VISION OF PLASMONICS FROM MICROWAVE AND OPTIC RANGES

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Abstract. Proposed article analyzes the various traditions that are applied in EM field theory by experts from different branches of physics, the same results of the study may look different. Here is compared the style of presentation of dispersion characteristics and also the frequency measurement units may not match entirely in microwave and optical ranges. There is explained the causes and logic of such discrepancies. Time-dependent and position-dependent solutions of the Helmholtz equation are often with opposite signs found in literature on microwave electrodynamics and optics including plasmonics. As a result, intermediate and final expressions may contain opposite signs. A detailed solution to the wave equation and the reasons for the appearance of opposite signs has been made here.

Key words: EM wave propagation, plane wave, light, microwaves, dispersion characteristic, Maxwell's equations, wave equation, Helmholtz equation, signs of solutions, units of frequency, electromagnetics, plasmonics, optics.

1. INTRODUCTION

We will consider here non-matching approaches that have appear and become traditional in the microwave electrodynamics based on the Maxwell's equations and the electromagnetic (EM) field theory, which is used in physical optics as well as in its branch such as plasmonics. The microwave electrodynamics is a part of EM field theory because the last one covers all frequency ranges.

Plasmonics is a modern field of physics that contains knowledge related to several in-touch areas such as condensed matter physics, physical optics, quantum mechanics, atomic and molecular physics [1] – [6]. Plasmonics studies the interaction between light (photons, EM waves) and free (conduction) electrons in conductors (metals, semimetals, semiconductors). Photon is the quantum of the EM field including light. Free electrons in conductors excited by electric field components of EM wave (light) can have collective oscillations with a frequency close to the frequency of acting light. Plasma oscillations are fast oscillations of the free electron density in conducting media. Plasmonics is based on a physical phenomenon called plasmons.

Plasmons by their physical nature are a composition of free electron plasma and the acting EM wave.

The surface plasmon waves are often used in next-generation modern devices. The surface plasmon wave consists of the upper and lower parts separated by the boundaries of the two environments. Both parts of the plasmonic wave propagate at the same speed which may be lower than the speed of an EM wave inside a dielectric (air) environment. We could imagine a wave of surface plasmons resembling a horse running through the shallow waters of a river. The lower part of the horse is in the water, thus causing concentric circles of water waves. The head of the moving horse creates an air swirl. The horse's body is in two environments at the same time. The horse, which we can compare with a plasmon wave, moves forward in the required direction as something single whole. The plasmonic wave moving along a dielectric–metal interface (surface) is also called the surface plasmon polariton (SPP) [7]. Polariton can be defined as the strong coupling of photon with another quasiparticle.

By definition microwave electrodynamics is the branch of physics that has deals with relatively fast changing EM fields. This corresponds to ranges: SHF (super high frequency), EHF (extremely high) and THF (tremendously high) with the general frequency range f = 3GHz (gigahertz, 10^9 Hz) – 3THz (terahertz, 10^{12} Hz) and wavelength λ = 10 cm to 100 μm (micrometer, 10^{-6} m) in free space (in a vacuum).

By definition optics is the branch of physics that studies the features of light, its interactions with matter and applications created on this base. Optics is the science of light which includes the EM wave spectrum as IR (infrared, 0.3 THz – 0.43 PHz), VIS (visible, 0.43 – 0.79 PHz), UV (ultraviolet, (0.79–30) PHz) and light is characterized by frequency f=0.3 – 3000 THz and wavelength λ = 1 mm to 10 nm (nanometer, 10^{-9} m) in free space. Let's remind you that 1 pentahertz (1 PHz) = 10^{15} Hz = 10^6 GHz = 10^3 THz.

The microwave electrodynamics [8], [9], [10] and physical optics [11], [12], [13] have deals with EM waves and they are both based on the Maxwell's equations. It is interesting to note that the signs in the expressions describing the phenomena of plasmonics obtained by experts of microwave and optical ranges are often opposite.

The purpose of this article is to show the differences caused by the traditions of the mentioned areas of physics, and to find out at what stage of the solution occur differences in the signs.

Here we will analyze a few important differences that have to face readers of technical literature concerning three issues: 1) dispersion characteristics; 2) units of frequency; 3) choosing opposite signs for position (spatial) terms as well for time (temporal) terms in solving the same wave equations. We are going to start parsing the differences from the simplest issue about graphs of characteristics.

2. GRAPHICS OF DISPERSION CHARACTERISTICS

Dispersion characteristics are usually turned upside down in the literature on plasmonics (optics) in comparison with the microwave electrodynamics. In the literature on plasmonics [14] – [20], is often used the dispersion characteristic in the form $f = F(k)$, where frequency f is the function on the propagation constant (wave number) k. Instead of the frequency f can also be used the angular (cyclic) frequency $\omega = 2\pi f$ or other normalized value such as ω/ω_p, where ω_p is the plasma frequency of the conduction electrons [1], [10], [13], [19].

In Fig.1 are given dispersion characteristics for the layered structure with parameters taken from [7]. The structure contains substances: the environment is air with the relative permittivity ε_1 = 1.0, the metal film with the thickness d = 30 nm is made of silver (Ag) at the relative permittivity that depends on the frequency $\varepsilon(\omega)$. The plasma frequency for silver is $\omega_p \approx 11.9989 \cdot 10^{15}$ s^{-1}. The bottom layer of the structure is made of the dielectric material with ε_2 = 2.25.

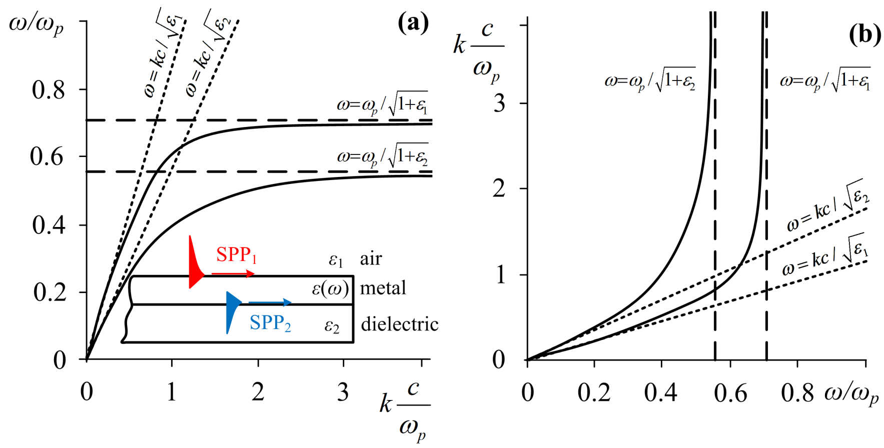

Fig. 1 – The dispersion characteristic of the SPP waves propagating on layered structure containing a metal – air and a metal – dielectric interfaces on which SPPs are excited for cases: (a) in the form commonly used in the optical range and (b) in the microwave range.

