only in a few physical settings. The ubiquitous self-focusing cubic (Kerr-type) nonlinearities lead to optical wave collapse, which completely destroy the corresponding localized objects; see for example a comprehensive review paper on wave collapse in physics with specific applications in optical and plasma waves [58]. The wave collapse can be either critical (in two dimensions) or supercritical (in three dimensions); see Ref. [58]. Fundamental (vorticity-less) solitons are destabilized by the wave collapse in two- and three-dimensional settings, whereas the vortex solitons are destroyed by the so-called splitting instability that breaks the axial symmetry of these topological objects, leading to their splitting into several fundamental solitons. The stabilization of either fundamental or vortex 2D and 3D solitons is a key issue from both theoretical and practical point of view and several generic physical mechanisms and settings to achieve soliton's stabilization in two and three dimensions have been put forward during the past years. The use of weaker nonlinearities that exist in either quadratic and saturable nonlinear optical media do not lead to wave collapse; the fundamental solitons are found to be stable, whereas the vortices cannot be stabilized by this mechanism. The effective trapping potentials, e.g., the photonic lattices (spatially periodic trapping potentials) may stabilize both 2D and 3D fundamental and vortex solitons. Also, the use of competing optical nonlinearities, e.g., the self-focusing cubic nonlinearities in combination with self-defocusing quintic nonlinearities can lead to stabilization of vortex solitons in some parts of their existence domains. Recently, it was elaborated a new method for the problem of stabilization of transversely localized vortex beams in Kerr media with self-focusing nonlinearity by the use of nonlinear absorption (multifoton absorption) [59]. The linear stability analysis and direct numerical simulations reveal regions of stability of single-vorticity nonlinear Bessel vortex beams against azimuthal breakup and collapse (such beams have the vorticity number $S = 1$); also, multiple-vorticity Bessel vortex beams have their stability domains too (such beams are characterized by the vorticity numbers $S > 1$), see Ref. [59]. Kartashov *et al.* [60] have recently investigated the existence and stability domains of rotating vortex clusters in optical media with inhomogeneous defocusing cubic nonlinearity growing toward the periphery. Such media can support a variety of stable vortex clusters nested in a common localized envelope. The rotation makes such vortex structures strongly asymmetric. Moreover, the rotation can stabilize some soliton families, such as vortex quadrupoles, see Ref. [60]. Special management techniques can also be used to stabilize 2D fundamental solitons; see Malomed *et al.* [36] for a more complete discussion of these main physical mechanisms to obtain robust multidimensional optical solitons.

As said above, saturation of the optical nonlinearities and the use of competing nonlinearities (e.g., self-focusing cubic nonlinearity in combination with self-defocusing quintic nonlinearity) may prevent the catastrofic beam self-focusing (the collapse of the optical beam) in two dimensions. We only mention here two recent relevant experimental results on robust trapping of fundamental [61] and vortex (2+1)-dimensional solitons in carbon disulfide [62]. Stable propagation of fundamental spatial solitons has been observed, which was supported by competing cubic and quintic nonlinearities [61]. Moreover, robust self-trapping of 2D vortex beams with topological charge $m = 1$ in carbon disulfide has been observed in a very recent work [62]. Such optical vortices remain self-trapped over five Rayleigh lengths and are excited using helical light beams at λ=532 nm and optical intensities ranging from 8 to 10 GW/cm^2. The numerical simulations reported in Ref. [62] were based on the (2+1)-dimensional nonlinear Schrödinger equation with three-photon absorption and nonpolynomial saturation of the refractive nonlinearity and demonstrated close agreement with the experimental data. Bouchard *et al.* [63] have investigated the polarization shaping of two different classes of light beams in order to control their nonlinear propagation dynamics. Stable propagation of space-varying polarized light beams in saturable self-focusing nonlinear media has been explored both experimentally and numerically and the results suggest that the spatial structure of the polarization plays a key role in preventing fragmentation of the optical beams, see Ref. [63].

2.1. LINEAR AND NONLINEAR LIGHT BULLETS: RECENT RESULTS

Next we briefly overview recent studies of a large variety of light bullets in both linear and nonlinear settings. Numerous experimental studies of nondiffracting optical wavepackets, e.g., Airy beams, Bessel beams, Airy-Bessel beams, etc., have been reported during the past few years; see, for example, Refs. [64–68]. It is also worth mentioning that Grillo *et al.* [69] generated nondiffracting electron Bessel beams in a middle energy range of transmission electron microscope by using a holographyc approach. Thus such electron Bessel beams were generated by diffracting electrons from a nanoscale phse hologram [69]. In a recent work, Amarande [70] has generated nondiffracting Bessel beams and has proposed a description of optical Bessel beams in the propagation range, that is, in the near-field, by using the so called self-convergent beam width technique [71], a powerful method that has been used before to characterize propagation of optical beams diffracted by hard-edge apertures.

