SEISMIC SOURCE AND EARTHQUAKE PARAMETERS FROM LOCAL SEISMIC RECORDINGS. EARTHQUAKES OF 28.10.2018 AND 23.09.2016, VRANCEA, ROMANIA

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A direct, practical and operative procedure is described for deriving the parameters of the seismic source and earthquakes from the ground displacement of the P and S waves recorded at a local site on Earth's surface. The procedure gives the seismic-moment tensor, the earthquake energy and magnitude, the orientation of the fault and the direction of the tectonic slip, the duration of the focal seismic activity and the dimension of the focal region (fault). The (previously published) theory underlying this procedure is briefly discussed. The aim of this paper is to provide a detailed description of how this theory can be used for practical purposes. Two specific examples of Vrancea earthquakes (October 10, 2018 and September 23, 2016) are presented.

Key words: seismic source; inverse problem; seismic waves; seismic moment; elasticity; seismic hyperbola.

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1. INTRODUCTION

The determination of the seismic source and earthquake parameters is a basic problem in seismology. [1]- [7] This problem is currently solved, almost in real time, by various international and national agencies. The Institute of Earth's Physics at Magurele provides such information for Vrancea earthquakes. The main quantity envisaged by these computation is the seismic-moment tensor and earthquake's magnitude. The solution is obtained by numerically fitting synthetic seismograms to various waveforms recorded at Earth's surface.

Recently, a new method was put forward for determining these parameters from local seismic recordings of the ground displacement produced by the P and S seismic waves at Earth's surface. [8] This theory has no fitting parameters. The components of the force density in a seismic focus localized at $R = 0$, with a seismic activity lasting a short time T at the initial moment $t = 0$, is given by

$$f_i = M_{ij} T \delta(t) \partial_j \delta(\mathbf{R}) \ , \tag{1}$$

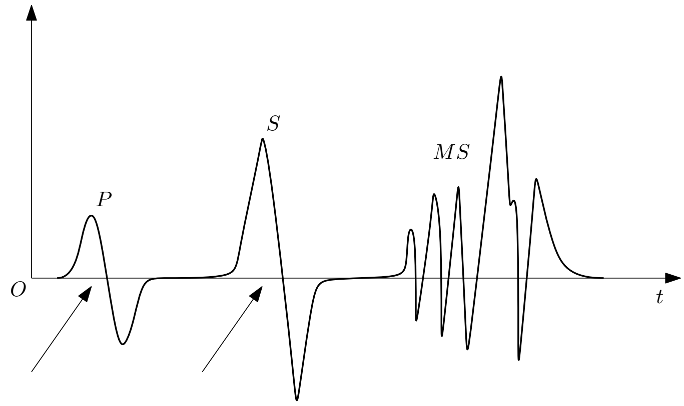

Fig. 1 – Sketch of a typical seismogram, displaying the P- and S-waves and the main shock MS. The arrows indicate the "same side" of the P- and S-waves discussed in text.

where $i, j = 1, 2, 3$ denote cartesian components and M_{ij} is the (symmetric) tensor of the seismic moment (see, for instance, Refs. [9]- [12]). The Dirac delta functions in equation (1) define an elementary earthquake, *i.e.* an earthquake with a localized focus and a short duration. The equation of the elastic waves in a homogeneous, isotropic body has been solved for this force, [13] and the static deformations produced by it in a homogeneous, isotropic half-space have been computed. [14] A homogeneous, isotropic elastic half-space with a plane surface is used as a model for Earth in the seismic regions of interest. It was shown that the force density given by equation (1) generates two spherical-shell waves in the far-field region, identified as the P (primary, longitudinal) and S (secondary, transverse) seismic waves. In addition, these primary waves produce wave sources on Earth's surface, with a cummulative elastic energy, which generate secondary waves; the wavefront of the secondary waves has a wall-like profile on Earth's surface, which is the main shock of the earthquakes. A highly simplified sketch of a typical seismogram displaying these features is shown in Fig. 1. The theory discussed herein makes use of the amplitudes of the P and S waves computed previously. [13]

Since the theory of determining the tensor M_{ij} may appear as being too technical, [8] we provide in this paper a direct, practical and operative procedure of applying this theory. In particular, the compatibility of the input data and the optimization of the errors are discussed. The procedure gives the seismic-moment tensor, the earthquake energy and magnitude, the orientation of the fault and the direction of the tectonic slip, the duration of the focal seismic activity and the dimension of the focal region (fault). The results are exemplified on two Vrancea earthquakes.