Fig.1 (a) corresponds to the tradition dispersion characteristic accepted in physical optics as [7] and Fig.1 (b) – represents to the same characteristic but in the form usually accepted in microwave electrodynamics. In Fig.1(a) on the ordinate (vertical) axis is placed the magnitude proportional to the normalized cyclic frequency ω/ω_p and on the abscissa (horizontal) axis is located a magnitude proportional to the propagation constant k.

In the literature on microwave electrodynamics [21] – [27] dispersion characteristics represent dependency $k = F(f)$, i.e. propagation constant k (or other value proportional to its) is located on the ordinate axis and frequency f (or other value proportional to its) is placed on the abscissa axis (see Fig.1 (b)). Although you can see sometimes the use of two approaches in the presentation of dispersion characteristics in the same article [28].

3. VARIETY OF FREQUENCY DESIGNATION UNITS

The different units can be used for the frequency of EM waves in examinations of appropriate physical values in the microwave range and the optical range in the relevant technical literature. Throughout history, the development of microwave waveguide and antenna technologies evolved from radio and microwave frequencies with a tendency to move into higher frequencies. In the microwave range, the frequency units are only in hertz (Hz) or multiples such as kHz, GHz, THz, and PHz [8] – [10].

In the optical range for the frequency can also be used the energy unit equivalents as electronvolts (eV), kelvin (K) and the reciprocal centimeter (cm^{-1}). It is easy to find "Energy Unit Conversions" on the Internet [29].

Historically, optical devices typically operate at relatively high frequencies f = 0.3 THz – 30 PHz of EM spectrum. In recent years there is a tendency in optics to develop devices that can also operate at lower frequencies. The optics was the basis for successful development of such advanced scientific research branches as plasmonics, photonics, spectroscopy. Therefore, in optics as well as in plasmonics the measurement of frequency additionally occurs in eV, which is the amount of kinetic energy obtained (or lost) by an electron accelerating from rest through an electric potential difference of one volt in vacuum, also in K which is the base unit of temperature and cm^{-1} that is a unit of wavenumber measured in cycles per unit distance or radians per unit distance [30] – [33]. Sometimes are used the normalized frequency units [34], [35].

Let's consider the reasons why additional units of measurement for frequency are used in optics.

As we know light is an EM wave that is oscillating with a frequency f in IR, VIS, UV ranges. But light is also a particle, which is called a photon and each photon carries a packet of energy that is proportional to the frequency f. Substance consist atoms and molecules and can absorb the energy from a photon. The photon is the smallest discrete amount or quantum of EM radiation (light). The energy of a photon E can be expressed such as:

$$E = h_{pl} f = \hbar\omega = h_{pl}\frac{c}{\lambda}, \quad (1)$$

where $h_{pl} \approx 4.13567 \cdot 10^{-15}$eV·s is the Planck's constant, $\hbar = h_{pl}/2\pi$ is the reduced Planck constant, $f\,\lambda = c$, c is the speed of light in a vacuum (m/sec), λ is the wavelength of EM wave (light) (m).

In solids, the atoms (molecules) vibrate about their equilibrium positions of their lattice nodes. Based on the law of equipartition of energy, the average energy of a simple harmonic oscillator is $E_k \to k_b T$, where k_b is the Boltzmann constant and T is the absolute temperature (K). The equipartition theorem relates the temperature of a system to its average energies [36], [37], (see Table 1).

Table 1.

Simple relations $f \propto E,\, k,\, T$.

Magnitude	Formula	Units	Equation
Energy, E	$E = h_{pl}\, f$	eV	(2)
Wavenumber, k	$k = 1/\lambda = f/c$	cm^{-1} (m^{-1})	(3)
Frequency, f	$f = c(1/\lambda)$	Hz	(4)
The average (kinetic) thermal energy, E_k	$E_k \to k_b T$ where $k_b = \tilde{8}.6 \cdot 10^{-5}$ eV/K is the Boltzmann constant, T is the temperature	K	(5)

We can express the frequency from Eq. (2) as $f = E/h_{pl}$, which means that the frequency f is proportional ($\propto$) to the energy E, i.e. $f \propto E$ in units eV, from Eq. (3) $f \propto k$ in units cm^{-1}, along with this using Eq. (5) $f \propto T$ in units kelvin (K). Here we give several relationships between the most commonly used in plasmonics (optics) units: 1THz = 0.00414eV, 1THz corresponds to the wavelength of 300 μm. 1eV = 241.8THz. 1eV corresponds to the wavelength $\lambda = 1.2398\mu$m and also 2eV corresponds to $\lambda = 0.620\mu$m, 6eV corresponds to $\lambda = 0.207\mu$m etc. Magnitudes equal 1cm^{-1} = 123.98μeV= 0.0299795THz. The relations between the various units which are used in the scientific literature on plasmonics are given in Table 2.

Table 2.

Conversion of values eV, Hz, cm^{-1}, and K.

	eV (electron volt)	Hz (Hertz)	cm^{-1}	K (kelvin)
eV	1	$2.41804{\cdot}10^{14}$	8065.73	11604.9
cm^{-1}	$1.23981{\cdot}10^{-4}$	$2.99793{\cdot}10^{10}$	1	1.42879
Hz	$4.13558{\cdot}10^{-15}$	1	$3.33565{\cdot}10^{-11}$	$4.79930{\cdot}10^{-11}$
K	$8.61705{\cdot}10^{-4}$	$4.79930\ {\cdot}10^{-11}$	0.695028	1

We use definitions and quantities here that are usually applied in quantum mechanics and optics. Quantum mechanics is a branch of physics which deals with physical phenomena that refer to structures with sizes of nanoscales (1–100 nm).

4. COMMENTS ABOUT THE IMAGINARY UNIT

Reading scientific literature we are faced with another ambiguity– this is a definition of an imaginary unit because of which can get opposite signs in expressions of EM field theory and other branches of physics.