Three-dimensional localized wavepackets of different types in free space have been recently investigated. Zhong *et al.* [72] studied 3D localized Airy-Laguerre-Gaussian wavepackets as exact solutions of the 3D potential-free Schrödinger equation, by using the method of separation of variables. These waveforms do not accelerate and are very robust during propagation over several Rayleigh lengths. Also, the same group [73] analytically constructed self-decelerating Airy-Bessel linear light bullet solutions of the 3D potential-free Schrödinger equation. Exact solutions with zero, integer, and half-integer topological charges (vorticities) were constructed. Such self-decelerating light bullets appear as disk-shaped fundamental rings, vortex rings, azimuthally-modulated rings, and either symmetric or asymmetric necklace-shaped patterns [73]. On the same line of research Yang *et al.* [74] studied the families of exact localized solutions of potential-free Schrödinger equation in spherical coordinates. The generic isointensity shapes of these 3D localized linear wavepackets were found to pertain to two main categories: (i) the combined disk-shaped and ring-shaped patterns and (ii) the necklace-ring-shaped patterns; see Ref. [74]. In another recent work [75] Airy-Tricomi-Gaussian compressed light bullets have been introduced as exact solutions of a 3D Schrödinger equation without external potential in cylindrical coordinates by using the method of separation of variables. During propagation over several Rayleigh lengths, such linear light bullets retain their waveform, showing a remarkable robustness [75]. Peng *et al.* [76] studied in detail different types of evolutions of self-accelerating Airy-Ince-Gaussian (AiIG) and Airy-Helical-Ince-Gaussian (AiHIG) light bullets in free space. The (3+1)-dimensional linear spatiotemporal evolution equation was solved analytically and the evolutions of different species of self-accelerating AiIG and AiHIG light bullets were investigated [76]. Also, we mention here the work of Zhang *et al.* [77] on controlable acceleration and deceleration of 1D and 2D obliquely incident Airy beams via variation of initial velocity. The problem of propagation of Airy beams with oblique incidence was addressed theoretically and numerically using classical Newtonian dynamics, see Ref. [77]. Bongiovanni *et al.* [78] reported extensive numerical simulations on a novel approach to generate nondiffractive and nondispersive Airy^3 light bullets with enhanced spatiotemporal energy confinement. It is expected that the results obtained by Bongiovanni *et al.* [78] may have direct applications in innovative optical techniques for imaging, tomography, and spatiotemporally resolved spectroscopy.

Spatiotemporal accessible solitons in *fractional dimensions* were also put forward by Zhong *et al.* [79]. These multidimensional solitons form in nonlocal nonlinear media of fractional dimension and are described by three "quantum numbers": the radial, orbital, and azimuthal ones. Such spatiotemporal solitons can be exactly written in terms of special functions, namely the Gegenbauer polynomials, the associated Laguerre polynomials, and the associated Legendre functions; see Ref. [79]. In a recent work by Baronio *et al.* [80], it was analytically predicted the existence of a novel family of spatiotemporal dark-lump solitons of the (2+1)-dimensional nonlinear Schrödinger equation. These exact solitary wave solutions describe nondiffractive and nondispersive spatiotemporal localized wavepackets propagating in optical Kerr media. Extensive numerical simulations confirmed the existence, stability, and peculiar elastic and anomalous interactions of such spatiotemporal dark-lump solitons [80]. Sazonov [81] has recently reported a detailed analytical study of the generation of spectral harmonics during the process of formation of spatiotemporal optical solitons ("light bullets") in nonlinear media.

The unique dynamics of nonfundamental (dipole and vortices) multidimensional localized structures in a series of physical settings have been also investigated both theoretically and experimentaly in recent works [82–84]. The dynamics of dipoles and vortices and various complexes of dipoles and/or vortices in nonlinearly coupled three-dimensional field oscillators trapped in a harmonic-oscillator potential was investigated in Ref. [82] through direct numerical simulations of the corresponding field equations. In that work, Driben *et al.* [82] studied in detail the relevant physical situation of repulsive cubic nonlinearity, which can be implemented in binary Bose-Einstein condensates. Zhu *et al.* [83] introduced Hermite-Gaussian vortex solitons of a (3+1)-dimensional partially nonlocal nonlinear Schrödinger equation with variable coefficients and investigated the dynamics of the corresponding spatiotemporal Hermite-Gaussian vortex solitons for different values of the topological charge (vorticity number) m and the integer parameter n that characterizes the corresponding Hermite-Gaussian function. In a seminal recent work [84] it has been reported the first experimental evidence of *spatiotemporal optical vortices*. These findings have been supported by extensive numerical simulations of the (3+1)-dimensional nonlinear propagation equation. This new kind of spatiotemporal vortical structure is an optical vortex with phase and energy circulation in a *spatiotemporal plane* (that is, not only in the transverse spatial dimensions) and forms a ring (or toroid) around the waist of an intense self-focusing laser pulse [84]. These genuine spatiotemporal optical vortices might be useful in diverse research areas such as all-optical transmission of information and microscopy.

In what follows we will briefly overview some innovative mechanisms, which have been recently introduced in the literature, to stabilize spatiotemporal solitons in cubic (Kerr type) nonlinear media. Kartashov *et al.* [85] studied the problem of stabilization of (2+1)-dimensional spatiotemporal solitons in Kerr media with focusing nonlinearity by using the dispersive coupling, i.e., the stabilization mechanism was based on the dispersion of linear coupling between the two field components forming the soliton states. Thus the coupling dispersion helps to generate 2D soliton families in self-focusing cubic media that are stable against the critical collapse [85]. Long-lived spatiotemporal optical solitons ("light bullets") in two-dimensional fiber arrays were studied in detail by Aceves *et al.* [86]. A detailed theoretical analysis on the existence and stability of these discrete-continuous light bullets was reported in Ref. [86] using a generic model that applies to multicore optical fibers or waveguide arrays. The direct numerical simulations of the localized light-bullet solutions were compared with the analytical derivation of the asymptotic solutions of the governing evolution equations, see Ref. [86]. Driben *et al.* [87] investigated to problem of creation of robust 2D and 3D vortices by torque in media with inhomogeneous defocusing nonlinearities. It is worth mentioning that the media with inhomogeneous defocusing nonlinearities support robust 2D and 3D localized structures that either do not exist or are unstable in other nonlinear systems, see Ref. [87]. In another relevant work by Driben *et al.* [88] a variety of multipoles and vortex multiplets in 2D and 3D nonlinear media with inhomogeneous defocusing nonlinearities were predicted. These remarkably robust nonlinear modes are self-trapped in a medium with a repulsive nonlinearity whose local strength grows from center to periphery. Such robust localized structures may be realized in both optical and matter-wave settings, in media with controllable cubic nonlinearities [88]. Adhikari [89] investigated stable spatial (2D) and spatiotemporal (3D) optical solitons that can exist in the cores of optical vortices. It was shown in Ref. [89] by extensive numerical simulations that robust, stable, mobile, either spatial or spatiotemporal optical solitons can form in the cores of optical vortices, while all nonlinearities are of the cubic (Kerr) type. In a subsequent work, by the same author [90], elastic collision and molecule formation of spatiotemporal optical solitons in cubic-quintic nonlinear media have been investigated. Both the statics and dynamics of stable, mobile 3D light bullets in a cubic-quintic nonlinear medium with a focusing cubic nonlinearity above a critical value and any defocusing quintic nonlinearity were studied by analytical methods (variational analysis) and extensive numerical simulations [90]. Three main collision scenarios were put forward: (a) at large velocities the collision is elastic, (b) at small velocities two optical bullets coalesce to form a optical bullet molecule, and (c) at a small range of intermediate velocities the localized light bullets could form a single entity, which expands indefinitely [90].