2. THEORY

The basic equations used in this paper relate (algebrically) the longitudinal displacement $\mathbf{v}_l$ (P wave) and the transverse displacement $\mathbf{v}_t$ (S wave), measured at a local site on Earth's surface, to the seismic-moment tensor M_{ij} and the duration T of the focal seismic activity. [13] We assume that the other ingredients entering these relations, like Earth's density and wave velocities, are known. Also, we assume that the position of the focus is known, such that we know the unit vector $\mathbf{n}$ from the focus to the origin of the local frame. Consequently, the data include one parameter of the longitudinal displacement (its magnitude) and two parameters of the transverse displacement; this makes three known parameters. In general, the seismic-moment tensor M_{ij} has six components, which, together with the duration T, make seven unknowns. However, for a fault, the Kostrov representation holds for the seismic-moment tensor, [15, 16] which reduces the number of components from six to four; the energy conservation relates, in fact, one of these components with the earthquake duration, such that the seismic moment has only three independent components for a fault. [8] It follows that we are left with four unknowns and three known parameters (equations). The fourth equation is provided by the covariance condition, which determines the problem. These relations have previously been derived. [8]

3. INITIAL INPUT. DATA COMPATIBILITY

We use a local reference frame with axes, denoted by $1, 2, 3$, correspondig to the directions North-South, West-East and the local vertical, respectively. Let θ_0 and φ_0 be the latitude and the longitude of the origin of this local frame, respectively. We assume that the latitude θ_E and the longitude φ_E of the epicentre are also known. Then, we determine immediately the coordinates of the epicentre

$$x_1 = -R_0\theta \ , \ x_2 = R_0 \cos\theta_E \cdot \varphi \ , \tag{2}$$

where $\theta = \theta_E - \theta_0$, $\varphi = \varphi_E - \varphi_0$ (in radians, *e.g.* $\theta = \theta^\circ \cdot \frac{\pi}{180}$) and $R_0 = 6370km$ is Earth's mean radius. Usually, the depth H of the focus is also given by the seismic measurements, such that we might know the unit vector $\mathbf{n}$ directed from the focus to the origin of the local frame. Unfortunately, the measured longitudinal displacement $\mathbf{v}_l$ is not always along the vector $\mathbf{n}$, which raises a problem of compatibility of the data. This is why we prefer to estimate the depth H of the focus. The experimental determination of the depth of the focus may be more affected by errors than the experimental determination of the epicentral coordinates.

The displacements $\mathbf{v}_l$ and $\mathbf{v}_t$ are measured from the P- and S-waves of the seismograms, respectively, for all three directions, as the maximum value of the displacement (with its sign) on the same temporal side of the seismogram recordings

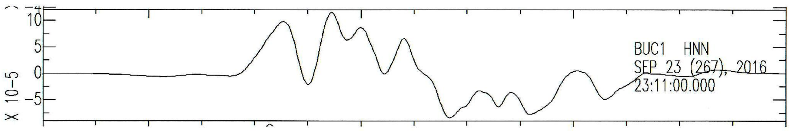

Fig. 2 – P- wave displacement along the South-North direction recorded at Bucharest for the Vrancea earthquake of 23.09.2016.

(along the time axis); these recordings have a scissor-like (double-shock) characteristic pattern. The "same temporal side" means either up to the point where these patterns change sign, or away from that point. The "same side" of the P- and S-waves is indicated by arrows in Fig. 1. The displacements used here are the envelope of the zoomed out oscillatory curves of the P- and S-waves recorded by seismograms. Instead of the maximum values, mean values may also be used. In addition, the sign of the longitudinal components should be compatible with the position of the focus. For instance, for Vrancea earthquakes recorded at Bucharest, the sign of the longitudinal components should be either $(+,-,+)$ or $(-,+,-)$, according to the coordinates of the epicentre and the station. We call this the "sign rule".

We include here an example of reading the displacement from a recording. The raw data are provided by accelerograms. A standard numerical code transforms the accelerations into displacements. The P-wave displacement recorded at Bucharest along the South-North direction for the Vrancea earthquake of 23.09.2016 is shown in Fig. 2. The ordinate scale (units 10^{-5}) should be multiplied by 10^3 for a displacement in cm. By zooming out this picture, we can see clearly the scissor-like pattern of the displacement. We can read easily the amplitudes on each part of the zero-point; they are $0.14cm$ and $0.8cm$. The mean value of these amplitudes is $0.11cm$. Since our axis points to the South, we must take this value with the negative sign. We assign this value to the left wing of the pattern in Fig. 2, and preserve this wing for all other seismograms (in order to observe the sign rule).