The complex number representation gives an alternative description that simplifies mathematical procedures. Complex exponential are used extensively in EM field theory, physical optics, classical and quantum mechanics. Euler's formula gives the fundamental relationship between the trigonometric functions and the complex exponential function:

$$e^{ix} = \cos x + i \sin x, \tag{6}$$

where e is the base of the natural logarithm, "i" is the imaginary unit, the argument x is the real number, and $\cos(\cdot)$ and $\sin(\cdot)$ are the trigonometric functions cosine and sine, respectively. The complex unit usually is taken as $i = \sqrt{-1}$, [11], [13], [38], [39].

First of all, [40] attracts attention to the discrepancy in literature regarding the use of different definitions of the imaginary unit. In [40] is shown that the designation of the imaginary unit in engineering is usually "j" and in physics/science the designation "i" is applied, that differ in sign and the ratio can be $j \leftrightarrow -i$.

In [41] is taken a sign coefficient $c = \pm 1$ which is a multiplier of the imaginary unit to show a possibility to choose the desired sign from the two signs offered by the solution of the wave equation.

It is usefully to note that in some publications, e.g. [42], the imaginary unit marked $i = \sqrt{-1}$, i.e. $j \leftrightarrow i$. These definitions can be changed by simply reversing the signs of the imaginary unit, i.e. $\pm i \rightarrow \mp i$ [43].

The different definition of the imaginary unit leads to the emergence of opposite signs in important expressions, such as the complex permittivity of materials, the one can be presented as $\varepsilon = \varepsilon' - i\varepsilon''$ or $\varepsilon = \varepsilon' + i\varepsilon''$, the same happened with the permeability, refractive index, propagation constant, terms for right- left-circular polarizations, Fourier series, Fourier transformation etc.

For this reason, we have to pay attention to the value of the imaginary unit in the publications.

5. COMMON REASONING

Since solutions to wave equations derived from Maxwell's equations may contain expressions with opposite signs, we will consider the stages of their obtaining. The Faraday's and Ampère's law equations in microwave and optics ranges are the same [8], [13], [21]:

- Maxwell-Faraday's equation, in microwave and optical ranges of frequency:

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}. \tag{7}$$

- Maxwell-Ampère's equation, in microwave and optical ranges of frequency:

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}. \tag{8}$$

where $\mathbf{E}(\mathbf{r},t)$ is the electric field strength of an EM wave, $\mathbf{H}(\mathbf{r},t)$ is the magnetic field strength, $\mathbf{D}(\mathbf{r},t)$ is the electric flux density, $\mathbf{B}(\mathbf{r},t)$ is magnetic flux density, $\mathbf{J}(\mathbf{r},t)$ is total electric current density. These values are depended on the coordinate $\mathbf{r}$ and time t. The equations are interesting to us because they are the base for receiving of the wave equations, the solutions of which we are going to study.

We will consider here the propagation of monochromatic plane EM waves in a homogeneous isotropic environment. The monochromatic wave characterized by oscillations that occur at only a single frequency at each spatial point. The simplest solution of the wave equations is for a plane wave, i.e. a wave whose surfaces of constant phase are infinite planes, perpendicular to the direction of propagation. There is a reason why the solution in the form of plane monochromatic waves plays a big role in electromagnetics. The fact is that using the Fourier transform by time and coordinates, any function of these variables can be decomposed into plane monochromatic waves, if only the function quickly descends on time (temporal) and space (spatial) infinity [44], [45]. We are going to explore expressions for the time–periodic case which are usually used in all branches of physics. In this case we can use simplifications for the differentiation and integration with respect to time t.

We can write time-harmonic Maxwell's equations in terms of vector field phasors $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$. Vector phasors contain the information on magnitude, direction, and phase, where radius vector is $\mathbf{r} = \sqrt{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z}$. The wave equations for the time-periodic case are $\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$ (and $\nabla^2\mathbf{H} + k^2\mathbf{H} = 0$), where $k = \omega/c$ is the wavenumber (propagation constant) in a medium. Phase speed of light in a vacuum is $c = k/\omega = 1/\sqrt{\varepsilon_0\mu_0}$, phase speed of light in an isotropic medium is $v = 1\big/\sqrt{\varepsilon\mu} = c\big/\sqrt{\varepsilon_r\mu_r}$, $\varepsilon = \varepsilon_r\varepsilon_0$ is the absolute permittivity, $\mu = \mu_r\mu_0$ is the absolute permeability, $\varepsilon_0 \approx 8.85 \cdot 10^{-12}$ (F/m) and $\mu_0 \approx 1.26 \cdot 10^{-6}$ (H/m) are the vacuum permittivity and permeability, ε_r and μ_r are the relative permittivity and permeability of an isotropic medium, respectively, $n = c/v$ is the index of refraction.

Because of the linearity of Maxwell's equations the solutions can be decomposed into a superposition of sinusoids or cosinusoids for the EM waves propagating in a vacuum. The convenience of complex notation has its origins in the Euler identity (formula) according to Eq. (6) such: $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ and $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$. By inverting Euler's expression we can receive the following representation of the cosine and sine functions: $\cos(\omega t) = \mathrm{Re}\left(e^{i\omega t}\right) = \left(e^{i\omega t} + e^{-i\omega t}\right)/2$ as well as $\sin(\omega t) = \mathrm{Im}\left(e^{i\omega t}\right) = \left(e^{i\omega t} - e^{-i\omega t}\right)/2i$. This is the basis for the Fourier transform method for the solution of the wave equations which are the second-order linear partial differential equations [46] – [48] .