We also briefly overview a few recent works in the area of *spatiotemporal dissipative structures* in different optical settings. It is well known that these unique physical objects were theoretically predicted long time ago in two pioneering works by Turing [91] and by Prigogine and Lefever [92]. Such dissipative structures have been observed in nonlinear systems far from equilibrium in diverse settings in biological, chemical, and optical systems; see, for example, the the overview paper by Rozanov [93] and a recent review by Tlidi *et al.* [21]. The prototype dynamical model for describing the peculiar properties of either spatial or spatiotemporal dissipative structures is the cubic-quintic complex Ginzburg-Landau (CQCGL) equation in either two or three dimensions. Recently, Cisternas *et al.* [94] demonstrated numerically the occurence of anomalous diffusion of dissipative solitons in the CQCGL equation in two spatial dimensions. Panajotov *et al.* [95] analyzed analytically and numerically the impact of delayed optical feedback on the spatiotemporal dynamics of another prototype model in this research field, the so-called Lugiato-Lefever equation, describing localized structures of light (often called *cavity solitons*) in nonlinear laser systems.

Rosanov and Fedorov [96] studied the topology of energy fluxes in 2D dissipative vortex solitons and their complexes in both wide-aperture lasers and exciton-polariton lasers. The same group [97] predicted and studied a new type of dissipative topological 3D optical solitons, namely asymmetric rotating and precessing localized structures in homogeneous active or pasive media with nonlinear saturable amplification and absorption. These dissipative localized optical structures ("dissipative precessons") can be formed, for example, in lasers with saturable absorption and long ring cavities; see Veretenov *et al.* [97]. The formation of 3D spatiotemporal dissipative solitons (dissipative "laser bullets") in dielectric media containing two-level quantum dots have been investigated by Gubin *et al.* [98]. The existence and properties of spatiotemporal optical similaritons in a dual-core waveguide with external source have been explored both analytically and numerically by Raju [99].

Next we briefly overview some relevant recent experimental results reported in the area of light bullets and spatiotemporal dynamics of optical solitons in different settings [100–103]. Majus *et al.* [100] reported a detailed experimental investigation of the nature of 3D spatiotemporal light bullets generated from the self-focusing of intense femtosecond pulses in bulk dielectric Kerr (cubic) nonlinear media in the anomalous group-velocity-dispersion regime. The self-focusing dynamics of 100 fs pulses (λ=1.8 μm) in sapphire was experimentally studied in the full 4D space by means of a 3D imaging method [100]. It was clearly demonstrated that the 3D light bullets consist of a sharply localized high-intensity core containing about 25% of the total energy and a delocalized ring-shaped low-intensity periphery, comprising a Bessel-like beam, see Ref. [100] for more details of this extremely relevant experimental work. The unique spatiotemporal dynamics of multimode optical solitons in graded-index fibers was studied both numerically and experimentally by Wright *et al.* [101]. It was explored experimentally and numerically the key properties and the complex spatiotemporal behavior of such multimode optical solitons. The formation, fission, and Raman dynamics of these multimode solitons have been studied in detail, see Ref. [101]. Shalaby and Hauri [102] recently demonstrated a low frequency 3D terahertz bullet with extreme brightness. In that work it was experimentally presented a λ^3 bullet in the terahertz range using low-frequency ultrabroadband terahertz pulses. The intensity of this terahertz bullet was of the order of 110 PW/m^2 and the electric and magnetic field strengths were 8.3 GV/m and 27.7 T, respectively [102]. Chekalin *et al.* [103] investigated both experimentally and numerically the complex scenarios of formation of light bullets from femtosecond filaments in the anomalous dispersion regime in two distinct media: fused silica and humid air. The light bullets identified in the work by Chekalin *et al.* [103] were short-lived objects formed in a femtosecond filament with a very high spatiotemporal optical field localization. Moreover, it was put forward that the generation of these light bullets is accompanied by the ejection of a supercontinuum light in the visible spectrum and an isolated anti-Stokes wing is formed in the visible area of the supercontinuum, see Ref. [103] for more details of these relevant studies. In a recent work by Gustave *et al.* [104] it has been reported the experimental observation of mode-locked spatial laser solitons in a vertical-cavity surface-emitting laser with frequency-selective feedback from an external cavity. The experimental results obtained Gustave *et al.* [104] may pave the way to the observation of trully mutually independent cavity light bullets.