Let $\mathbf{f}=(f_1,f_2,f_3)$ be the longitudinal displacement as read from the seismogram and let $\mathbf{g}=(f_1/f,f_2/f,f_3/f)$. Then, the coordinates of the epicentre should be given by $-Rg_1$, $-Rg_2$, where R is the distance to the focus; they should be as close as possible to the coordinates $x_{1,2}$, respectively. Therefore, we minimize the quadratic form $(Rg_1+x_1)^2+(Rg_2+x_2)^2$ and get an estimate

$$R_1=-\frac{g_1x_1+g_2x_2}{g_1^2+g_2^2} \tag{3}$$

for the focal distance, with a relative error

$$\chi_1=1-\frac{(g_1x_1+g_2x_2)^2}{(g_1^2+g_2^2)(x_1^2+x_2^2)}\ . \tag{4}$$

Making use of equation (3) we get an estimate

$$H_1 = -\sqrt{R_1^2 - (x_1^2 + x_2^2)} \tag{5}$$

for the depth of the focus.

Let $\mathbf{v}_t$ be the transverse displacement, measured as described above, and let $\mathbf{t} = \mathbf{v}_t / v_t$. It may happen that $\mathbf{f}$ and $\mathbf{v}_t$ are not perpendicular to each other; we define $\sin\phi = \mathbf{gt}$, where ϕ may be different from zero. This rises again a problem of data compatibility. We define the vector

$$\mathbf{n} = \frac{1}{\cos\phi}(\mathbf{g} - \mathbf{t}\sin\phi) \ , \tag{6}$$

which is perpendicular to $\mathbf{t}$, and take the longitudinal displacement as

$$\mathbf{v}_l = f\mathbf{n} \ . \tag{7}$$

We have now the possibility to get another estimate

$$R_2 = -\frac{n_1 x_1 + n_2 x_2}{n_1^2 + n_2^2} \tag{8}$$

of the focal distance and another estimate

$$H_2 = -\sqrt{R_2^2 - (x_1^2 + x_2^2)} \tag{9}$$

of the depth of the focus, with a relative error

$$\chi_2 = 1 - \frac{(n_1 x_1 + n_2 x_2)^2}{(n_1^2 + n_2^2)(x_1^2 + x_2^2)} \ . \tag{10}$$

Finally, we use the mean values $R = (R_1 + R_2)/2$ and $H = (H_1 + H_2)/2$ for the focal distance and the depth of the focus. In practice, if the angle ϕ is too far from zero, the input data lead to large errors.

4. RESULTS: EARTHQUAKE ENERGY AND MAGNITUDE; FOCAL VOLUME, FAULT SLIP

According to the theory, [8] the reduced magnitude of the seismic moment is given by

$$M = (M_{ij}^2/2)^{1/2} = 4\pi\sqrt{2}\rho R^{3/2}\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4} \tag{11}$$

and the earthquake energy is

$$E = M/2 \tag{12}$$

(the magnitude of the seismic moment is $\overline{M} = \sqrt{2}M = (M_{ij}^2)^{1/2}$. Using the Gutenberg-Richter (Hanks-Kanamori) law

$$\lg E = 1.5 M_w + 15.65 \tag{13}$$

(or $\lg\overline{M} = \frac{3}{2}M_w + 16.1$), we derive the (moment) magnitude of the earthquake

$$M_w = \frac{1}{1.5}\left(\lg E - 15.65\right) \ . \tag{14}$$

In these equations R is the focal distance and $\mathbf{v}_l$ is the longitudinal displacement (P-wave, equation (7)) as determined above; $\mathbf{v}_t$ is the transverse displacement (S-wave), as measured experimentally; ρ is Earth's mean density (we can take $\rho = 5g/cm^3$) and $c_{l,t}$ are the velocities of the longitudinal and transverse waves (for instance, we may take $c_l = 7km/s$ and $c_t = 3km/s$). All the equations are written in units cm,g,s.