Reading technical literature in electromagnetics we notice that the solution of wave equations may contain dependencies on time and coordinates with the plus or minus signs for every term, for example as

$$E(\mathbf{r},t) \sim e^{-i\,(\omega t-(kr))}, \tag{9}$$

and

$$E(\mathbf{r},t) \sim e^{i\,(\omega t-(\mathbf{kr}))} = e^{-i\,((\mathbf{kr})-\omega t)}. \tag{10}$$

Let's consider the reason for the inconsistency of the signs in solving the same problems by specialists of different branch of physics. About agreements of the signs (plus or minus) in solutions of the same EM problems are discussed in [38], [40], [50]. In [38] provides reasoning and research on agreements and definitions regarding signs in the literature on ellipsometry. (We remind that ellipsometry is known as an optical technique for investigating the dielectric properties of films and layers. Ellipsometry let us to measure changes of an EM wave polarization when the wave reflects or transmits through a material structure.) As noted in this publication many parameters which appear in the theory of ellipsometry crucially depend on the choice of arbitrary conventions and definitions. Here are analyzed two alternatives for the complex formulation of sinusoidal oscillations. The paper highlights that the negative temporal (time) exponent is preferred here because of its use in most of modern physics, particularly quantum mechanics, although appositive alternative is firmly established in electrical engineering. A direct consequence of the choice in sign for the exponent in the time-factor is the sign of the imaginary part of the complex refractive index n. In [38] noted the people studying of ellipsometry are confronted with a confusing multiplicity of conventions and definitions employed. In the articles on the subject, and this problem persists in the present literature. In an effort to untangle this situation, nine conventions and definitions have been singled out, where arbitrary choices between two alternatives have to be made in the theory of ellipsometry.

In [40] specialists from Massachusetts Institute of Technology (MIT) identified that opposed signs are used in Engineering, i.e. "the Negative Sign Convention", and physics (probably, in optics), i.e. "the Positive Sign Convention" for EM wave propagating in $+z$ –direction. MIT presented a table with expressions for the Maxwell's equations, wavenumber, propagation constant, complex permittivity ε, permeability μ, and the index of refraction n, Fourier transform etc. Article [50] address the issues of signs in exponential terms containing time and coordinates. In [42] is shown that for plane forward waves moved in $+k$ – direction can be described as

$$E = E_0\, e^{i(-\omega t + kz)}, \tag{11}$$

or

$$E = E_0\, e^{i(\omega t - kz)}. \tag{12}$$

We can write for the first case of EM forward wave:

$$E = \mathrm{Re}\{E_0\, e^{i(-\omega t + kz)}\} = E_0 \cos(-\omega t + kz), \tag{13}$$

and the peak of EM wave has moved to a positive location:

$$\Delta z = (\omega/k)\,\Delta t, \tag{14}$$

where Δz is a positive shift for the plane forward waves moved in $+k$ direction.

On other hand, we can write for the backward waves also two variants:

$$E = E_0\, e^{i(\omega t + kz)}, \tag{15}$$

or

$$E = E_0\, e^{i(-\omega t - kz)}. \tag{16}$$

We can write for the first case of the backward waves:

$$E = \mathrm{Re}\{E_0\, e^{i(\omega t + kz)}\} = E_0 \cos(\omega t + kz). \tag{17}$$

The peak of EM wave moves toward the negative direction:

$$\Delta z = -(\omega/k)\,\Delta t, \tag{18}$$

where Δz is a negative shift for EM plane waves moved in $-k$ direction, i.e. this represents the backward wave.

In [13], [39], [51] the electric field is taken in the form $E(\mathbf{r},t) \propto e^{i((\mathbf{kr}) - \omega t)}$. It fits the meaning that $e^{-i\omega t}$ corresponds to the positive frequency and the exponent with the opposite sign, i.e. with the minus, such:

$$e^{i\omega t} = e^{-i(-\omega)t}, \tag{19}$$

corresponds to the negative frequency $(-\omega)$.

The physical field can be presented as the sum of the positive and negative frequency parts [39], [51]: $E(\mathbf{r},t) = \left(E_0 e^{i((\mathbf{kr}) - \omega t)} + E_0^{*} e^{-i((\mathbf{kr}) - \omega t)}\right)/2$. We can obtain the real parts of the field by adding their respective complex conjugates E_0^{*} and dividing the result by 2.

We remind here that the values of the wavenumber (propagation constant) k and angular frequency ω are not independent values, because the product of wavenumber

k on the speed of EM wave in a matter v was marked by us as $\omega = kv$ and we can re-write $\omega - kv = 0$. So we see that in expressions ω and k have the opposite signs.

As an example we take and show the dependencies on time and coordinates for the electric field $\mathbf{E}$, of course, we can do the same for all of other fields $\mathbf{H}$, $\mathbf{B}$, $\mathbf{D}$ etc.

6. SOLUTIONS OF WAVE EQUATIONS AND SIGNS

Now we examine the reason why opposite signs are occurred in the solutions of the wave equation. For this reason, we will analyze these solutions of the wave equation [39] – [42], [49] – [56]. The wave equation is a second-order linear partial differential equation for the characterization of EM waves. In [8], [57] – [60] is given the EM wave equation and its solutions. The homogeneous form of the wave equation, written in terms of the electric field $\mathbf{E}$ can be expressed:

$$\left(v^2 \nabla^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0, \tag{20}$$

where the Laplace operator (Laplacian) $\nabla^2 = \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is a second-order differential operator in a Cartesian coordinate system, t is time, $\partial^2/\partial t^2$ is a second order derivative of time.

Converting Eq. (20) gives:

$$\nabla^2 E(\mathbf{r},t) - \frac{1}{v^2}\frac{\partial^2 E(\mathbf{r},t)}{\partial t^2} = 0. \tag{21}$$

The wave equation (21) describes the propagation of EM waves through a homogeneous isotropic medium. Three-dimensional form of the wave equation often arises in the study of physical problems involving partial differential equations in both space and time. We use for a solution the method of the separation of variables [57] – [61]:

$$E(\mathbf{r},t) = T(t)\,R(\mathbf{r}). \tag{22}$$

We put expression (22) to Eq. (21) and get:

$$\Delta\left(T(t)\,R(\mathbf{r})\right) - \frac{1}{v^2}\frac{\partial^2 T(t)}{\partial t^2} R(\mathbf{r}) = 0. \tag{23}$$

We divide the right and left side of the equation into the production $T(t)\,R(\mathbf{r})$:

$$\frac{\Delta R(\mathbf{r})}{R(\mathbf{r})} - \frac{1}{v^2}\frac{\partial^2 T(t)/\partial t^2}{T(t)} = 0. \tag{24}$$

The first term of Eq. (24) depends only on the radius-vector $\mathbf{r}$, and the second term depends only on the time of t. This is only possible if both components are equal to the same constant.

Let's define this constant as $-k^2$, then we can write:

$$\frac{\Delta R(\mathbf{r})}{R(\mathbf{r})} = -k^2, \tag{25}$$

if the constant $-k^2$ turns out to be positive, then let's assume that k is a purely imaginary number.