2.2. 2D AND 3D SOLITONS IN CARBON NANOTUBES

The peculiar dynamics of ultrashort optical pulses and the formation of both 2D and 3D solitons in carbon nanotubes (CNTs) have been investigated in detail during the past few years [105–118]. Leblond and Mihalache [105] studied the formation of ultrashort spatiotemporal optical solitons in arrays of CNTs, using a short-wave approximation to derive a generic 2D sine-Gordon equation, describing evolution of ultrashort light bullets in such nanomaterials. Diffractionless and dispersionless robust propagation over large distances (with respect to the corresponding optical wavelength) of few-cycle (2+1)-dimensional spatiotemporal solitons in the form of optical breathers has been numerically investigated [105]. Zhukov *et al.* [106] studied 3D electromagnetic breathers in CNTs with the field inhomogeneity along their axes. It was found that the propagation of such ultrashort electromagnetic pulses induces a redistribution of the electron density in the CNT. Konobeeva and Belonenko [107] analyzed analytically and numerically the wave equation for the short electromagnetic pulse propagating in *chiral* CNTs and studied the dynamics of such pulses. It was derived a phenomenological propagation equation similar to the sine-Gordon equation [107] derived using the multiscale expansion method [105]. Dynamics of ultrashort light pulses in CNTs placed in media with spatially modulated refractive indices (Bragg media) have been also studied by Belonenko and Nevzorova [108]. The influence of multi-level impurities on the dynamics of ultrashort electromagnetic pulses in arrays of CNTs has been investigated analytically and numerically by Zhukov *et al.* [109]. The effects of the hoping integrals and band gap of deep impurities on the ultrashort pulse tail decay have been also studied, see Ref. [109] for more details. Konobeeva and Belonenko [110] studied the interesting problem of the dynamics of ultrashort electromagnetic pulses in chiral carbon-nanotube waveguides in the presence of an external *alternating* electric field. The same authors [111] investigated the peculiar behaviour of ultrashort electromagnetic pulses in chiral CNTs in the presence of an external *static* electric field. It was put forward an evolution equation for the ultrashort pulses that is similar to the sine-Gordon equation [111]. Zhukov *et al.* [112] investigated the problem of interaction of 2D electromagnetic breathers with an electron inhomogeneity in arrays of CNTs. The possibility of stable propagation of electromagnetic pulses in heterogeneous arrays of CNTs has been put forward and it was established that, depending on its speed of propagation, the ultrashort laser pulse can pass through the area of increased electron concentration or be reflected therefrom.

Recently, a detailed study of the interaction of a 2D electromagnetic pulse with electron inhomogeneities in arrays of CNTs in the presence of field inhomogeneity has been reported by Zhukov *et al.* [113]. The propagation dynamics of 2D extremely short electromagnetic pulses in Bragg media containing immersed arrays of CNTs has been reported by Zhukov *et al.* [114]. The possibility of stable propagation of such ultrashort light bullets in this setting has been revealed, see Ref. [114]. Other interesting theoretical studies in this area deal with the problem of formation of discrete solitons in a Bragg envirnonment containing CNTs [115], the study of 2D light bullet formation in Bragg media with harmonically modulated refractive indices, which contain arrays of CNTs [116], and the special opto-acoustic effects in arrays of CNTs [117]. A comprehensive study of collision of 3D bipolar optical solitons in arrays of CNTs has been reported in a recent work by Zhukov *et al.* [118]. The extensive numerical simulations reveal the possibility of stable post-collision propagation of extremely short pulses over distances much greater than their characteristic sizes [118].

2.3. ULTRASHORT (FEW-CYCLE) LOCALIZED OPTICAL STRUCTURES

During the past two decades a lot of experimental and theoretical studies of the physics and applications of short optical pulses with widths ranging from tens of nanoseconds to only a few optical cycles (that is, several femtoseconds) have been reported in the literature; for a few review papers in this fast growing research area see Refs. [119–129]. The high power laser system and the planned experiments to be performed at the Extreme Light Infrastructure-Nuclear Physics (ELI-NP) facility to be built in Magurele, Romania, have been reviewed in a recent paper by Ursescu *et al.* [128]. This unique experimental facility will deliver ultrashort 10 PW laser pulses with intensities reaching 10^{23} W/cm^2. The planned high field physics and quantum electrodinamics experiments to be achieved at the ELI-NP facility have been overviewed in a comprehensive paper by Turcu *et al.* [129].

One of the main questions in this area is how the propagation of ultrashort (few-cycle) pulses can be adequately modeled theoretically. Though a lot of ultrashort propagation theoretical models centered on the slowly-varying-envelope approximation (SVEA), which is a perturbative approach that is valid only when the pulse duration is much longer than the optical cycle, other realistic non-SVEA models have been put forward, see Refs. [130–141]. These non-SVEA propagation models are mainly based on modified Korteweg-de Vries equation, sine-Gordon equation, short-pulse equation, and modified Korteweg-de Vries-sine-Gordon equation; see the comprehensive review paper by Leblond and Mihalache [17].

The series of theoretical papers published during the past year in the area of ultrashort pulses [142–151] investigated several physical problems ranging from the important issue of interplay of diffraction and nonlinear effects in the propagation of very short light pulses [142] to the potential use of focused ultrashort pulses for far-field light nanoscopy [151]. We also mention here the recently published relevant papers by Arkhipov *et al.* [152], which theoretically investigated a new possibility of unipolar pulse generation in Raman-active media excited by a series of few-cycle optical pulses and by Pakhomov *et al.* [153], which defined a general framework to produce unipolar half-cycle pulses of controllable form in resonant media with nonlinear field coupling.

The problem of the coupling of two optical waveguides in the few-cycle regime has been investigated in a recent paper by Leblond and Terniche [154]. The theoretical and numerical analysis has been done in the framework of a generalized Kadomtsev-Petviashvili model that can be reduced to a system of two coupled modified Korteweg-de Vries equations describing the nonlinear propagation in the coupled waveguides. In a subsequent work by Terniche *et al.* [155], the problem of few-cycle soliton propagation in two parallel optical waveguides, in the presence of linear nondispersive coupling, has been addressed. It was shown that the few-cycle vector solitons in such coupled waveguides are *optical breathers*. Moreover, it was revealed spatial oscillations of amplitude and energy of these solitons, in addition to the usual breathing due to carrier-envelope velocity mismatch, see Ref. [155]. Leblond *et al.* [53] reported comprehensive analytical and numerical studies of few-cycle solitons in supercontinuum generation dynamics by using generic propagation models that do not rely on the SVEA. Such non-SVEA models numerically predict supercontinuum generation over several octaves. It is worth noting that a definite advantage of the non-SVEA models is that they do not require a large number of terms and coefficients in the governing propagation equations, see Ref. [53].

Next we briefly overview a few recent relevant experimental results in the active research area of ultrashort (femtosecond) light-bullet formation and associated spatiotemporal phenomena. Panagiotopoulos *et al.* [156] experimentally investigated the super high power femtosecond light bullet and its formation and robust propagation in the atmosphere. It was shown that such mid-infrared wavelength light bullets are resistant to uncontrolled multiple filamentation and progate over long distances [156].