Similarly, the focal volume is given by

$$V = \frac{M}{2\rho c_t^2} \ , \tag{15}$$

whence we may infer the dimension of the focal region and the magnitude of the fault slip $l = V^{1/3}$.

5. RESULTS: SEISMIC-MOMENT TENSOR; FOCAL STRAIN, FOCAL-ACTIVITY DURATION

The seismic-moment tensor is given by

$$M_{ij} = \frac{M}{1-m_4^2}\left[m_i n_j + n_i m_j - m_4\left(m_i m_j + n_i n_j\right)\right] \ , \tag{16}$$

where

$$m_i = -\frac{c_l^3 v_{li} + c_t^3 v_{ti}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/2}} \ ,$$
$$m_4 = -\frac{c_l^3 v_l}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/2}} \tag{17}$$

and $\mathbf{n}$ is given by equation (6). [8] As discussed in **Introduction**, the components M_{ij} can be viewed as generalized force couples, while the vector $\mathbf{m}$ may be viewed as indicating the direction of a "force" acting in the focus; m_4 is a measure of the "force" acting along the observation radius (longitudinal "force"). We can check the traceless condition $M_{ii} = 0$ and the covariance condition $m_i^2 = 1$.

The focal strain is given by

$$u_{ij}^0 = \frac{M_{ij}}{2M} \ , \tag{18}$$

where $u_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$ are the strain components for the displacement vector $\mathbf{v}$ (the superscript 0 stands for the focus). The duration of the seismic activity in the focal region is given by (Apostol 2019)

$$T = (2R)^{1/2} \frac{\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4}} \; ; \tag{19}$$

it is related to the focal volume by

$$V = \frac{4\pi R^2}{c_t^2 T} \left(c_l v_l^2 + c_t v_t^2\right) \; ; \tag{20}$$

hence we may estimate the rate of the focal strain u_{ij}^0/T and the rate of the focal slip l/T (during the seismic activity).

It is useful to have a quick and simple estimation of the order of magnitude of the various quantities introduced here. To this end we use a generic velocity c for the seismic waves and a generic vector $\mathbf{v}$ for the displacement in the far-field seismic waves. From the covariance equation $m^2 = 1$ (equation (17)) we get immediately $cT \simeq \sqrt{2Rv}$, which provides an estimate of the duration T of the seismic activity in the focus in terms of the displacement measured at distance R. The focal volume can be estimated as $V \simeq \pi (2Rv)^{3/2} \simeq \pi (cT)^3$, as expected (dimension l of the focal region of the order cT; the rate of the focal slip is $l/T \simeq c$). The earthquake energy is $E \simeq \mu V \simeq M/2 \simeq 2\rho c^2 V$, where $\mu = \rho c^2$ is the Lame coefficient and M is the reduced magnitude $\left(M_{ij}^2\right)^{1/2} = \sqrt{2}M$ of the seismic moment (and the magnitude of the vector $M_{ij}n_j$). The focal strain is of the order unity, as expected. The magnitude of the earthquake is given immediately by equation (14). In addition, we can see the relationship $\lg v = M_w + 10.43 - \lg\left[(2R)(2\pi\rho c^2)^{2/3}\right]$, or $\lg(2Rv) = M_w + 1.83$ (for $\rho = 5g/cm^3$, $c = 5km/s$ and v measured in cm). Hence, we may see that the displacement measured at Bucharest for a Vrancea earthquake of magnitude $M_w = 7$ is of the order $v \simeq 30cm$. Similar estimations can be made for other magnitudes, by using equation (14).

6. RESULTS: FAULT GEOMETRY

The geometry of the seismic activity in a fault is characterized by the normal $\mathbf{s}$ to the fault (unit vector) and the slip unit vector $\mathbf{a}$ lying on the fault; these two vectors are mutually orthogonal. According to Kostrov representation (and the covariance condition), [8] the seismic-moment tensor is given by

$$M_{ij} = M(s_i a_j + a_i s_j) \; . \tag{21}$$

We can see the conditions $M_{ii} = 0$ and $M_{ij}s_i s_j = 0$ (or $M_{ij}a_i a_j = 0$), which, together with the covariance condition $m_i^2 = 1$ (where $m_i = M_{ij}n_j/M$; and $m_4 = M_{ij}n_i n_j/M$), lead to three independent components of the tensor M_{ij}. From equation (21) we can see that, apart from the (simultaneous) symmetry operations $\boldsymbol{s} \to -\boldsymbol{s}$ and $\boldsymbol{a} \to -\boldsymbol{a}$, which indicate merely a reflection of the fault and the slip, there exists another symmetry, given by $\mathbf{s} \longleftrightarrow \mathbf{a}$, which indicates an important uncertainty. Indeed, any fault slip is accompanied by another fault slip, along an orthogonal direction, as a consequence of mattter conservation. It follows that we are not able to make the difference between the direction of the fault and the direction of the slip, because, actually, we have another fault oriented along the slip, and, of course, another slip oriented along the original fault.