After converting Eq. (25), we get:

$$\Delta R(\mathbf{r}) + k^2 R(\mathbf{r}) = 0. \tag{26}$$

In Eq. (26) the function $R(\mathbf{r})$ can be the electric field $E(\mathbf{r})$ or the magnetic field $H(\mathbf{r})$. In mathematics, the eigenvalue problem for the Laplace operator Δ is called Helmholtz equation. For the spatial (space) part of the solution of the linear partial differential wave equation (21), we will use Eq. (26) which is the Helmholtz equation. Term k is the eigenvalue and in Eq. (26) also means the wave number.

For the time (temporal) part of the solution of the wave equation (21), we will get:

$$\frac{1}{v^2}\frac{\partial^2 T(t)/\partial t^2}{T(t)} = -k^2. \tag{27}$$

After transformation of Eq. (27) we get:

$$\frac{\partial^2 T(t)}{\partial t^2} + (kv)^2 T(t) = 0. \tag{28}$$

We introduce here for brevity the designation $\omega = kv$.

$$\frac{\partial^2 T(t)}{\partial t^2} + \omega^2 T(t) = 0, \quad (\text{Equation of harmonic oscillations}). \tag{29}$$

The solutions of Eqs. (26) and (29) in the complex form are simpler comparing with the solutions in the real form. Therefore, it is advisable to look for solutions in a complex form, and then consider the physical part of a complex solution. We use the complex algebra but we recognize that only the real part of the final answer has physical significance [39], [52]. For a linear differential equation with physical coefficients, the real part of the general complex solution is a final physical solution.

It has been applied the underlining of the symbol or letter to designate the complex value $\underline{T}(t)$ and real value $T(t)$. The general complex solution to the harmonic oscillation equation is linear combination:

$$\underline{T}(t) = \underline{T}_{01}\, e^{i\omega t} + \underline{T}_{02}\, e^{-i\omega t}, \tag{30}$$

where $\underline{T}_{01}$ and $\underline{T}_{02}$ are arbitrary complex integrated constants. The question of which of the two terms of Eq. (30) with the plus or minus sign to choose as a solution is your personal choice. You can choose $\underline{T}_{01}\, e^{i\omega t}$ or another $\underline{T}_{02}\, e^{-i\omega t}$. The general solution is just the superposition of all possible solutions of the certain differential equation. At this stage, these decisions are tantamount to each other. It is a matter of agreement.

A common solution to the harmonic oscillation equation can be obtained as a linear combination of species solutions: $\underline{T}(t) = \underline{T}_0 e^{i\omega t}$ where ω can have two possible values with the same module but different signs. $\underline{T}_0$ arbitrary integrated constant, different for positive and negative values ω.

Let's go back now to the spatial part of the solution of the wave equation to the Helmholtz equation (26). We will continue to search for particular solutions to the wave equation by separating variables. We are looking for a solution to the Helmholtz equation for the spatial part in the form of three functions, each of which depends only on one spatial coordinate:

$$R(\mathbf{r}) = X(x)\,Y(y)\,Z(z)\,. \tag{31}$$

We'll put Eq. (31) into the Helmholtz equation (26) and get:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(X(x)\,Y(y)\,Z(z)) + k^2\,(X(x)\,Y(y)\,Z(z)) = 0. \tag{32}$$

Denote $X = X(x),\ Y = Y(y),\ Z = Z(z)$:

$$\left(Y\,Z\frac{\partial^2 X}{\partial x^2} + X\,Z\frac{\partial^2 Y}{\partial y^2} + X\,Y\frac{\partial^2 Z}{\partial z^2}\right) + k^2 X\,Y\,Z = 0. \tag{33}$$

Denote derivatives by the appropriate coordinates $X'' = \partial^2 X/\partial x^2$, $Y'' = $ $= \partial^2 Y/\partial y^2$, $Z'' = \partial^2 Z/\partial z^2$:

$$\left(X''\,Y\,Z + Y''X\,Z + Z''\,X\,Y\right) + k^2 X\,Y\,Z = 0. \tag{34}$$

Two strokes indicate the second derivative in each case by corresponding variable.

We will divide Eq. (34) by the product X, Y, Z and get:

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2, \tag{35}$$

the first term of Eq. (35) depends only on the x coordinate, the second term depends only on the y coordinate and the third term depends only on the z coordinate. This is only possible if each of these components is a constant. We identify these constants as $\left(-k_x^2\right),\ \left(-k_y^2\right),\ \left(-k_z^2\right)$. Then:

$$k^2 = k_x^2 + k_y^2 + k_z^2, \tag{36}$$

where $k_x,\ k_y,\ k_z$ can be seen as a projection of the wavenumber vector $\mathbf{k}$.

$$\frac{X''}{X} = -k_x^2, \tag{37}$$

$$\frac{Y''}{Y} = -k_y^2, \tag{38}$$

$$\frac{Z''}{Z} = -k_z^2, \tag{39}$$

This equation of harmonic oscillations $\partial^2 X \big/ \partial x^2 = -k_x^2 X$ (see Eq. (37) is dependent from the coordinate but do not from time. A complex solution to Eq. (37) can be taken as a linear combination of particular solutions:

$$\underline{X} = \underline{X}_{01}\, e^{ik_x x} + \underline{X}_{02}\, e^{-ik_x x}. \tag{40}$$

We can choose solution $\underline{X} = \underline{X}_{01}\, e^{ik_x x}$ or $\underline{X} = \underline{X}_{02}\, e^{-ik_x x}$. We will now make a choice as in the microwave electrodynamics and engineering $\underline{X} = \underline{X}_0\, e^{-ik_x x}$, where the projection k_x of the wavenumber vector $\mathbf{k}$ can take two possible values with the same module, but different signs. The term with the minus component is a matter of agreement.