In a recent work [157], the filamentation process and light bullet formation dynamics in dielectric media with weak, moderate, and strong anomalous group velocity dispersion (GVD) have been experimentally investigated. In a series of measurements, the filamentation dynamics of intense ultrashort laser pulses in the spacetime domain propagating in sapphire and fused silica in the wavelength range from 1.45 μm to 2.25 μm has been investigated in detail in the above mentioned three GVD regimes [157]. At the wavelength $\lambda = 1.45$ μm, in the regime of weak anomalous GVD, it was observed a pulse splitting into two sub-pulses producing a pair of ultrashort light bullets, while in the regimes of moderate (at $\lambda = 1.80$ μm) and strong (at $\lambda = 2.25$ μm) anomalous GVD, a much different transient dynamics was observed. In these latter GVD regimes the formation of a single self-compressed quasistationary ultrashort light bullet with a peculiar spatiotemporal shape was observed. This light bullet is comprised of an extended ring-shaped periphery and a localized intense core that carries most of the self-compressed light pulse energy, see Ref. [157] for more details of this very relevant experimental work. Recently, Chekalin *et al.* [158] studied the regular oscillations of the field-peak amplitude of a near-single-cycle light bullet during propagation in a mid-infrared filament. Both experimental and numerical results of the temporal evolution of a light bullet formed in isotropic LiF by mid-infrared femtosecond pulses with wavelenghts in the range from $\lambda = 2.60$ μm to $\lambda = 3.35$ μm were reported in Ref. [158]. It is worth noting that the pulse power was slightly larger than the critical power for self-focusing and that the anomalous GVD was necessary for the formation of the light bullet in the mid-infrared filament [158].

3. NONLINEAR LOCALIZED STRUCTURES IN PARITY-TIME-SYMMETRIC SYSTEMS

Two comprehensive reviews on the unique nonlinear features of a variety of parity-time-symmetric physical systems have been recently published by Konotop *et al.* [159] and Suchkov *et al.* [160]. This fast-growing research field emerged about twenty years ago, when Bender and Boettcher [161] proposed the space-time reflection symmetry, the so-called *parity-time (PT) symmetry* in quantum mechanics, which can be responsible for purely real spectra of non-Hermitian operators. A simple example of a PT-symmetric complex-valued potential is $H_0 = p^2 + x^2 + ix$, whose real positive eigenvalues are $E_n = 2n + 5/4$. The spectrum of the Hamiltonian $H = p^2 - (ix)^N$, where N is a real number, was studied in Ref. [161] using detailed numerical calculations and a semiclassical analysis based on the WKB approximation. When $N = 2$, the Hamiltonian H corresponds to that of the harmonic oscillator with the eigenvalues $E_n = 2n + 1$. However, when $N \geq 2$, the spectrum of the Hamiltonian H is infinite, discrete, and entirely real and positive. A sort of "phase transition" occurs at $N = 2$, because when $1 < N < 2$, there are only a finite number of real positive eigenvalues and an infinite number of complex conjugate pairs of eigenvalues of the above written Hamiltonian H; when $N \leq 1$, there are no real eigenvalues, see Ref. [161]. Recently, Bender [162] published a short viewpoint on the seminal paper by Dorey *et al.* [163], stressing the fact that it was that comprehensive work published in 2001 [163] that gave a complete and rigorous proof of the seminal results reported by Bender and Boettcher in 1998 [161].

The PT-symmetry can be implemented in optical settings, e.g. in planar slab waveguides [164] and in coupled optical structures [165]. This theoretical concept has been demonstrated experimentally in 2009 [166] and in 2010 [167], in two pioneering works. In most situations, the PT-symmetric optical system is built in the form of a periodic optical structure (an optical lattice described by a PT-symmetric complex-valued periodic potential). Several unique physical phenomena emerge in such PT-symmetric optical systems: nonreciprocal diffraction patters, optical power oscillations, double refraction etc. Also, we mention here the relevant recent papers by Yan *et al.* [168–171] on the unique features of optical solitons that form in different types of PT-symmetric and non-PT-symmetric complex-valued external potentials. The theoretical studies reported during the past few years on dynamics of spatial solitons in PT-symmetric optical lattices have been recently overviewed by He and Malomed [172] and by He *et al.* [173].

The problem of stability of optical solitons (either spatial or temporal ones) in the PT-symmetric nonlinear coupler with gain in one waveguide and loss in the other one have been studied by Alexeeva *et al.* [174]. PT-symmetric couplers with competing cubic self-focusing and quintic self-defocusing nonlinearities have been investigated by Burlak *et al.* [175]. Note that the physical setting explored in detail in Ref. [175] may be realized in optical waveguides, in the spatial and temporal domains alike. A PT-symmetric dual-core system with the sine-Gordon nonlinearity and derivative coupling has been studied by Cuevas-Maraver *et al.* [176] as a relevant extension of the class of nonlinear PT-symmetric models with the proper balance of gain and loss.

Barashenkov and Gianfreda [177] have investigated an exactly solvable PT-symmetric dimer from a Hamiltonian system of nonlinear oscillators with gain and loss. It was put forward that the nonlinearity softens the PT-symmetry breaking transition in the nonlinearly-coupled dimer. Thus stable periodic and quasiperiodic states with large enough amplitudes can persist for arbitrarily large values of the gain-loss coefficient, see Ref. [177]. In a subsequent paper, Barashenkov *et al.* [178] studied in detail the problem of integrability and PT-symmetry restoration in nonlinear Schrödinger dimers with gain an loss.