The vectors $\mathbf{s}$ and $\mathbf{a}$ are given by [8]

$$\begin{aligned} \mathbf{s} &= \frac{\alpha}{\alpha^2-\beta^2}\mathbf{m} - \frac{\beta}{\alpha^2-\beta^2}\mathbf{n} \ , \\ \mathbf{a} &= -\frac{\beta}{\alpha^2-\beta^2}\mathbf{m} + \frac{\alpha}{\alpha^2-\beta^2}\mathbf{n} \ , \end{aligned} \tag{22}$$

where

$$\begin{aligned} \alpha &= \sqrt{\frac{1+\sqrt{1-m_4^2}}{2}} \ , \\ \beta &= sgn(m_4)\sqrt{\frac{1-\sqrt{1-m_4^2}}{2}} \ ; \end{aligned} \tag{23}$$

the vector $\mathbf{n}$ is given by equation (6) and the vector $\mathbf{m}$ and the scalar m_4 are given by equation (17). These relations ensure the identity of equation (16) with equation (21). If we define two orthogonal coordinates $u = a_i x_i$ (along the slip) and $v = s_i x_i$ (along the normal to the fault), then the quadratic form $M_{ij}x_ix_j = const$ defines a hyperbola $uv = const/2M$; its asymptotes are directed along the normal to the fault $\mathbf{s}$ and the slip in the fault $\mathbf{a}$. We call it the seismic hyperbola. [8] For high values of the reduced magnitude M of the seismic moment the seismic hyperbola is tight. Actually, for various $const$ in $M_{ij}x_ix_j = const$ we get a hyperboloid directed along the third axis $\mathbf{s} \times \mathbf{a}$.

A similar hyperbola may be derived from equation (16) by using the coordinates $\xi = m_i x_i$ (along the vector $\mathbf{m}$) and $\eta = n_i x_i$ (along the vector $\mathbf{n}$); its equation is $2\xi\eta - m_4(\xi^2+\eta^2) = const$. We recall that $\mathbf{m}$ indicates the direction of a "force" acting in the focus; the angle made by the vectors $\mathbf{m}$ and $\mathbf{n}$ is given by $\cos\chi = m_4$, the angle made by $\mathbf{n}$ and $\mathbf{s}$ (observation radius and the fault direction) is given by $\sin\psi = \sqrt{\left(1+\sqrt{1-m_4^2}\right)/2}$ and the angle made by $\mathbf{n}$ and $\mathbf{a}$ (observation radius and the fault slip) is $\pi/2-\psi$.

7. EXPLOSIONS

For explosions, which are isotropic, the moment tensor is a scalar. We write it as $M_{ij} = -M\delta_{ij}$. We have only a longitudinal displacement. The above formulae reduce to

$$M = 2\pi\rho c_l^2 (2Rv_l)^{3/2} \ , \ V = \pi(2Rv_l)^{3/2} \ ,$$
$$T = \frac{\sqrt{2Rv_l}}{c_l} \ . \tag{24}$$

The "focal" region for explosions is a sphere. The minus sign in the definition of the moment tensor indicates the fact that the slip on a point of the surface of the "focal" sphere is opposite to the direction of the surface element at that point.

Also, we note that both a seismic shear faulting and an explosion produce a longitudinal displacement, such that their distinct contribution cannot be resolved (a superposition of a seismic shear faulting and an isotropic mechanism - the so-called "hybrid" mechanism - cannot be resolved).