Similarly coordinates y and z:

$$\underline{Y} = \underline{Y}_0\, e^{-ik_y y}, \tag{41}$$

$$\underline{Z} = \underline{Z}_0\, e^{-ik_z z}. \tag{42}$$

We'll put terms (40) – (42) in Eq. (31) and get:

$$\underline{R}(\mathbf{r}) = \underline{X}_0\, \underline{Y}_0\, \underline{Z}_0\, e^{-ik_x x}\, e^{-ik_y y} e^{-ik_z z} = \underline{R}_0(\mathbf{r})\, e^{-i\,(k_x x + k_y y + k_z z)} \tag{43}$$

Eq. (43) is a particular solution to the Helmholtz equation (26) which we can rewrite:

$$\underline{R}(\mathbf{r}) = \underline{R}_0(\mathbf{r})\, e^{-i\,(\mathbf{kr})}. \tag{44}$$

The complete solution of wave equation Eq. (21) on the base of Eq. (22) can be written:

$$\underline{E}(\mathbf{r}, t) = \underline{T}(t)\, \underline{R}(\mathbf{r}) = \underline{T}_0\, e^{i\omega t}\, \underline{R}_0(\mathbf{r})\, e^{-i\,(\mathbf{kr})}. \tag{45}$$

A particular solution to the wave equation Eq. (21) in the form of plane monochromatic waves is:

$$\underline{E}(\mathbf{r}, t) = \underline{E}_0 e^{i\,(\omega t - (\mathbf{kr}))}, \tag{46}$$

where $\underline{E}_0$ is the arbitrary integrated constant. This solution is as usually use in EM problems of the microwave range.

Analyzing of technical literature we see that in [10], [11], [59] use dependency on coordinates and time in form $e^{i\,(\omega t - (\mathbf{kr}))}$ and in [13], [20], [39] use in form $e^{i\,((\mathbf{kr}) - \omega t)}$.

Signs in expressions that are used in different branches of physics:

- Microwave electrodynamics, Engineering:

- The electric and magnetic field strengths of the time-periodic plane wave: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\, e^{i\omega t}$, $\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\, e^{i\omega t}$.
- We can replace: $\frac{\partial e^{i\omega t}}{\partial t} = i\omega\, e^{i\omega t}, \frac{\partial}{\partial t} \to i\omega$
- We can replace: $\frac{\partial^2 e^{i\omega t}}{\partial t^2} = (i\omega)^2\ e^{i\omega t}, \quad \frac{\partial^2}{\partial t^2} \to -\omega^2$.
- For differentiation with respect to time t: $\partial \mathbf{E}(x,y,z,t)/\partial t = i\omega \mathbf{E}(x,y,z)$, the same for the magnetic field strengths.
- Faraday's law for the Time–Periodic Case: $\nabla \times \left(\mathbf{E}(\mathbf{r})\, e^{i\omega t}\right) = -\frac{\partial \mathbf{B}(\mathbf{r}) e^{i\omega t}}{\partial t}$, $\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}$.
- Ampère's law for the Time–Periodic Case: $\nabla \times \left(\mathbf{H}(\mathbf{r})\, e^{i\omega t}\right) = \mathbf{J}(\mathbf{r}) e^{i\omega t} + \frac{\partial \mathbf{D}(\mathbf{r}) e^{i\omega t}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} + i\omega\varepsilon\mathbf{E}$.

- Optics, Plasmonics:
  - The electric and magnetic field strengths of the time-periodic plane wave: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\, e^{-i\omega t}$, $\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\, e^{-i\omega t}$.
  - We can replace: $\frac{\partial e^{-i\omega t}}{\partial t} = -i\omega\, e^{-i\omega t}, \frac{\partial}{\partial t} \to -i\omega$
  - We can replace: $\frac{\partial^2 e^{-i\omega t}}{\partial t^2} = (-i\omega)^2\ e^{-i\omega t}, \quad \frac{\partial^2}{\partial t^2} \to -\omega^2$.
  - For differentiation with respect to time t: $\partial \mathbf{E}(x,y,z,t)/\partial t = -i\omega \mathbf{E}(x,y,z)$, the same for the magnetic field strengths.
  - Faraday's law for the Time–Periodic Case: $\nabla \times \left(\mathbf{E}(\mathbf{r})\, e^{-i\omega t}\right) = -\frac{\partial \mathbf{B}(\mathbf{r}) e^{-i\omega t}}{\partial t}$, $\nabla \times \mathbf{E} = +i\omega\mu\mathbf{H}$.
  - Ampère's law for the Time–Periodic Case: $\nabla \times \left(\mathbf{H}(\mathbf{r})\, e^{-i\omega t}\right) = \mathbf{J}(\mathbf{r}) e^{-i\omega t} + \frac{\partial \mathbf{D}(\mathbf{r}) e^{-i\omega t}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} - i\omega\varepsilon\mathbf{E}$.

Signs between the relative and imaginary parts in the complex relative permittivity $\underline{\varepsilon}_r = \underline{\varepsilon}/\varepsilon_0$ and index of refraction $\underline{n}$:

- Microwave electrodynamics, Engineering:
  - Relative permittivity: $\underline{\varepsilon}_r = \varepsilon_r' - i\varepsilon_r''$.
  - Refractive index: $\underline{n} = n' - in'' = \sqrt{\underline{\varepsilon}_r} = \sqrt{(\varepsilon_r' - i\varepsilon_r'')}$; at $\underline{\mu}_r = 1$, where $\varepsilon_r' = (n')^2 - (n'')^2$; $\varepsilon_r'' = 2n'n''$.

- Optics, Plasmonics:
  - Relative permittivity: $\underline{\varepsilon}_r = \varepsilon_r' + i\varepsilon_r''$.
  - Refractive index: $\underline{n} = n' + in'' = \sqrt{\underline{\varepsilon}_r} = \sqrt{(\varepsilon_r' + i\varepsilon_r'')}$; at $\underline{\mu}_r = 1$, where $\varepsilon_r' = (n')^2 - (n'')^2$; $\varepsilon_r'' = 2n'n''$.

In the scientific literature on the engineering and microwave electrodynamics the time dependency is usually expressed by the complex exponential function $e^{i\omega t}$ while in physics literature and especially in optics and plasmonics this dependence is usually taken by $e^{-i\omega t}$. It should be noted that in some works this approach is not true, e.g. "Electromagnetic Wave Theory" [8] is taken $E(\mathbf{r},t) \propto e^{i((\mathbf{kr})-\omega t)}$, in "The plane wave spectrum representation of electromagnetic fields" is presented $E(\mathbf{r},t) \propto e^{i(\omega t \pm (\mathbf{kr}))}$.

7. CONCLUSIONS

The approach to the processes of EM wave propagation in the area of plasmonics from the point of view of experts in the microwave and optical range specialists is often different.