Next we overview a few recent relevant theoretical works on 2D solitons supported by complex-valued PT-symmetric optical potentials. Two-dimensional solitons supported by mixed linear-nonlinear complex-valued optical lattices were investigated by Ren *et al.* [179]. Also, the peculiar physical properties of 2D linear modes and solitons that form in PT-symmetric Bessel-type complex-valued potentials have been put forward by Chen and Hu [180]. The asymmetric linear modes and solitons in 2D PT-symmetric optical potentials have been also studied in a recent work by the same group [181]. Wang *et al.* [182] studied in detail the dynamics of 2D solitons in PT-symmetric optical lattices with nonlocal defocusing nonlinearity. It was revealed that the PT-symmetry can lead to fission and shifting of solitons; these peculiar phenomena are affected by the degree of nonlocality of the corresponding defocusing nonlinearity, see Ref. [182]. Liu *et al.* [183] investigated both analytically and numerically the existence, stability, and robustness (to initial random perturbations) of vector soliton solutions in PT-symmetric coupled waveguides. In two subsequent works the same group investigated the problem of nonlinear parity-time-symmetry breaking in optical waveguides with complex-valued Gaussian-type potentials [184] and the existence and stability of both symmetric and asymmetric solitons that form in self-focusing optical waveguides with PT-symmetric double-hump Scarff-II potentials [185].

Three-dimensional solitons supported by diverse PT-symmetric complex-valued potentials have been also investigated theoretically in very recent works [186–188]. Xu *et al.* [186] constructed explicit exact solutions describing spatiotemporal solitons supported by PT-symmetric external potentials in the presence of competing nonlinearities. Localized structures in the (3+1)-dimensional nonlinear Schrödinger equation with inhomogeneous diffraction and dispersion coefficients and power-law nonlinearity in the presence of PT-symmetric external potentials have been also studied by Wang *et al.* [187]. Kartashov *et al.* [188] investigated in detail the peculiar properties of 3D topological solitons in 2D PT-symmetric optical lattices with focusing Kerr nonlinearity. It was uncovered that the domain of stability for fundamental solitons can extend nearly up to the PT-symmetry breaking point, where the linear spectrum becomes complex-valued and that 2D PT-symmetric lattices can support continuous families of stable 3D fundamental and vortex solitons even in self-focusing Kerr media. To the best of our knowledge, the results reported by Kartashov *et al.* [188] constitute the first example of continuous families of stable 3D propagating solitons supported by PT-symmetric complex-valued external potentials.

During the past few years, the concept of PT-symmetry has been demonstrated experimentally in diverse areas of optical physics. The first experiments in optical settings [166, 167] used a pair of coupled waveguides (a waveguide with loss and the other one with an equivalent gain). Recent experimental works deal with PT-symmetric microwave cavities, PT-symmetric lasers, whispering-gallery microcavities, optical solitons in PT-symmetric lattices, observation of Bloch oscillations in complex PT-symmetric photonic lattices etc., see, for example, the recent consistent review paper [159]. Wimmer *et al.* [189] reported the first observation of optical solitons in PT-symmetric lattices. The same group investigated the Bloch oscillations in complex-valued PT-symmetric photonic lattices [190]. Zhang *et al.* [191] recently reported the observation of PT-symmetry in optical induced atomic lattices. PT-symmetric optical lattices with periodical gain and loss profiles in a coherently prepared four-level N-type atomic system have been experimentally demonstrated in a paper by Zhang *et al.* [191]. This recent seminal work paves the way for exploiting and better understanding the unique physical properties of non-Hermitian physical systems in atomic settings.

4. ROGUE WAVES IN MULTIDIMENSIONAL PHYSICAL SETTINGS

In this Section we overview some of the recent theoretical and experimental advances in the area of extreme wave events and patterns that are known as rogue waves ("freak waves"). These special wave structures emerge in a broad class of physical systems ranging from optics to Bose-Einstein condensates; see the comprehensive review papers [192–194]. It is worth mentioning that a lot of theoretical and experimental studies of such extreme events that were reported during the past few years have mainly focused on optical and hydrodynamical environments; see, for example, Refs. [195–216].

It is well known that the many theoretical works published in the past few years in the area of rogue waves were strongly influenced by the seminal papers by Peregrine [217] and Akhmediev *et al.* [218] in which the exact rational solutions of the complete integrable nonlinear Schrödinger (NLS) equation were investigated. In addition to the prototype NLS equation, there are many other integrable evolution equations admitting Peregrine-type soliton solutions, Akhmediev-type breather solutions, and Kuznetsov-Ma breather solutions [219, 220]: the Hirota equation, the modified Korteweg-de Vries equation, the Sasa-Satsuma equation, the Manakov system, and the derivative NLS equation, to name only a few such completely integrable partial differential equations.

In what follows we will briefly overview some recent theoretical works that investigated a large variety of rogue waves in two- and three-dimensional settings. Various exact rational solutions of the 2D Maccari's system have been recently investigated by Yuan *et al.* [201]. In that work, a general form of rational solutions for the 2D Maccary's system of three coupled partial differential equations was given in terms of Gram determinants using the bilinear method. Liu *et al.* [202] derived an explicit form of parallel line rogue waves of the third-type Davey-Stewartson equation using the bilinear transformation method. It was shown that the simplest (fundamental) rogue wave is a line-type rogue wave that arise from a constant background, while the higher-order rogue waves are parallel line rogue waves that describe the interaction between several fundamental rogue waves and which consist of several wavefronts resembling parallel lines, see Ref. [202] for a detailed study of these issues. Chen *et al.* [203] presented in a compact form families of exact explicit nonsingular rational soliton (lump) solutions of any order to the Kadomtsev-Petviashvili I partial differential equation, which is an integrable (2+1)-dimensional extension of the classic Korteweg-de Vries equation. It is worth mentioning that the exact explicit lump solutions of any order to the Kadomtsev-Petviashvili I equation resemble rogue waves in their characteristic profiles but with variable peak amplitudes [203]. Chen *et al.* [199] introduced the concept of *rogue-wave bullets*, that is, nonlinear wavepackets localized in two dimensions with characteristic rogue wave profiles, propagating in the third dimension with significant stability. It is worth mentioning that this unique feature makes these waves quite similar to light bullets, with the additional characteristics that they propagate on a finite background, see Ref. [199]. Nonautonomous rogue wave solutions and numerical simulations for a 3D NLS equation have been reported recently by Yu [221]. Multi-rogue wave exact solutions were given employing the generalized Darboux transformation method [221].