8. EARTHQUAKE OF 28.10.2018, VRANCEA

We apply here the algorithm described above to two earthquakes. For the Vrancea earthquake of 28.10.2018 the epicentre coordinates are $\theta_E = 45.61°$, $\varphi_E = 26.41°$ and the depth of the focus is $H = -147.8Km$ $(= x_3)$. We use the data from Cernavoda station with coordinates $\theta_0 = 44.3°$, $\varphi_0 = 28.3°$ (coordinates $x_1 = -145.64km$, $x_2 = -125.99km$). The position vector is

$$\boldsymbol{n} = (0.60, 0.52, 0.61) \ . \tag{25}$$

Within the accuracy used here, the vector $\boldsymbol{v}_l$ is directed along the vector $\boldsymbol{n}$, with magnitude $v_l = 0.18cm$, so there is no need to estimate other values of the focus depth and vectors $\boldsymbol{n}$. Noteworthy, the sign rule for Cernavoda is $(+,+,+)$ (or $(-,-,-)$). The vector of the transverse displacement is

$$\boldsymbol{v}_t = (-0.30, 0.40, -0.08)cm \tag{26}$$

(magnitude $v_t = 0.51cm$) and the angle made by $\boldsymbol{v}_l(\boldsymbol{n})$ with $\boldsymbol{v}_t$ is $\simeq 92°$ (which may be viewed as an acceptabale departure from orthogonality).

Making use of equations (11)-(15) we get the energy $E = 4.65 \times 10^{23} erg$, the magnitude of the seismic moment $\overline{M} = 1.30 \times 10^{24} erg$, the magnitude of the earthquake $M_w = 5.33$ and the focal volume $V = 9.6 \times 10^{11} cm^3$. The Institute for Earth's Physics, Magurele, announced the magnitude $M_w = 5.5$ (www.infp.ro, with an error $\Delta M_w = 0.7$). We can see that the dimension of the focal volume (the focal slip) is $\simeq 100m$.

Making use of equations (16)-(20) we get the "force" vector

$$\boldsymbol{m} = (-0.46, -0.68, -0.56) \ , \ m_4 = -0.98 \ , \tag{27}$$

the seismic moment

$$(M_{ij}) = \begin{pmatrix} 1.4 & -7.5 & -1.6 \\ -7.5 & 1.6 & -4.8 \\ -1.6 & -4.8 & -2.8 \end{pmatrix} \times 10^{23} erg \tag{28}$$

and the duration of the focal activity $T = 8.7 \times 10^{-3} s$; the focal strain is of the order $10^{-1} cm$, the rate of the focal strain is of the order $10 cm/s$ and the rate of the focal slip is of the order $10^6 cm/s$. The deviation of M_{ii} from zero in equation (28) is a measure of the error of these estimations.

Using equations (22) and (23), we get the parameters $\alpha = 0.78$, $\beta = -0.63$ and the fault and the slip vectors

$$\begin{array}{c} \boldsymbol{s} = (0.09, -0.94, -0.26) \ , \\ \boldsymbol{a} = (0.84, -0.09, 0.57) \ ; \end{array} \tag{29}$$

these vectors pierce the Earth's surface at $\theta = 46.05°$, $\varphi = 33.38°$ ($\boldsymbol{s}$) and $\theta = 43.67°$, $\varphi = 26.18°$ ($\boldsymbol{a}$) (see **Appendix**).

9. EARTHQUAKE OF 23.9.2016, VRANCEA

The epicentre coordinates for this earthquake are $\theta_E = 45.71°$, $\varphi_E = 26.62°$ and the depth of the focus is $H = -92 km$ $(= x_3)$. We use the data from Magurele station with coordinates $\theta_0 = 44.35°$, $\varphi_0 = 26.03°$ (coordinates $x_1 = -151.12 km$, $x_2 = 45 km$). The position vector is

$$\boldsymbol{n} = (0.83, -0.25, 0.50) \ . \tag{30}$$

The direction of the vector $\boldsymbol{v}_l$ is clsoe to the direction of the vector $\boldsymbol{n}$; its magnitude is $v_l = 0.13 cm$. The sign rule for Magurele is $(+,-,+)$ (or $(-,+,-)$). The vector of the transverse displacement is

$$\boldsymbol{v}_t = (-0.30, -0.30, -0.17) cm \tag{31}$$

(magnitude $v_t = 0.46 cm$) and the angle made by $\boldsymbol{v}_l(\boldsymbol{n})$ with $\boldsymbol{v}_t$ is $\simeq 106°$ (this is a rather large deviation from orthogonality).