In fact, the choice of signs in expressions is more related to the traditions of scientific school, which include experts who write articles and books in different branches of physics.

It is important to draw readers attention to a different approach to certain issues in technical literature in the microwave and optical ranges.

REFERENCES

1. S. Enoch, N. Bonod, "Plasmonics From Basics to Advanced Topics", (Springer, Heidelberg, New York, Dordrecht, London, 2012).
2. Y. Li, "Plasmonic Optics: Theory and Applications", (SPIE, 2017).
3. S. A. Maier, H. A. Atwater, "Plasmonics: Localization and guiding of electromagnetic energy in metal/dielectric structures," Journal of Applied Physics, Vol. 98, pp. 011101-(1-10), 2005.
4. B. Dastmalchi, P. Tassin, Th. Koschny, and C. M. Soukoulis, "A New Perspective on Plasmonics: Confinement and Propagation Length of Surface Plasmons for Different Materials, and Geometries," Advanced Optical Materials, Vol. 4, no. 1, Jan. 2016, pp. 177-184.
5. M. I. Stockman, "Nanoplasmonics: From Present into Future," Ch. 1 of "Plasmonics: Theory and Applications", Edited byT. V. Shahbazyan, M. I. Stockman, Springer, 2013.
6. M. S. Tame, K. R. McEnery,Ş. K. Özdemir, J. Lee, S. A. Maier, and M. S. Kim, "Quantum plasmonics", Nature Physics, vol. 9, no. 6, pp.329–340, Jun 2013.
7. A. V.Zayats,I. I.Smolyaninov, and A. A.Maradudin, "Nano-optics of surface plasmon polaritons", ELSEVIER, Science Direct, Physics Reports, vol. 408, no. 3–4, pp. 131-314, March 2005.
8. J. A. Kong, "Electromagnetic Wave Theory," EMW Publishing, Cambridge, Massachusetts, USA, 1016 p., 2008, ISBN 0-9668143-9-8.
9. J. R. Reitz, F. J. Milford, and R. W. Christy, "Foundations of Electromagnetic Theory," Addison-Wesley Publishing Company, Reading & etc, 630 p., 1993.
10. L. Nickelson, "Electromagnetic Theory and Plasmonics for Engineers," Springer, Singapore, 749 p., 2019.
11. E. Hecht, "Optics," Addison Wesley Longman, Inc. Reading&etc, 694 p., 1998.
12. P. Padley, D. Suson, "Waves and Optics," Rice University, Houston, Texas, 181 p., Nov. 2005.

http://cnx.org/content/col10279/1.33/

13. J. Peatross, M. Ware, “Physics of Light and Optics,” in Frontiers in Optics 2010/Laser Science XXVI, 248 p., 2015 https://doi.org/10.1364/FIO.2010.JWA64
14. H. Yoon, K.Y.M. Yeung, P. Kim, D. Ham, “Plasmonics with two-dimensional conductors,” Phil. Trans. R.Soc., pp. 1-13, Mar 2014.
15. M. A. van de Haar, R. Maas, B. Brenny, A. Polman, “Surface plasmon polariton modes in coaxial metal-dielectric-metal waveguides,” New J. Phys., vol. 18, no. 4, pp. 043016 (1-14), Apr. 2016
16. L. Shen, X. Chen, T.-J. Yang, “Terahertz surface plasmon polaritons on periodically corrugated metal surfaces,” Optics Express, vol. 16, no 5, pp. 3326-3333, March 2008.
17. A.V. Zayats, I. I. Smolyaninov, “Near-field photonics: surface plasmon polaritons and localized surface plasmons,” Journal of Optics A: Pure and Applied Optics, vol. 5, no. 4, pp. S1-S35, Jun. 2003.
18. W.L. Barnes, A. Dereux, T.W. Ebbesen, “Surface plasmon subwavelength optics,” Nature, vol. 424, pp. 824–830, Aug. 2003.
19. B. Dastmalchi, Ph. Tassin, Th. Koschny, C.M. Soukoulis, “A New Perspective on Plasmonics: Confi nement and Propagation Length of Surface Plasmons for Different Materials and Geometries,” Adv. Optical Mater, vol. 4, no. 1, pp. 177–184, Jan. 2016.
20. R. Mansoor, A. H. AL-Khursan, “Numerical modelling of surface plasmonic polaritons,” Results in Physics, ELSEVIER, vol. 9, pp. 1297-1300, Jun. 2018.
21. S. Drabowich, A. Papiernik, H. Griffiths, J. Encinas, B. L. Smith, “Modern Antennas,” Chapman-Hall, London, Weinheim, New York, Tokyo, Melbourne, Madras, 1998.
22. S. Yang, J. Song, “Analysis of Guided and Leaky TM0n and TE0n Modes in Circular Dielectric Waveguide,” Progress In Electromagnetics Research B, vol. 66, pp. 143–156, 2016.
23. S. K. Raghuwanshi, B. M. A. Rahman, “Proagation and Characterization of Novel Graded and Linearly Chirped Type’s of Refractive Index Profile Symmetric Planar Slab Waveguide by Numerical Means”, Progress In Electromagnetics Research B, vol. 62, pp. 255–275, Jan. 2015.
24. M. Riaziat, R. Majidi-Ahy, I-J. Feng, ”Propagation Modes and Dispersion Characteristics of Coplanar Waveguides,” IEEE Transactions on Microwave Theory and Techniques, vol. 38, no. 3, pp. 245-251, March 1990.
25. K.Y. Kim, “Fundamental guided electromagnetic dispersion characteristics in lossless dispersive metamaterial clad circular air hole waveguides,” Journal of Optics a Pure and Applied Optics, vol. 9, no. 11, 24 p., Nov. 2007.
26. A. Eroglu, J. K. Lee, “Wave Propagation and Dispersion Characteristics for a Nonreciprocal Electrically Gyrotropic Medium,” Progress In Electromagnetics Research, vol. 62, pp. 237–260, 2006.
27. D. Plonis, J. Bucinskas, R. Pomarnacki etc. “Electric field and dispersion characteristic calculations of glass tube waveguides filled with biological substances,” Electronics. Basel, MDPI AG. vol. 8, no. 3, art. 301, pp. 1-14, 2019.
28. A. Polemi, E. Rajo-Iglesias, S. Maci, “Analytical Dispersion Characteristic of a Gap-Groove Waveguide,” Progress In Electromagnetics Research M, vol. 18, pp. 55–72, Jan. 2011.
29. Energy Units Converters to Hz, eV, K, cm^-1 etc. available at https://sjbyrnes.com/convert.html
30. B. M. Moskowitz, “Fundamental physical constants and conversion factors”, In Handbook of Physical Constants: American Geophysical Union, editor by T. J. Ahrens, vol. 1, pp. 346-355, 1995.
31. J. Tamayo-Arriola, E. M. Castellano, M. M. Bajo, A. Huerta-Barberà, E. Muñoz, V. Muñoz-Sanjosé, A. Hierro, “Controllable and Highly Propagative Hybrid Surface Plasmon–Phonon Polariton in a CdZnO-Based Two-Interface System,” ACS Photonics, vol. 6, no. 11, pp. 2816-2822, 2019.