In a recent work, Liu *et al.* [222] proposed a scheme in a cold atomic gas to demonstrate the existence of optical Peregrine rogue waves and Akhmediev and Kuznetsov-Ma breathers with very low light power and to realize their active control via the phenomenon of electromagnetically induced transparency. Also, it was put forward in Ref. [222] that in (2+1)-dimensions the Akhmediev and Kuznetsov-Ma breathers can be actively manipulating by using external magnetic fields. A detailed numerical study of spatiotemporal extreme events occuring in the transverse section of the optical field emitted by a broad-area semiconductor laser with a saturable absorber has been reported by Rimoldi *et al.* [223]. The existence of rogue waves in this setting has been shown by extensive numerical simulations and the typical temporal and spatial sizes of such extreme events have been determined [223]. Gibson *et al.* [224] put forward a spatiotemporal mechanism to produce 2D optical rogue waves via vortex turbulence, that is, in the presence of turbulent states with creation, interaction, and annihilation of optical vortices. Rogue waves in ultracold bosonic seas have been recently explored both analytically and numerically by Charalampidis *et al.* [225] by using both the 1D nonpolynomial nonlinear Schrödinger equation and the 3D Gross-Pitaevskii equation.

Roger *et al.* [226] reported experimental results that showed extreme events in the statistics of resonant radiation emitted from 3D light bullets. Extensive numerical simulations reproduced the experimental findings, which were found to be intrinsically linked to the simultaneous occurrence of both temporal and spatial self-focusing dynamics, see Ref. [226]. Experimental evidence of rogue-wave formation in normal-dispersion ytterbium fiber lasers was reported in a paper by Liu, Zhang, and Wise [227]. In that work, it was observed rogue statistics in the noise bursts from a dissipative soliton laser; a large fraction of extremely high-energy pulses was observed. The pulse amplitude can be six times the significant wave height, corresponding to pulse energies as high as 50 nJ [227].

5. MATTER-WAVE LOCALIZED STRUCTURES

In this Section we briefly overview recent theoretical studies on peculiar physical properties of different types of multidimensional matter-wave fundamental solitons and vortex structures in a series of physical settings of atomic BECs; see also a viewpoint on multidimensional soliton structures and their legacy in contemporary atomic, molecular, and optical physics [36].

The results surveyed in this Section stem from intrinsically two- and three-dimensional numerical and analytical analyses, but reduced low-dimensional equations (see, e.g., the polynomial and non-polynomial equations in Refs. [228–235]) can also provide significant insights into the emergence and subsequent dynamics of three-dimensional structures. We notice, however, that these equations of reduced computational load do not yet describe dipolar or spin-orbit interactions, where one usually relies on fully three-dimensional simulations. Moreover, let us note that the surge of papers dedicated to high-dimensional localized structures in matter-wave media can be directly related to the parallelization of numerical solvers of the Gross-Pitaevskii equation [41–48] and a series of landmark results have been reported during the recent years, such as the first creation of spin-orbit coupled BECs [236].

Modulational instabilities such as the ones reported experimentally in Refs.

[237, 238] are known to excite density waves that give a fragmented appearance to an initially localized condensate [239]. This phenomenon has been predicted theoretically for a large set of experimental (one- and two-dimensional) configurations, using both as one- [240, 241] and two-component BECs [242], considering contact [243, 244] and long-range interactions [245–247]. Similar results have been obtained for superfluid Fermi gases where it was shown that Faraday-type instabilities can turn unstable an initially localized state that is energetically stable [248].

In a recent paper [249] the problem of self-trapped BECs with spin-orbit coupling and the existence of metastable solitons in 3D free space without the ground state have been investigated by means of variational methods and systematic numerical simulations. It was shown that depending on the relative strength of the intra- and intercomponent attraction, the stable 3D solitons feature a *semivortex* or mixed-mode structure. These 3D localized structures were found to be stable against small perturbations, motion, and collisions, in spite of the fact that the local cubic self-attractive interaction gives rise to the supercritical collapse in three dimensions [249]. The very important issues of storage and retrieval of (3+1)-dimensional weak-light bullets and vortices in a coherent atomic gas via electromagnetically induced transparency have been studied by Chen *et al.* [250]. Stable, mobile, dipolar or nondipolar 3D matter-wave solitons in the vortex cores of uniform BECs have been investigated by Adhikari using numerical techniques [251]. Also, the 3D dynamics of the so-called *oscillons* of atomic BECs in dynamic traps has been analyzed by Rosanov [252].

Bisset *et al.* [253] investigated the dynamics of vortex lines, vortex rings, and Hopfions in 3D atomic BECs. The exotic state called Hopfion can be stabilized in parabolic traps, without the need for trap rotation or inhomogeneous interactions. These stationary states are robust against perturbations of the input state; see Ref. [253] for a detailed numerical study of these unique localized matter-wave structures. The same group [254] studied the bifurcation and the stability of both single and multiple vortex rings in 3D atomic BECs. It was shown numerically that the vortex rings are stable in isotropic traps and in the large-chemical-potential regime [254]. Bidasyuk *et al.* [255] have shown that Hopf solitons (Hopfions) can be stabilized in rotating atomic BECs. The Hopfions are matter-wave vortex complexes that are characterized by two distinct integer winding numbers (topological indices). Both energetic and dynamical stability of Hopfions have been put forward under experimentally feasible conditions; see Ref. [255] for details of these numerical simulations.

Yakimenko *et al.* [256] reviewed recent theoretical works on the generation and decay of persistent currents in toroidal atomic BECs. Also, some theoretical results concerning the hysteresis effects in 2D ring-shaped BECs were presented, which are very important for the ongoing experiments on the realization of atomtronic circuits [256]. In a paper by Sudharsan *et al.* [257] stable multiple vortex solitons in collisionally inhomogeneous attractive BECs have been investigated in detail by solving the corresponding Bogoliubov-de Gennes equations and running direct numerical simulations of the 2D Gross-Pitaevskii evolution equation for the single-atom wave function. It was found that vortex solitons with topological charges up to $S = 6$ are stable above a critical value of the chemical potential [257]; to the best of our knowledge, this work provides a first example of a physical setting giving rise to stable vortex solitons with winding numbers $S > 1$ in trapped self-attractive BECs, that is, stable higher-order 2D vortex solitons. Recently, Sakaguchi *et al.* [258] investigated vortex solitons in 2D spin-orbit coupled BECs by including the effects of the Rashba-Dresselhaus coupling and Zeeman splitting. Combined analytical and numerical methods were used to identify the existence domains and stability boundaries for the 2D matter-wave soliton families [258].