Making use of equations (11)-(15) we get the energy $E = 1.1 \times 10^{23} erg$, the magnitude of the seismic moment $\overline{M} = 3.1 \times 10^{23} erg$, the magnitude of the earthquake $M_w = 4.92$ and the focal volume $V = 2.2 \times 10^{11} cm^3$. The Institute for Earth's Physics, Magurele, announced the magnitude $M_w = 5.5$ (with an error $\Delta M_w = 0.4$). We can see that the dimension of the focal volume (the focal slip) is $\simeq 60 m$. The rather large difference in magnitudes arises from the deviation of the vectors $\boldsymbol{v}_{l,t}$ from mutual orthogonality.

Making use of equations (16)-(20) we get the ”force“ vector

$$\boldsymbol{m} = (-0.19, -0.57, -0.71)\ ,\ m_4 = -0.97\ , \tag{32}$$

the seismic moment

$$(M_{ij}) = \begin{pmatrix} 0.89 & 1.21 & 0.25 \\ 1.21 & 0.23 & 1.45 \\ 0.25 & 1.45 & -0.8 \end{pmatrix} \times 10^{23} erg \tag{33}$$

and the duration of the focal activity $T = 6 \times 10^{-3}s$; the focal strain is of the order $10^{-1}cm$, the rate of the focal strain is of the order $10cm/s$ and the rate of the focal slip is of the order $10^6 cm/s$. The deviation of M_{ii} from zero in equation (33) is a measure of the error of these estimations.

Using equations (22) and (23), we get the parameters $\alpha = 0.79$, $\beta = -0.61$ and the fault and the slip vectors

$$\begin{aligned} \boldsymbol{s} &= (0.29, 0.77, -0.21)\ , \\ \boldsymbol{a} &= (0.69, 0.07, 0.88)\ ; \end{aligned} \tag{34}$$

these vectors pierce Earth's surface at $\theta = 45.88°$, $\varphi = 22.24°$ ($\boldsymbol{s}$) and $\theta = 44.1°$, $\varphi = 26.7°$ ($\boldsymbol{a}$).

10. CONCLUDING REMARKS

The practical application of the theory of determining the seismic source and earthquake parameters from local seismic recordings of the P and S seismic waves [8] is presented, with a detailed emphasis on the specific points of its implementation. Special attention is given to the input parameters, as read from seismograms; these parameters, which are the amplitudes of the ground displacement, should satisfy certain compatibility conditions. The optimization procedure described above can be employed to improve compatibility. It is shown how the earthquake energy and magnitude can be derived, as well as the volume of the focal region and the focal slip. Also, it is shown how to determine the tensor of the seismic moment, the focal strain and the duration of the seismic activity in the earthquake focus. We describe how to deduce the orientation of the fault and the direction of the focal slip. The particular case of an isotropic explosion is also presented. The procedure is applied to two Vrancea earthquakes (October 28, 2018, and September 23, 2016), both with magnitude $M_w = 5.5$. The errors implied by the practical application of this theory are discussed. Particularly interesting is a rapid estimation, by hand, of the earthquake parameters, which is described herein. The procedure presented in this paper can be implemented by means of a numerical computing program with modest resources, and the results can be obtained in real time. The procedure is routinely applied in the

Institute of Earth's Physics at Magurele, Romania.

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Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

APPENDIX

It may be of interest to determine the points where the vectors $\mathbf{s}$ and $\mathbf{a}$ pierce Earth's surface. For this it is necessary to express all the vectors in the reference frame of the Earth (a sphere). We have the vector which determines the origin of the local frame, the vector which determines the focus and the vector $\mathbf{s}$ (or $\mathbf{a}$) with the origin in the focus. The point of interest on Earth's surface corresponds to a vector $\lambda\mathbf{s}$ (or $\lambda\mathbf{a}$), where λ has a well-determined value. We express this vector in Earth's frame and requires it to be on Earth's surface; this condition leads to the equation

$$\lambda^2 + 2\lambda\left[R_0 s_3 - R(\mathbf{ns})\right] - 2R_0 H = 0 \; ;$$

we need to choose for λ the smallest absolute value of the roots.

A simplified version of these calculations can be done for points close to the local observation point and the epicentre, such that we may approximate the Earth's surface by a plane surface. The corresponding equations are

$$H\frac{s_1}{s_3} + x_1 = -R_0\theta \; ,$$

$$H\frac{s_2}{s_3} + x_2 = R_0 \cos\theta' \cdot \varphi$$

($s_3 > 0$); the coordinates of the intersection point are $\theta' = \theta_0 + \theta$ and $\varphi' = \varphi_0 + \varphi$.

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