32. S. Svanberg, “Atomic and Molecular Spectroscopy: Basic Aspects and Practical Applications,”Springer-Verlag Berlin and Heidelberg, New York, 1992, 603 p., ISBN-13: 978-3-642-98109-8.
33. A. Derkachova, K. Kolwas, I. Demchenko, Dielectric Function for Gold in Plasmonics Applications: Size Dependence of Plasmon Resonance Frequencies and Damping Rates for Nanospheres, Plasmonics, vol. 11, pp. 941– 951, June 2016.
34. E. Pisano, C. E. Garcia-Ortiz, F. Armenta-Monzon, M. Garcia-Mendez, V. Coello, “Efficient and Directional Excitation of Surface Plasmon Polaritons by Oblique Incidence on Metallic Ridges,” Plasmonics”, vol. 13, pp.1935–1940, Jan. 2018.
35. R. Yang, Z. Lu, “Subwavelength Plasmonic Waveguides and Plasmonic Materials”, Nanoplasmonics and Metamaterials , vol. 2012, Article ID 258013, 12 p., Jul 2012.
36. S. Štaras “Introduction to semiconductor devices,” Technika, Vilnius 244 p., 2012.
37. O. A. Awoga, A.N. Ikot, A.E. Essiett, L. E. Akpabio, “Thermodynamic Properties of the Harmonic Oscillator and a Four Level System”, Applied Physics Research, vol. 3, no. 1, pp.47-59, May 2011.
38. R. H. Muller, “Definition and conventions in ellipsometry”, University of California, Lawrence Berkeley National Laboratory. LBNL Report : UCRL18585, 37 p., Nov. 1968.
39. D. J. Griffith, “Introduction to Electrodynamics,” Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1999..
40. Sign conventions for EM waves, MIT, Department of Electrical Engineering and Computer Science, EE 3321 Electromagnetic Field Theory, https://empossible.net/wp-content/uploads/2018/03/Summary-of-EM-Sign-Conventions.pdf (was available on June 2, 2020)
41. R.T. Holm, Convention confusions, in Handbook of Optical Constants of Solids II, edited by E.D. Palik, Academic Press, Boston, pp. 21-55, 1991.
42. K. Iizuka, Elements of Photonics, Vol. I: In Free Space and Special Media, Editors: K. Iizuka, B. E. A. Saleh, John Wiley & Sons, Inc., New York, Jun. 6, 2002.
43. H. Fujiwara, “Spectroscopic Ellipsometry: Principles and Applications” John Wiley and Sons, Ltd., 368 p., 2007.
44. P. C. Clemmow, “The plane wave spectrum representation of electromagnetic fields,” The Institute of Electrical and Electronics Engineers, Inc., New York, Sept. 1996.
45. R. Snieder, K. van Wijk, “A Guided Tour of Mathematical Methods for the Physical Sciences, ”Cambridge University Press, New York, 559 p., 2015.
46. A. L. Schoenstadt, “An Introduction to Fourier Analysis Fourier Series. Partial Differential Equations and Fourier Transforms”, Department of Applied Mathematics, Naval Postgraduate School, Monterey, California 93943, 268 p., Aug. 1992.
47. S. T. Karris, “ Signals and Systems with MATLAB, Computing and Simulink, Modeling,” Orchard Publications, 688 p., 2008.
48. M. C. Pereyra, L. A. Ward, “Harmonic Analysis From Fourier to Wavelets,” American Mathematical Society, Institute for Advanced Study, Princeton, New Jersey,410 p., 2010.
49. C. E. Mungan, “Sign convention for backward traveling waves,” The Physics Teacher, vol. 57, no. 3, p. 132, Feb. 2019.
50. J. Perkins, M. J. Ruiz, and A Reliable, “Wave Convention for Oppositely Traveling Waves”, The Physics Teacher, vol. 56, no. 9, p. 622 , Nov. 2018.
51. D. A. Steck, “Classical and Modern Optics”, Department of Physics, University by of Oregon, 357 p. Jun. 2006.
52. D. Cheng, “Field and Wave Electromagnetics,” Addison-Wesley Publishing Company, Reading,

Massachusetts etc, 703 p, 1989.
53. B. Deconinck, "Partial Differential Equations and Waves," University of Washington, Seattle, WA, 98195, USA, 118 p., 2017.
54. A. Tveito, R. Winther, "Introduction to Partial Differential Equations: A Computational Approach," Springer, New York, Tokyo, 387 p., 1998.
55. J. D. Jackson, "Classical Electrodynamics," John Wiley & Sons, Inc., New York, London, Sydney, 1962.
56. M. S. Tame, K. R. McEnery, S. K. Özdemir, J. Lee, S. A. Maier, M. S. Kim, "Quantum plasmonics," Nature Physics, vol. 9, no. 6, p. 329–340, Jun. 2013.
57. S. Ramo, J.R. Whinnery, T.Van Duzer, "Fields and Waves in Communication Electronics," John Willey & Sons, New York, Singapore, 817 p., 1984.
58. G. R. Fowles, "Introduction to Modern Optics," Dover Publications, Inc., New York, 328 p.,1975.
59. J. Bauck, "A Note on Fourier Transform Conventions Used in Wave Analyses," 3 p., Jul. 2018.
60. S. J. Orfanidis, "Electromagnetic Waves and Antennas," ECE Department Rutgers University, 94 Brett Road Piscataway, NJ 08854-8058, 1413 p.
61. Q-Han Park, "Optical antennas and plasmonics," J. Contemporary Physics, vol. 50, no. 2, pp. 407-423, May 2009.