The unique collision dynamics of skyrmions in two-component BECs was numerically investigated by Kaneda and Saito [259] in the framework of the mean-field theory. It was revealed that when two skyrmions collide with each other, they first merge and then scatter into various states. For head-on collisions, skyrmions with winding numbers equal to one are scattered [259]. The $SO(2)$-induced breathing patterns in multicomponent 1D, 2D, and 3D BECs have been studied by Charalampidis *et al.* [260]. Vector solitons and a variety of vortex structures were constructed in this setting [260]. In a recent paper [261] 3D vortex structures in rotating dipolar BECs were investigated by using the mean-field approximation. The stable vortex lattice structures were analyzed by numerically solving the 3D time-dependent Gross-Pitaevskii equation in imaginary time. The stability of different vortex states was also examined by evolution in real time; see Ref. [261].

Driben *et al.* [262] theoretically investigated the novel dynamics of 3D non-coaxial matter-wave vortices that are trapped in parabolic potentials and interact via repulsive nonlinearities. The unique precession and nutation phenomena of these nonlinearly coupled matter-wave vortex solitons have been studied in detail via numerical simulations, see Ref. [262]. In the theoretical work by Wang *et al.* [263] the existence, stability, and dynamics of dark spherical shell solitons in 3D BECs have been studied both analytically and numerically. Such spherical shell solitons can be stable sufficiently close to the linear limit of the isotropic BECs, see Ref. [263] for more details. Recently, 2D symbiotic solitons and vortices in binary BECs have been analytically investigated by a variational technique; also, detailed numerical calculations have been performed [264]. The domains of existence and the stability of such symbiotic solitons and vortices in 2D two-component spinor atomic systems with repulsive intra-species and attractive cross-component interactions, in the presence of a 2D lattice harmonic potential have been studied in detail [264]. The influence of stochastic (thermal) noise on phase slips in toroidal BECs was recently studied by Snizhko *et al.* [265] and it was revealed that the added thermal noise lowers the corresponding phase slip threshold. The resonant excitation of sound modes from Josephson oscillations in BECs has been studied by Bidasyuk *et al.* [266] using the 3D time-dependent Gross-Pitaevskii equation. The obtained results were also supported by a simplified model based on the equivalent Josephson equations [266]. Stable self-trapped giant vortex annuli in microwave-coupled atomic BECs were recently investigated by Qin *et al.* [267] and conditions for experimental realization of such vortices with large values of the topological charge S were also discussed.

Zezyulin *et al.* [268] theoretically studied the problem of stationary throughflows in BECs with gain and loss. These stationary states with asymptotically nonvanishing fluxes were described by solutions of the PT-symmetric Gross-Pitaevskii equation with nonvanishing boundary conditions, see Ref. [268]. Stable necklace-like states and persistent flows in 2D toroidally trapped BECs in the presence of Rashba-type spin-orbit coupling have been recently investigated by White *et al.* [269]. It was shown in Ref. [269] that the number of "petals" composing the necklace-like state is highly tuneable and depends on the spin-orbit coupling strength, thus allowing a wide range of stable states to be created in this physical setting.

Wang and Kevrekidis [270] have revisited in a recent work [270] the problem of formation and stability of two-component dark-bright solitons in 3D atomic BECs and have explored in detail the dynamical evolution of these waveforms via direct numerical simulations. Also, the $SO(2)$ rotations of the considered soliton families have been investigated by Wang and Kevrekidis [270], leading to pairs of planar and spherical dark-bright states. Recently, Gautam and Adhikari [271] have studied the bright vortex solitons in a quasi-2D spin-orbit-coupled hyperfine spin-one three-component BEC using both a variational method and a direct numerical solution of a mean-field model. The ground state of these bright vortex solitons is radially symmetric for weak ferromagnetic and polar interactions. However, for a sufficiently strong ferromagnetic interaction, the emergence of asymmetric bright vortex solitons has been put forward, see Ref. [271].

In a recent paper by Danaila *et al.* [272], the experimental evidence for the existence, stability, and dynamics of dark-antidark solitary waves in two-component BECs confined in elongated dipole traps has been presented. Theoretical and numerical investigations of vector dark-antidark states in the 2D setting have been also reported in Ref. [272] and the concept has been generalized to vortex-antidark solitary waves and ring-antidark-ring solitary waves.

6. CONCLUSION

In this paper we have focused on the current state-of-the-art of the continuously growing research area of multidimensional localized structures in optical and matter-wave media, in a large variety of physical settings. We hope that our article fills a gap in the topical reviews dedicated to the results published over the past couple of years. Here we have briefly described several relevant experimental and theoretical studies that have been recently published in these very broad research areas. Thus we have overviewed important theoretical and experimental results obtained in the field of linear and nonlinear light bullets, the formation and subsequent stability of two- and three-dimensional ultrashort solitons in solid state materials such as carbon nanotubes, recent studies of ultrashort (few-cycle) localized optical structures, the formation and the unique properties of nonlinear localized structures in parity-time-symmetric systems, the problem of rogue (freak) waves in multidimensional physical settings, and recent studies of formation and stability of a variety of localized structures in Bose-Einstein condensates.

The intense theoretical efforts in the above mentioned fields and the impressive results obtained over the past few years have inspired and triggered recent experimental investigations in these research areas, and a few significant experimental results have been already reported. We conclude with the hope that this overview on recent exciting theoretical and experimental developments in the field of multidimensional localized structures in optical and matter-wave media will inspire further studies on these subjects.

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